國立中山大學九十三學年度碩士班招生考試試題

科目： 工程數學【海下技術研究所碩士班】

1. (10%) Determine the solution of the following initial-valued problem, and plot the results:

$$4\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 3y = 0, \quad y(0) = 2, \quad y'(0) = \frac{1}{2}$$

2. (10%) Find the directional derivative of $f(x, y, z) = \frac{1}{\sqrt{x^2+2y^2+z^2}}$ at the point $P = (1, 2, 4)$ in the direction of the vector $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

3. (10%) Solve the following linear system by the method of Gaussian elimination:

$$\begin{cases} x - y + 3z = 2 \\ 3x - 3y + z = -1 \\ x + y = 3 \end{cases}$$

4. (10%) Give the definition of the Dirac delta function: $\delta(t - t_0)$, and then show: $\int_{-\infty}^{\infty} f(t)\delta(t - t_0)dt = f(t_0)$.

5. (15%) Solve the following ordinary differential equation:

$$x^2y'' + xy' + 4y = 2\sin(\ln x)$$

6. (15%) Consider the ordinary differential equation: $x^2y'' + xy' + x^2y = 0$

   (a) Write down the solutions in terms of Bessel functions. (5%)

   (b) Sketch the solutions, and discuss the behaviors of the solutions for small and large x, respectively. (10%)

7. (10%) Find the eigenvalues and the corresponding *orthogonal* eigenvectors of the matrix: $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

8. (20%) Solve the non-homogeneous heat equation: $\frac{\partial T}{\partial t} = k\nabla^2 T + q(r, t)$ inside the circle $(r < a)$ with $T = 0$ at $r = a$ and initially $T = 0$, where k is a constant, and a is the radius of the circle.

國立中山大學九十三學年度碩士班招生考試試題

科目：應用力學【海下技術研究所碩士班（選考）】

說明：本試卷共六題，六題之中請任選四題作答，至多不得超過四題。若答題超過四題，閱卷時將依答題次序僅評閱前四題，超過四題部分不予計分。每題 25 分，總分 100 分。

1. (25%) 一鉛塊重 51.75 kg 以滑輪懸掛於長度 20 公尺的繩子上，如圖 1 所示，請問在平衡狀態下，高度 y 爲多少？（在 A 點的滑輪尺寸忽略不計）

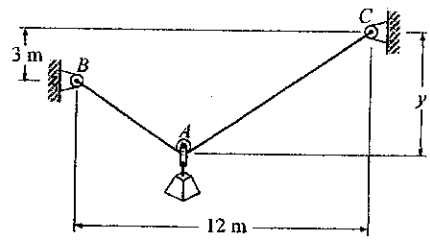


圖 1

2. (25%) 一個重量 12.5 lb 的圓盤與彈簧連接，而且當圓盤旋轉時，彈簧末端會沿著圓盤周圍纏繞，如圖 2 所示。假設圓盤沒有任何外力作用時，彈簧並未受到拉伸，請問當圓盤受到切向力 F = 6 lb 以及耦力矩 M = 10 lb·ft 的作用時，圓盤會旋轉幾度？

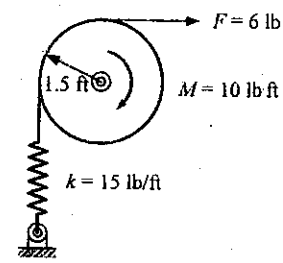


圖 2

3. (25%) 一支 6 kg 的木棒靜止懸吊於 O 點，如圖 3 所示。若有一顆 3 g 重的子彈以 45 度角射向木棒 A 點，而且子彈的速度爲 300 m/s，當子彈完全嵌進木棒的那一瞬間，木棒的旋轉角速度爲何？

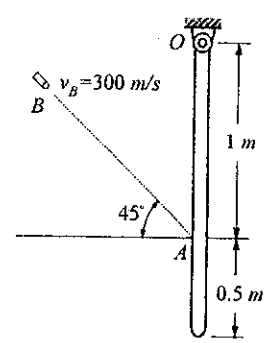


圖 3

4. (25%)

(a) 一支材質均勻、長度 L、質量 M 的細棒，若其旋轉軸選在質心 o 點上，由旋轉慣量的定義爲

$$I_o = \int_M x^2 dm$$

請證明此細棒的旋轉慣量 $I_o = \frac{1}{12} L^2 M$ 。

(b) 平行軸定理：當旋轉軸從質心平移至另一點 P 時，則旋轉慣量可表示爲 $I_P = I_o + Md^2$，其中 d 爲 P 點到質心的距離。藉由此定理，請計算圖 4 中由 6 根長度 L、質量 M 的細棒組成的「田」字形狀，以中心爲旋轉軸的旋轉慣量。

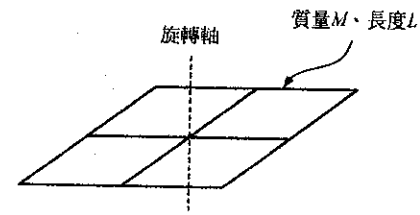


圖 4

5. (25%) 如圖 5 所示，長度爲 L、質量爲 M 的擺，隨著轉動而上揚。若桿子的質量不計，請問：

(a) 穩定轉速 ω 與夾角 θ 之關係。

(b) H 點外力的大小及方向。

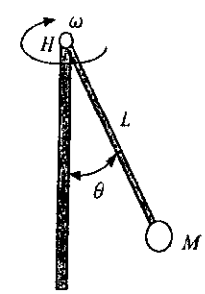


圖 5

6. (25%)解釋名詞

(a) 保守力

(b) 內力與外力

(c) 科氏力

(d) 質心

(e) 彈性碰撞係數

1. (30%, 5 points each) Define and explain the following terms:
   (a) Pressure prism
   (b) Pressure head
   (c) Capillary action
   (d) Manometer
   (e) Buckingham Pi theorem
   (f) Reynolds stress

2. (10%) A spherical buoy has a diameter of 1.5 m, weighs 8.5 kN, and is anchored to the sea floor with a cable. If the buoy is completely immersed in the water, what is the tension of the cable? (Hint: seawater specific weight is 10.1 kN/m^3)

3. (10%) A mountain lake has an average temperature of 10°C and a maximum depth of 40 m. For a barometric pressure of 598 mm Hg, determine the absolute pressure (in pascals) at the deepest part of the lake.

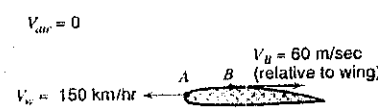
4. (10%) An open 1-m-diameter tank contains water at a depth of 0.7 m when at rest. As the tank is rotated about its vertical axis, the center of the fluid surface is depressed. At what angular velocity will the bottom of the tank first be exposed? No water is spilled from the tank.

5. (10%) Give the required assumptions, and then derive the following Bernoulli's equation:

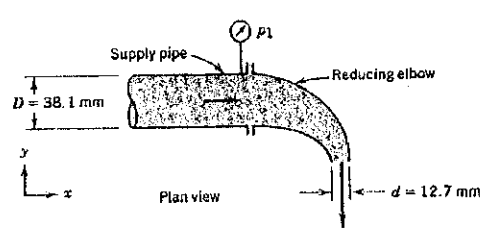
$$-\frac{\partial \phi}{\partial t}+\frac{p}{\rho}+\frac{1}{2}\left[\left(\frac{\partial \phi}{\partial x}\right)^2+\left(\frac{\partial \phi}{\partial z}\right)^2\right]+gz=C(t)$$

   where ϕ is the velocity potential.

6. (15%) The wing section of an airplane in flight at 150 km/hr at 1000 m altitude in standard air is shown in the figure. Point A is the stagnation point, and the speed at the point B is 60 m/sec relative to wing. Find the stagnation pressure at point A, and the static pressure at point B. (Note that at 1000 m, $p/p_{SL}=0.8870$, and $\rho/\rho_{SL}=0.9075$, where the subscript SL stands for "Sea Level". $p_{SL}=1.01325\times10^5$ N/m^2, $\rho_{SL}=1.225$ kg/m^3.)



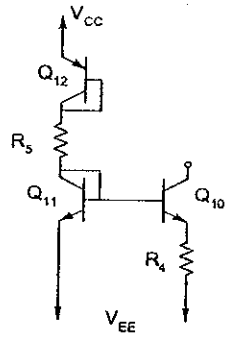
7. (15%) Water flows steadily through the reducing elbow shown. The elbow is smooth and short, and the flow accelerates, so the effect of friction is small. The volume flow rate is $Q=1.27$ L/sec. The elbow is in a horizontal plane. Estimate the gage pressure at section 1, i.e., p_1. Calculate the x component of the force exerted by the reducing elbow on the supply pipe.



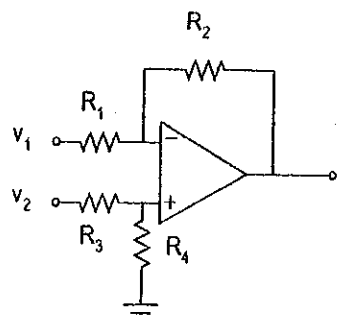
1. For the bias circuit shown, the transistors have the same parameter : $I_S = 10^{-14}A$. Also we have R_5=40kΩ, R_4=5kΩ, V_{CC}=15V, and V_{EE}=-15V. Calculate : (20%)

   (a) the reference current through R_{ref}.

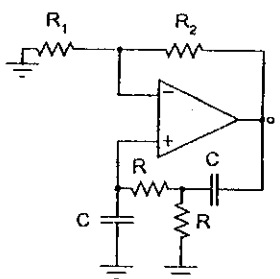
   (b) the collector current I_{C10}.



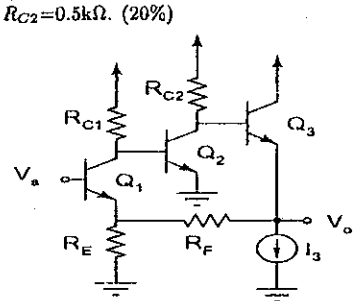
2. Consider the difference amplifier shown. Let $R_1 = R_3 = 10k\Omega$ and $R_2 = R_4 = 1M\Omega$. If the op amp has $V_{os} = 2mV$, $I_B = 0.3\mu A$, and $I_{os} = 50nA$, find the worst case (largest) dc offset voltage at the output. (20%)



3. For the oscillator circuit shown, determine the oscillating frequency and the ratio of $\frac{R_2}{R_1}$ for oscillation. (20%)



4. For the series-shunt feedback amplifier shown, determine A, β, R_{in}, and R_{out}. The transistors have h_{fe}=100 with Q_1 biased at 1mA, Q_2 at 2mA, and Q_3 at 5mA. The circuit paremeters are : R_E= 50Ω, R_F=950Ω, R_{C1}=1kΩ, R_{C2}=0.5kΩ. (20%)



5. The TTL circuit has parameters : V_{CC} = 5 V, R_1 = 6 kΩ, R_2 = 2 kΩ, R_3 = 800 Ω, R_4 = 1.5 kΩ, β_F = 25 and β_R = 0.1. (20%)

   (a) Determine the volatges at each node and currents in each transistor (i_B , and i_C,) for V_X = logic 0.

   (b) Repeat for V_X = logic 1.