國立中山大學100學年度碩士班招生考試試題

科目：應用數學【物理系碩士班】

(每題 10 分, 共 10 題)

1. Evaluate the integral $\int \delta\left[(x-a)(x-b)\right] f(x)dx$, where $\delta[\cdots]$ is the Dirac delta function and the range of integration includes the points a, and b $(a \neq b)$. (10%)

2. If a vector $\vec{F}$ is given by $\vec{F} = (x^2+y^2+z^2)^n (x\hat{e}_x + y\hat{e}_y + z\hat{e}_z)$, where $\hat{e}_i$ is the unit vector along the i direction, find (a) $\nabla \cdot \vec{F}$. (5%) (b) $\nabla \times \vec{F}$. (5%)

3. Resolve the circular cylindrical unit vectors $\hat{e}_\rho$, $\hat{e}_\varphi$, and $\hat{e}_z$ into their Cartesian components $\hat{e}_x$, $\hat{e}_y$, and $\hat{e}_z$. (10%)

4. Evaluate the function of matrix $\exp\{i\sigma_2\theta\}$, where $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ in which i is the imaginary unit. (10%)

5. For a resistance-inductance circuit Kirchhoff's law leads to the differential equation $L\frac{dI(t)}{dt} + RI(t) = V$ for the current $I(t)$, where L is the inductance, R is the resistance, and V is the time-independent input voltage, all constant. If the initial condition is $I(0)=0$, find the solution of the current $I(t)$. (10%)

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6. Find the Fourier series expansion of the function $f(x)=x$ in the specific interval $-\pi \le x \le \pi$. (10%)

7. Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{1-2\alpha\cos\theta+\alpha^2}$, $\alpha \in (0,1)$.

(10%)

8. Find the Fourier transformation of the function

$f(t)=\sin\omega_0 t$, where ω_0 is a constant. (10%)

9. Find the Laurent series of the function of a complex variable

$f(z)=\frac{1}{z^2(z-i)}$ in the intervals

(a) $0<|z-i|<1$, (5%) (b) $|z-i|>1$. (5%).

10. Find the general solution of the partial differential

equation
$$\begin{cases} \frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}, & x\in(0,l),\ t>0, \\ u_x(0,t)=u_x(l,t)=0, & t>0, \\ u(x,0)=\varphi(x), & x\in(0,l), \end{cases}$$

where $u_x=\frac{\partial u}{\partial x}$. (10%)

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科目：電磁學【物理系碩士班】

1. Find the poynting vector on the surface of a long straight conducting wire (of radius b and conductivity σ) that carries a direct current I. (8%) Verify poynting theorem. (8%)

2. The circuit in Fig.1 is situated in a magnetic field

$$\vec{B}(t) = \hat{z}B_0 cos\left(\omega t - \frac{2\pi}{3}x\right)\ (\mu T),$$

where $\omega = 3\pi \times 10^7$ /sec and $B_0 = 3\mu T$. Assume $R_1 = 30\ \Omega$, $R_2 = 15\ \Omega$ and $L_1 = 60\ cm$, $L_2 = 30\ cm$, find the current $i(t)$. (10%)

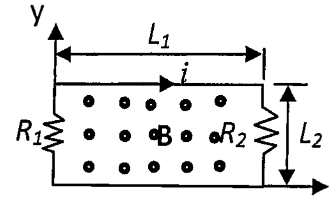

Fig. 1

3. When a spacecraft reenters the earth's atmosphere, its speed and temperature ionize the surrounding atoms and molecules, and create plasma. It has been estimated that the electron density is in the neighborhood of 2×10^8 per cm^3. Discuss the plasma's effect on frequency usage in radio communication between the spacecraft and the mission controllers on earth. (10%)

4. Two charges ($+q$ and $-q$) are arranged along the z-axis at $z=d/2$ and $z=-d/2$, respectively, as shown in Fig. 2. Determine the electric field and potential at a distance point $p(r, \theta, \phi)$. (16%)

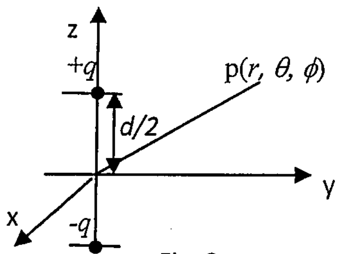

Fig. 2

5. An air coaxial transmission line has a solid inner conductor of radius a and a very thin outer conductor of inner radius b. (a) Determine the magnetic flux density in all space. (6%) (b) Determine the inductance per unit length of the line. (6%) (c) How much magnetic energy per unit length is stored in the system? (6%)

6. Two grounded, semi-infinite, parallel-plane electrodes are separated by a distance b. A third electrode perpendicular to and insulated from both is maintained at a constant potential V_0 (see Fig. 3). Determine the potential distribution in the region enclosed by the electrodes. (16%)

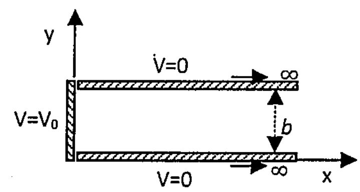

Fig. 3

7. A cylindrical bar magnet of radius b and length L has a uniform magnetization $\mathbf{M} = \hat{z}M_0$ along its axis. (a) Determine the equivalent magnetization charge density. (6%) (b) Determine the equivalent magnetization current density $\mathbf{J}_m$ and $\mathbf{J}_{ms}$. (8%)

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科目：近代物理【物理系碩士班】

1. **(20%)** An electron is confined in the ground state in a one-dimensional box of width 10^{-10} m. Its energy is 38 eV.
   (a) **(10%)** Write the time-independent wave function corresponding to the state of the lowest possible energy.
   (b) **(10%)** Calculate the energy of the electron in its first excited state.
2. **(20%)** At the time t=0 the wave function for hydrogen atom is

$$\varphi(r,0)=\frac{1}{\sqrt{10}}(2\varphi_{100}+\sqrt{3}\varphi_{210}+\varphi_{211}+\sqrt{2}\varphi_{21-1})$$

where the subscripts are values of the quantum numbers *n, l, m*. Ignore spin and radiative transitions.

   (a) **(8%)** Find the expectation value for the *z* component of the angular momentum, $\langle \vec{L}_Z \rangle$, of this system.
   (b) **(8%)** Find the expectation value for the energy of this system.
   (c) **(4%)** What is the probability of finding the system with $l=+1$, $m=-1$ as a function of time?
3. **(15%)** The famed sodium doublet arises from the spin-orbit splitting of the sodium 3*p* level. The fine-structure splitting of the $2P_{3/2}$ and $2P_{1/2}$ levels in hydrogen is 2.13×10^{-3} eV. Estimate the magnetic field that the 2*p* electron in hydrogen experience. Assume $\vec{B}$ is parallel to z axis.
4. **(20%)** An electron in an atom has orbital angular momentum $\vec{L}_1$ with quantum number l_1=1, and a second electron has orbital angular momentum $\vec{L}_2$ with quantum number l_2=2.
   (a) **(10%)** What are the possible quantum number for the total orbital angular momentum $\vec{L}=\vec{L}_1+\vec{L}_2$.
   (b) **(10%)** What are the possible quantum number *j* for the combination $\vec{J}=\vec{L}+\vec{S}$.
5. **(25%)** Investigate the Zeeman spectrum produced by hydrogen atoms initially in the *n*=2 state. Assume the atoms to be in a magnetic field of magnitude $\vec{B}$=2.00 T, and choose the *z*-axis along the direction of $\vec{B}$.
   (a) **(6%)** Express and calculate the total magnetic energy of an electron with orbital and spin contributions. $[\frac{e\hbar}{2m_e}=9.27\times10^{-24}(J/T)=5.79\times10^{-5}(eV/T)]$
   (b) **(7%)** Draw the energy levels for *n*=1 and *n*=2 without spin consideration. Write each energy state by (n, l, m_l) and each energy level in the unit of eV.
   (c) **(7%)** Draw the energy levels for *n*=1 and *n*=2 with spin consideration. Write each energy state by (n, l, m_l, m_s) and each energy level in the unit of eV.
   (d) **(5%)** Indicate the possible transition in (b) and (c) for an electron excited to the *n*=2 state of hydrogen. Selection rules should be taken into account.