國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：基礎數學【應數系碩士班甲組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
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- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學 113 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：基礎數學【應數系碩士班甲組】　　題號：424001

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　共1頁第1頁

答題時，每題都須寫下題號與步驟。請依題號順序作答，不會作答題目請寫下題號並留空白。

1. (10%) Evaluate $\lim_{x\to 0^+} \tan(x)\ln(\sin^2(x))$

2. (20%) Evaluate the following integrals:

   (a) (10%) $\int \frac{xe^{3x}}{(3x+1)^2}dx$

   (b) (10%) $\int_0^{\ln 4} \frac{e^x}{\sqrt{e^{2x}+9}}dx$

3. (15%) Let $F(x) = \int_0^x \ln(1+\frac{t^2}{2})dt$. Find the Maclaurin series for $F(x)$ and its radius of convergence.

4. (10%) Evaluate the following expressions:

$$\int_{\frac{1}{\sqrt{2}}}^{1}\int_{\sqrt{1-x^2}}^{x} 1\,dydx + \int_1^{\sqrt{2}}\int_0^x 1\,dydx + \int_{\sqrt{2}}^{2}\int_0^{\sqrt{4-x^2}} 1\,dydx$$

5. (15%) Let V be the vector space of all polynomials $p(t)$ of degree less or equal to 15.

   (a) (10%) Show that the map $T: V \to \mathbb{R}$ given by $T\big(p(t)\big) = \int_0^1 p(t)\,dt$ is a linear transformation.

   (b) (5%) Find the dimension of the kernel of T.

6. (15%) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 5 & -2 \\ 1 & -1 & 0 \end{bmatrix}$ and $V = \{x = (x_1, x_2, x_3)^T \in \mathbb{R}^3 | x_1 - 2x_2 + x_3 = 0\}$

   (a) (5%) Define $T(x) = Ax$, show that $T: V \to V$.

   (b) (10%) Find the eigenvectors and eigenvalues of T acting on V.

7. (15%) Assume $f(x)$ is a polynomial whose coefficients are integers, and we know that

$$\int_4^\infty \frac{f(x)}{(x+1)^2(4x^2+1)}dx = \ln\frac{20}{13} + \frac{1}{5}$$

   (a) (5%) Use the limit comparison test for improper integrals, what is the maximum degree of $f(x)$?

   (b) (5%) Using the partial fraction method, we have $\frac{f(x)}{(x+1)^2(4x^2+1)} = \frac{A}{(x+1)^2} + \frac{B}{x+1} + \frac{Cx+D}{4x^2+1}$. Evaluate $\int \frac{f(x)}{(x+1)^2(4x^2+1)}dx$ (Express the results in terms of A, B, C, D and x).

   (c) (5%) According to (a) and (b), find the values of A, B, C, D and find $f(x)$.

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：微積分【應數系碩士班乙組】

—作答注意事項—

考試時間：100分鐘

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國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：微積分【應數系碩士班乙組】 題號：424002

※本科目依簡章規定「不可以」使用計算機(問答申論題) 共1頁第1頁

計算題：共7題，子題分數平均分配。答題時，每題都必須寫下題號與詳細步驟。

[1]. (16%) Evaluate the following limits.

(a) $\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{x} - 1}$

(b) $\lim_{n \to \infty} \left[\frac{4}{\sqrt{2n^2 - 1^2}} + \frac{4}{\sqrt{2n^2 - 2^2}} + \frac{4}{\sqrt{2n^2 - 3^2}} + \cdots + \frac{4}{\sqrt{2n^2 - n^2}} \right]$

[2]. (15%)
Let $f(x, y, z) = \sqrt{xy^3 z} + \cos(xy) + z \ln x$ be defined for $x, y, z > 0$.
Compute f_x, f_{xy}, f_{xyz}.

[3]. (20%) Evaluate the following integrals.

(a) $\int_3^\infty (x-3)^2 e^{-x} dx$ (b) $\int_0^\pi \frac{1}{3 - \cos x} dx$

[4]. (13%) Find the Taylor series of $\sqrt{1+x}$ at $x = 0$.

[5]. (12%) Solve the differential equation $y' + 3y = e^{2x}$ with $y(0) = 1$.

[6]. (12%)
Show that the Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$ in the coordinates (r, θ), where $x = r\cos\theta$ and $y = r\sin\theta$, is

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} = 0.$$

[7]. (12%)
Air is pumped into a spherical balloon so that its volume increases at a rate of $120 cm^3/s$. How fast is the radius of the balloon increasing when the radius is $30 cm$?

========全卷完========

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：線性代數丙【應數系碩士班丙組】

—作答注意事項—

考試時間：100分鐘

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1. [20%] Let
$$A = \begin{bmatrix} 1 & 4 & 4 & 4 & 25 \\ -1 & -4 & -4 & -3 & -20 \\ -4 & -16 & -16 & -11 & -75 \end{bmatrix}.$$
Find a basis for each of the row space $\mathrm{Row}(A)$, the column space $\mathrm{Col}(A)$ and the kernel $\ker(A)$ of A. What are their dimensions?

2. [20%] Let $E_{i,j}$ be the 2×2 matrix whose i, j-entry is 1 while other entries are 0. Let $\mathbb{R}^{2\times 2}$ be the space of all 2×2 real matrices and $\beta = \{E_{1,1}, E_{1,2}, E_{2,1}, E_{2,2}\}$ its basis. Define a linear function $f : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$ by $X \mapsto JXJ$, where J is the 2×2 all-ones matrix. Find the matrix representation $[f]_\beta^\beta$ of f with respect to the bases β and β. Then use it to find a basis of the kernel $\ker(f)$ of f.

3. [20%] For $p, q \geq 1$, let
$$A_{p,q} = \begin{bmatrix} O_{p,p} & J_{p,q} \\ J_{q,p} & O_{q,q} \end{bmatrix},$$
where $O_{m,n}$ and $J_{m,n}$ are the $m \times n$ zero matrix and all-ones matrix, respectively. Find $\det(A_{p,q} - xI)$.

4. [20%] Let $x = x(t)$ and $y = y(t)$ be functions from $\mathbb{R}$ to $\mathbb{R}$. Solve the system of differential equations
$$\begin{cases} \dot{x} = 2x + y, \\ \dot{y} = x + 2y. \end{cases}$$
Here $\dot{x} = \frac{dx}{dt}$ and $\dot{y} = \frac{dy}{dt}$ are the derivatives.

5. [20%] Let $\langle \cdot, \cdot \rangle$ be an inner product defined on $\mathbb{R}^n$. Show that there is a basis β of $\mathbb{R}^n$ such that
$$\langle \mathbf{x}, \mathbf{y} \rangle = [\mathbf{y}]_\beta^\top [\mathbf{x}]_\beta$$
for any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, where $[\mathbf{v}]_\beta$ is the vector representation of $\mathbf{v} \in \mathbb{R}^n$ with respect to β.

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：機率與統計【應數系碩士班甲組】

—作答注意事項—

考試時間：100分鐘

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- Please **answer the questions in order** and **write down the question number** for **each** question. If you are not able to answer the question, leave it blank.
- Notations: i.i.d., independent and identically distributed; cdf, cumulative distribution function; pdf, probability density function; Z is a standard normal random variable.

1. (15%) Let X and Y be two nondegenerate random variables with finite variances. It is readily seen that $\hat{\theta} = X$ is an unbiased estimator for $\theta = E(X)$. Instead of using $\hat{\theta}$, we consider an alternative estimator $\hat{\theta}_{\alpha,\beta} = X - \beta(Y - \alpha)$, where $-\infty < \alpha < \infty$ and $\beta \neq 0$ are parameters. In the subsequent questions, please express your answers in terms of E, Var, and Cov.
   (a) (5%) Find α such that $\hat{\theta}_{\alpha,\beta}$ is an unbiased estimator.
   (b) (5%) Find β such that $\operatorname{Var}(\hat{\theta}_{\alpha,\beta})$ is minimized.
   (c) (5%) Find the percentage of variance reduction obtained by using $\hat{\theta}_{\alpha,\beta}$ instead of $\hat{\theta}$, that is, $\{\operatorname{Var}(\hat{\theta}) - \operatorname{Var}(\hat{\theta}_{\alpha,\beta})\}/\operatorname{Var}(\hat{\theta}) \times 100\%$ based on α and β you found in (a) and (b).

2. (30%) Let $X_1, \dots, X_n$ be i.i.d. uniform random variables on the unit interval $[0,1]$.
   (a) (5%) Find the joint cdf of the random vector (X_1, X_1).
   (b) (5%) Find the joint cdf of the random vector $(X_1, 1 - X_1)$.
   (c) (10%) Find the pdf of $Y = \prod_{i=1}^{n} X_i$.
   (d) (10%) Find the pdf of $Y = \max(X_1, \dots, X_n) - \min(X_1, \dots, X_n)$.

3. (15%) Let $X_1, \dots, X_n$ be i.i.d. continuous random samples from the cdf $F(x)$. Define the empirical cdf $F_n(x) = n^{-1} \sum_{i=1}^{n} I(X_i \le x)$, where $I(.)$ is the indicator function. Let s be a constant.
   (a) (5%) Find the asymptotic distribution of $n^{1/2}\{F_n(s) - F(s)\}$.
   (b) (5%) Find an asymptotic 95% confidence interval for $F(s)$ based on the result in (a). Please express your answer by using z_α that satisfies $P(Z > z_\alpha) = \alpha$.
   (c) (5%) Find the covariance $\operatorname{Cov}\{F_n(s), F_n(t)\}$, where t is another constant and $s \neq t$.

4. (10%) Let $X_1, \dots, X_n$ be i.i.d. random samples from the inverse Gaussian pdf
$$f_{\mu,\lambda}(x) = \left(\frac{\lambda}{2\pi x^3}\right)^{1/2} \exp\left\{-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right\}, \quad x > 0, \mu > 0, \lambda > 0.$$
   Find the maximum likelihood estimators for μ and λ.

5. (10%) Let $X_1, \dots, X_n$ be i.i.d. normal random variables with mean μ_X and variance σ^2. Also, let $Y_1, \dots, Y_m$ be i.i.d. normal random variables with mean μ_Y and variance σ^2. Suppose that X_i's and Y_i's are independent and the parameters μ_X, μ_Y, and σ^2 are unknown. Find the two-tailed t-test for testing the null hypothesis $H_0\colon \mu_X = \mu_Y$ against the alternative hypothesis $H_1\colon \mu_X \neq \mu_Y$ with the significance level α. Please express your answer by using $t_{\alpha,\nu}$ that satisfies $P(T_\nu > t_{\alpha,\nu}) = \alpha$, where T_ν is a Student t random variable with ν degrees of freedom.

6. (10%) Suppose that there are two people. Each of them tosses a fair coin n times. Find the probability that they will toss the same number of heads. You must simplify your answer.

7. (10%) Prove that $P(Z > z) < \frac{1}{z}\phi(z)$ if $z > 0$, where $\phi(z)$ is the pdf of Z.

- End -

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：線性代數乙【應數系碩士班乙組】

—作答注意事項—

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科目名稱：線性代數乙【應數系碩士班乙組】　　題號：424005

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　共1頁第1頁

Please write down all the detail of your computation and solution.

1. (15%) Find the third column of the following product matrix

$$\begin{bmatrix} \sqrt{e} & \frac{1}{3} & \sqrt{2} & \pi \\ 10^5 & 7 & 0 & 3.7 \\ i & \sin 3 & -1 & \ln 2 \end{bmatrix} \begin{bmatrix} 0.2 & -0.3 & 0.1 \\ 2 & -1 & -1 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ -\sqrt{3} & 0 & \sqrt{3} \end{bmatrix} \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} -e & 2.3 & 1 & -\pi \\ \frac{1}{7} & \sqrt{5} & 1 & 2 \\ \pi & -2 & 1 & -\sqrt{2} \end{bmatrix}.$$

2. (15%) Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Find the necessary and sufficient conditions on a, b, c and d such that A is symmetric positive definite, and then prove it.

3. (15%) Find the least squares plane $z = ax + by + c$ for (x, y, z) data: $(0,0,2), (0,1,2), (1,0,1)$ and $(1,1,5)$.

4. (15%) Let A be a real $n \times n$ matrix where $n \geq 2$. Porve that the followings are equivalent:

(1) A is orthogonally diagonalizable,
(2) A has a real orthonormal set of n eigenvectors.

5. (20%) Let

$$A = \begin{pmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ -1 & 2 & 3 & -1 \\ 3 & -1 & -1 & 2 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 7 \\ 5 \\ -6 \\ 8 \end{pmatrix}.$$

(1) Find permutation matrix P, lower triangular matrix L and upper triangular matrix U such that $A = PLU$.

(2) Use (1) to solve $A\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$.

6. (20%) Let $n \times n$ matrix A have all entries -3 where $n \geq 2$. Find all of its eigevalues, corresponding eigenvectors, and its Jordan canonical form.

國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：高等微積分【應數系碩士班丙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

1. (10%) Use $\epsilon - \delta$ argument to show that $\lim_{x\to 1}(x^2 + 2x) = 3$.

2. (10%) Give an example of a function that is NOT integrable on $[0, 1]$. Support your argument.

3. (10%) Is the sequence of functions $f_n(x) = \frac{x^n}{1+x^n}$ uniformly convergent on the interval $[0.5, 1.5]$? Prove or disprove.

4. (10%) Consider the vector-valued function $G : \mathrm{R}^2 \to \mathrm{R}^2$ defined by

$$G(x, y) = (x \cos y, x \sin y).$$

Show that G is locally invertible about every point in $\mathrm{R}^2 \setminus \{0\}$.

5. Let $f : E \to \mathrm{R}$ be continuous on E, a closed and bounded subset of R^n.

   (a) (5%) Show that the image set $f[E]$ has a maximum point M and a minimum point m.

   (b) (10%) Show that $f[E]$ is a closed and bounded interval.

6. Let $g(x, y, z) = x^3 - 3xy - y^3 - z^2$.

   (a) (6%) Compute the gradient vector $\nabla g(x, y, z)$ and Hessian matrix $H_g(x, y, z)$.

   (b) (4%) Determine if the critical point $(-1, 1, 0)$ is a local maximum of g.

   (c) (5%) Find the Taylor polynomial of order 2 based at the point $\mathrm{x}_0 = (-1, 1, 0)$.

7. (15%) State and prove one of the two Fundamental Theorems of Calculus.

8. The mean value theorem for integrals states that for a continuous function f defined on $[a, b]$, there exists some $c \in (a, b)$ such that

$$\int_a^b f(t)\, dt = f(c)(b - a).$$

   (a) (6%) Letting $F(x) = \int_a^x f(t)\, dt$, prove the above theorem as an application of the (usual) mean value theorem.

   (b) (9%) Show that the mean value theorem for integrals implies the mean value theorem too, when the function f is continuously differentiable on $[a, b]$.

~ 全卷完 ~