1. (10%) Assume that an initial amount of money, S_0, is deposited in a bank that pays interest at an annual rate r *compounded continuously*. Furthermore, assume that a deposit or withdraw takes place at a constant rate k. Find the amount of money $S(t)$ at any time.

2. (15%) Consider the following non-linear first-order ODE:

$$y' + p(t)y = q(t)y^n, \quad n \neq 0, 1$$

   (a) Show that, by appropriate substitution, the above equation may become linear ODE. (5%)

   (b) Solve the equation in a symbolical form. (10%)

3. (20%) Consider the Dirac delta function $\delta(x - x_0)$ defined as:

$$\delta(x - x_0) = 0, \text{ for } x \neq x_0 \text{ and } \int_{-\infty}^{\infty} \delta(x - x_0)dx = 1.$$

   (a) Show that $\int_{-\infty}^{\infty} f(x)\delta(x - x_0)dx = f(x_0)$. (10%)

   (b) Solve the ODE: $y'' + 4y = \delta(t - \pi) - \delta(t - 2\pi)$, with the initial conditions: $y(0) = 0, \quad y'(0) = 0$. (10%)

4. (15%) The Newton-Raphson (or simply Newton's) method is one of the most-frequently employed numerical method for solving a root-finding problem: $f(x) = 0$.

   (a) Derive the iterative scheme based upon graphical method. (5%)

   (b) Set up the Newton iteration for computing $\sqrt{3}$, starting from $x_0 = 1$, and calculating x_1, x_2, and x_3. (10%)

5. (20%)

   (a) $\mathbf{A}$ is known to be similar to the matrix $\mathbf{B} = \begin{bmatrix} 4 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$. Find the eigenvalue of $\mathbf{A}$. (10%)

   (b) Find the explicit formulas for $\mathbf{C}^k$ in terms of the positive integer k, where $\mathbf{C} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$. (10%)

6. (20%) For an axisymmetric sound field, its sound pressure $p(r, z, t)$ may be shown to satisfy the following wave equation:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) + \frac{\partial p}{\partial z^2} \right],$$

   where c is a constant.

   (a) Let p be represented by: $p(r, z, t) = \int_{-\infty}^{\infty} p_\omega(r, z)e^{i\omega t}d\omega$, show that $p_\omega(r, z)$ satisfies the following PDE:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p_\omega}{\partial r} \right) + \frac{\partial p_\omega}{\partial z^2} + k^2 p_\omega = 0,$$

   where $k = \omega/c$. (5%)

   (b) Again, let $p_\omega(r, z)$ be represented by: $p_\omega(r, z) = \int_0^{\infty} \tilde{p}_\omega(z) J_0(k_r r) k_r dk_r$, where J_0 is the zeroth-order Bessel function of the first kind, show that $\tilde{p}_\omega$ satisfies:

$$\frac{d^2 \tilde{p}_\omega}{dz^2} + k_z^2 \tilde{p}_\omega = 0,$$

   where $k_z^2 = k^2 - k_r^2$. (10%)

   (c) Now, solve for $\tilde{p}_\omega(z)$ for constant k_z, and then write down the solution for $p(r, z, t)$ in an integral form. (5%)

國立中山大學９５學年度碩士班招生考試試題

科目：應用力學【海下所碩士班選考】

說明：本試卷共四題，每題 25 分，總分 100 分。

1. (25%) 一質量 5m 的楔形塊靜止於水平面上，如圖 1 所示。當一個質量 2m 的圓球體以垂直速度 v 撞擊楔形塊時，若恢復係數爲 e，試求出圓球體與楔形塊在碰撞後瞬間的速度。

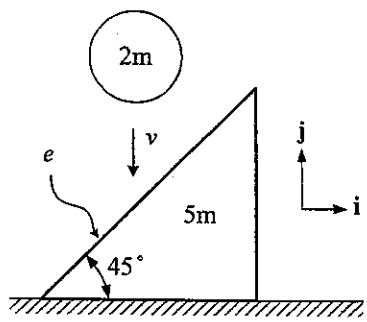


圖 1

2. (25%) 重量 W 的圓錐體其尺寸與重心位置如圖 2 所示。在圓錐頂 B 繫一條繩索，繩索另一端固定在光滑牆面 A 點上。試求能夠使圓錐體基底（Base）靜靠於光滑牆面且保持平衡的最大繩索長度 L 爲何？

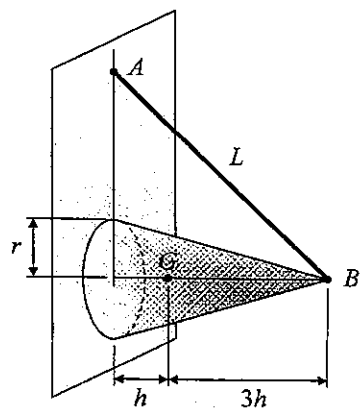


圖 2

3. (25%) 一個開口爲正方形的量杯站立於 15°光滑斜面上，將水徐徐注入，如圖 3 所示。求 H 爲多少時，量杯會傾倒。（不計量杯的質量；量杯開口 5 cm ×5 cm ，擋板 h 高度爲 0.5 cm。）

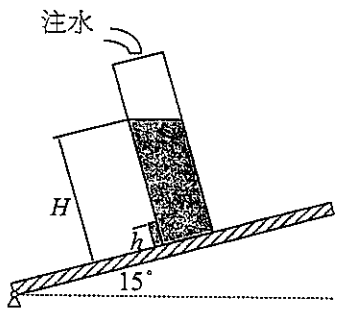
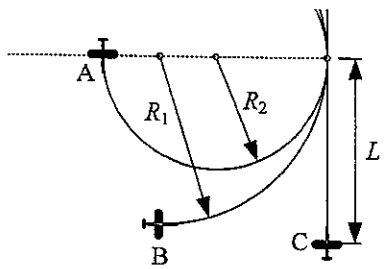


圖 3

4. (25%) 三架特技機在不同高度上進行高速會合表演，如圖 4 所示。A 的軌跡爲半徑 R_1=2700 m, B 的軌跡爲半徑 R_2=1800 m, C 沿直線飛行。A, B 分別在圓周的 9 點鐘及 6 點鐘位置，逆時鐘飛行；C 則距離會合點尚有 2800 m。若三機在 20 秒後相會，請問 (1) 各機速度爲多少；(2) A 相對於 B、A 相對於 C 的速度各爲多少；(2) A 相對於 B、A 相對於 C 的加速度各爲多少。



上視圖

圖 4

國立中山大學９５學年度碩士班招生考試試題

科目：流體力學【海下所碩士班選考】

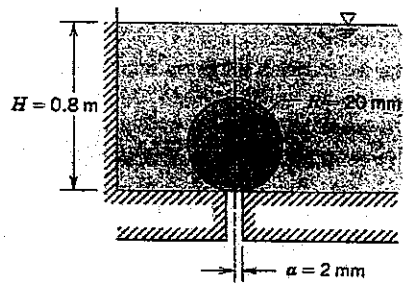
1. (30%; 3 points each) Explain the following terms: (a) Non-Newtonian fluid (b) Streakline (c) Cavitation (d) Reynolds stress (e) Material derivative (f) Isentropic process (g) Idea gas law (h) Drag coefficient (i) Hydraulic jump (j) Pitot tube

2. (15%) 0.3-m-diameter soccer ball, pressurized to 20 kPa, develops a small leak with an area equivalent to 0.006 mm^2. If viscous effects are neglected and the air is assumed to be incompressible, determine the flow rate through the hole. Would the ball become noticeably softer during a 1-hr soccer game? Explain. Is it reasonable to assume incompressible flow for this situation? Explain.

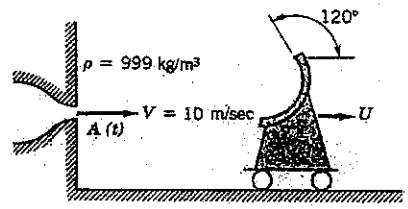
3. (15%) An airplane moves forward at a speed of 971 km/hr, and the frontal intake area of the jet engine is 0.80 m^2 with the entering air density of 0.736 kg/m^3. A stationary observer determines that relative to the earth, the jet engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is 0.558 m^2 and the exhaust gas density is 0.515 kg/m^3. Estimate the mass flow rate of fuel into the engine in kg/hr.

4. (10%) The velocity components in a steady, incompressible, two dimensional flow field are $u = 2y$ and $v = 4x$. Determine the corresponding stream function and show on a sketch several streamlines. Indicate the direction of flow along the streamlines.

5. (15%) A sphere of radius R, made from material of specific gravity SG, is submerged in a tank of water. The sphere is placed over a hole, of radius a, in the tank bottom. Develop a general expression for the range of specific gravities for which the sphere will float to the surface. For the dimension shown in the figure, determine the minimum SG required for the sphere to remain in the position shown.



6. (15%) The wheeled cart shown rolls with negligible resistance. The cart is to accelerate to the right at a constant rate of 2 m/sec^2. This is to be accomplished by "programming" the water jet area, $A(t)$, that reaches the cart. The jet speed remains constant at 10 m/sec. Obtain an expression for $A(t)$ required to produce the motion. Sketch the area variation for $t \leq 4$ sec. Evaluate the jet area at $t = 2$ sec.



1. As far as the devices shown in Figure 1 are concerned,
   (a). Please draw and explain what Base-Width Modulation is and what Channel Length Modulation is. *10%*
   (b). Please plot the concentration profile of the minority carrier in (i) a cut-off, (ii) a forward-active, and (iii) a saturated npn bipolar transistor respectively. *10%*

2. Determine and sketch the steady output voltage V_o for the circuits in Figure 2.a and Figure 2.b respectively. Their input signals are given by its figure respectively and assume the cut-in voltage V_γ = 0V and on resistance r_f = 0 for the diode. *2*10%*

3. Please use a Zener diode with a breakdown voltage of 5.6V to design an Op-amp voltage reference source as shown in Figure 3 with an output of 10.0V. Assume the voltage regulation will be within specifications if the Zener diode is biased between 1-1.2mA. In other words, please design the values of R_1, R_2, R_3, R_4 and R_F. *20%*

4. Please design an astable circuit as shown in Figure 4 Using a 680pF capacitor to obtain a square wave with a 50-kHz frequency and a 75% duty cycle. Specify the values of R_A and R_B. *2*10%*

5. For the circuit in Figure 5,
   (a). Please break the loop at node X and find the loop gain.
   (b). For R = 10kΩ, please find C and R_f to obtain sinusoidal oscillations. *2*10%*

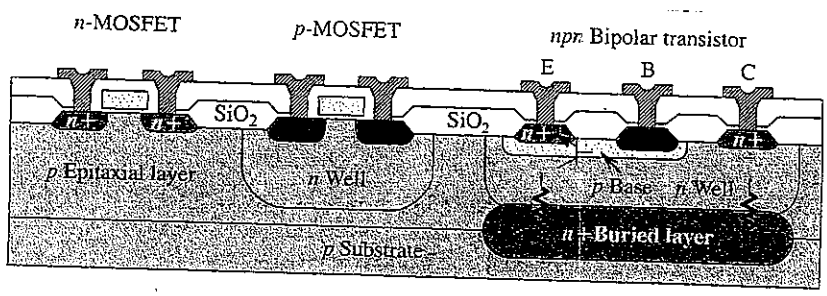
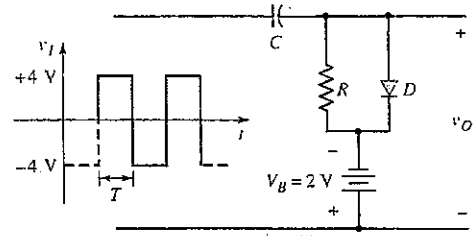


Figure 1

Figure 2.a



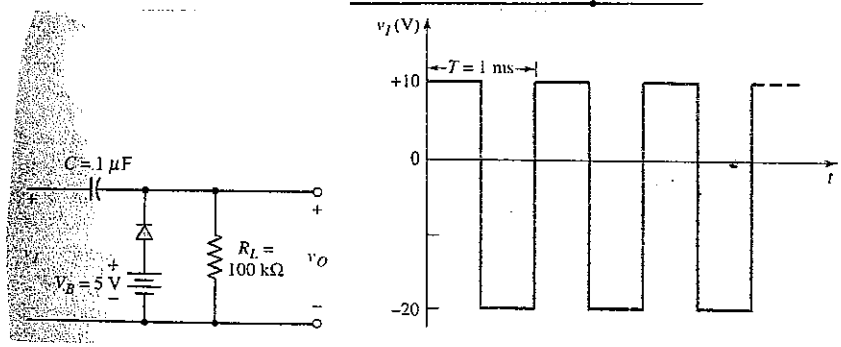


Figure 2.b

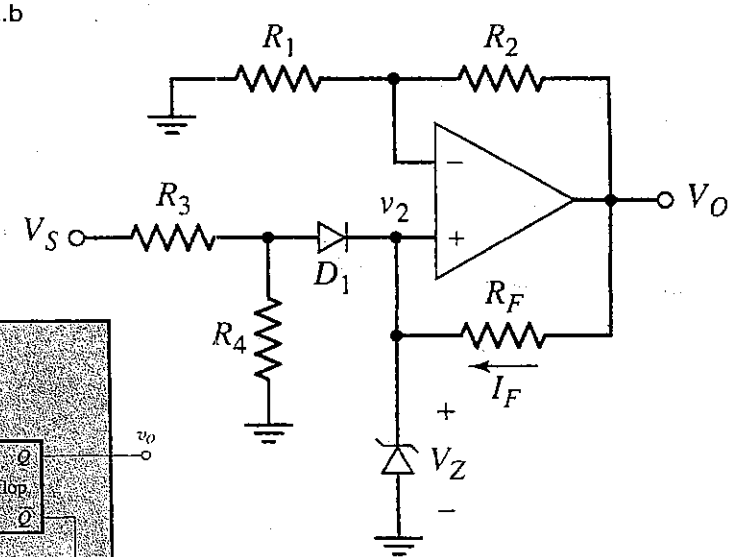


Figure 3

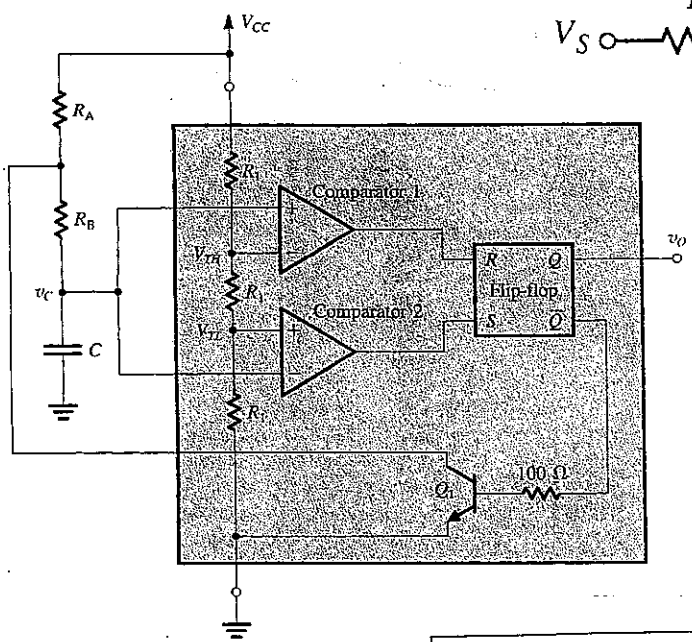


Figure 4

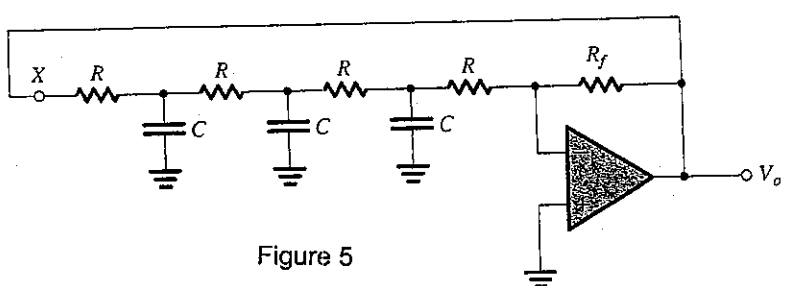


Figure 5