國立中山大學104學年度碩士暨碩士專班招生考試試題

科目名稱：基礎數學【應數系碩士班甲組】
※本科目依簡章規定「不可以」使用計算機(問答申論題)

題號：424001

答題時，每題都必須寫下題號與步驟。

1. (10%) Evaluate $\displaystyle\int_0^\infty \frac{\arctan x}{1+x^2}\,dx$.

2. (10%) Evaluate $\displaystyle\lim_{t\to 0^+}\left(\int_0^t \frac{dx}{\sqrt{1-x^2}}\right)^t$.

3. (10%) Evaluate $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} (1+\sqrt{x^2+y^2})\,dy dx$.

4. (15%) Determine the set of real numbers x for which $\displaystyle\sum_{k=1}^\infty \left(1+\frac{1}{k}\right)^k (x-1)^k$ converges.

5. (15%) Minimize $2x+y+4z$ subject to $x^2+y^2+z^2=7$.

6. (20%) Let $M_2(\mathbb{R})$ be the collection of 2×2 matrices with real-number entries and $L: M_2(\mathbb{R}) \to M_2(\mathbb{R})$ be a linear transformation defined by

$$L(X) = \begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix} X - X \begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix}.$$

(a) Find the dimension of the image of L.

(b) Find a basis for the kernel of L.

7. (20%) Let $f(x,y,z) = \lambda(x^2+y^2-z^2) + z^2 + (2-4\lambda)xz$ be a quadratic form in x, y and z.

(a) Find a 3×3 symmetric matrix A such that $f(x,y,z) = vAv^t$, where $v = [x,y,z]$ and v^t is the transpose of v.

(b) Find the real values of λ such that f is positive definite.

~全卷完~

國立中山大學104學年度碩士暨碩士專班招生考試試題

科目名稱：微積分【應數系碩士班乙組】 題號：424002

※本科目依簡章規定「不可以」使用計算機(問答申論題) 共1頁第1頁

計算題：共7題，子題分數平均分配。答題時，每題都必須寫下題號與詳細步驟。

[1]. (14%) Evaluate the following limits.

(a) $\lim_{x\to\infty}\left(\sin\frac{1}{x}\right)^{1/x}$ (b) $\lim_{x\to 2^+}\left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$

[2]. (12%) State the Newton's method for approximating the zeros of a function.

[3]. (20%) Evaluate the following integrals.

(a) $\int_0^\infty \frac{1}{e^x+e^{-x}}dx$ (b) $\int \frac{2x}{x^2+4x+8}\,dx$

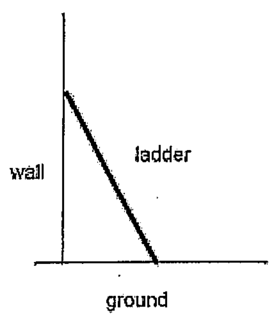
[4]. (12%) Find the area of the region inside $r=2(1+\cos\theta)$ and outside $r=2\cos\theta$.

[5]. (14%) Find the interval of convergence for the power series $\sum_{n=1}^{\infty}\frac{5^n}{n}(2x-1)^n$.

[6]. (12%) Evaluate the iterated integral by converting to polar coordinates.

$$\int_0^2\int_0^{\sqrt{4-x^2}}\sin\sqrt{x^2+y^2}\,dy\,dx$$

[7]. (16%) A ladder 20 meters long is leaning against the wall (see figure). The base of the ladder is pulled away from the wall at a rate of 1 meter per second. (a) How fast is the top of the ladder moving down the wall when its base is 12 meters from the wall? (b) Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 12 meters from the wall.



========= 全卷完 =========

國立中山大學 104 學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【應數系碩士班丙組】　　　　　　　　　　題號：424003

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　　　　　　共 1 頁第 1 頁

1 (10 pts) Find A^{-1} by Gauss-Jordan elimination with $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

2 (10 pts) A matrix $M \in R^{n\times n}$ is called skew-symmetric if $M^T = -M$. Prove the skew-symmetric matrices form a subspace of $R^{n\times n}$.

3 (10 pts) Let $A = \begin{bmatrix} 1 & 0 \\ a & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & c & 0 \\ 0 & d & 1 \end{bmatrix}$, where a, b, c, d are non-zero real numbers.

(a) Find bases for the row and column spaces of A.

(b) Is A invertible?why?

4 (10 pts) Find the Jordan canonical form of matrix A, where $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

5 (10 pts) Prove that

$$\begin{vmatrix} \mathrm{O} & \mathrm{C} \\ \mathrm{A} & \mathrm{B} \end{vmatrix} = (-1)^m |\mathrm{A}||\mathrm{C}|,$$

where O is the $m \times m$ zero matrix and A, B and C are $m \times m$ matrices.

6 (50 pts) Prove or disprove.

(a) If A and B share the same column space, row space, null space, left null space then $A = B$.

(b) If rows of A are linearly dependent, so are columns.

(c) Let $A \in R^{n\times n}$ and $Ax = 0 \Rightarrow x = 0$, then $rank(A) = n$.

(d) Let W_1, W_2 are subspaces of V then $W_1 + W_2 = \{w_1 + w_2 | w_1 \in W_1, w_2 \in W_2\}$ is a subspace of V.

(e) Let A be a real $n \times n$ matrix, then A and its transpose A^t have the same minimal polynomial.