1. (10%) Solve the following initial-valued problem using Laplace transform:

$$y'' + 2y' + 5y = \sin(3t), \quad y(0) = 1, \; y'(0) = -1$$

2. (15%) The vibration of a mechanical system with damper may be represented by the following ODE:

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + ky = 0$$

where m, c, k are *positive* constants. Show that $\lim_{t\to\infty} y(t) = 0$. What is the physical significance?

3. (20%) Consider the following second-order linear nonhomogeneous ODE:

$$x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = x^2, \; x > 0$$

One may easily show that one of the *homogeneous* solutions is $\frac{\sin x}{\sqrt{x}}$.

(a) Find the second *homogeneous* solution. (10%)

(b) Find the particular solution. (The solution may be represented in an integral form.) (10%)

4. (15%) This problem is about line integration:

(a) Use Green's theorem to evaluate the line integral: $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$, where $\mathbf{F} = 3x^2\,\mathbf{i} - 4xy\,\mathbf{j}$, and C is the contour along the boundary of the rectangle, $0 \le x \le 4$, $0 \le y \le 1$, in counterclockwise sense. [10%]

(b) Find the length of the semicubical parabola: $\mathbf{r} = t\,\mathbf{i} + \sqrt{t^3}\,\mathbf{j}$ from $(0,0,0)$ to $(4,8,0)$. [5%]

5. (15%) Consider the following matrix A:

$$A = \begin{bmatrix} 1 & -\sqrt{3} & 0 \\ -\sqrt{3} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the diagonal matrix Λ and the orthogonal matrix P so that $A = P\Lambda P^T$ (the superscript T stands for transpose). Also, outline the procedure that may efficiently evaluate A^N, where N is an integer, which may be large.

6. (25%) The acoustic sound wave may be described by the wave equation:

$$\frac{\partial^2 p}{\partial t^2} = c^2\frac{\partial^2 p}{\partial x^2}$$

where p is the acoustic pressure, and c is the sound speed.

(a) Show that *any* function of the form $f(x-ct)$ or $f(x+ct)$ is the solution of the PDE. What is the significance of these functions? [10%]

(b) If the initial and boundary conditions are:

$$\text{BC's} \quad p(0,t) = t, \;\; p(1,t) = 0$$
$$\text{IC's} \quad p(x,0) = 0, \;\; \frac{\partial p}{\partial t}(x,0) = 0$$

determine the solution for $p(x,t)$. [15%]

1. (20%) *Define and give the physical significance* of the following non-dimensional parameters:
   (a) Reynolds number (5%)
   (b) Strouhal number (5%)
   (c) Froude number (5%)
   (d) Mach number (5%)

2. (10%) The velocity of a two-dimensional fluid flow is given by:

$$\mathbf{v}(x,y) = 4xy\,\hat{\mathbf{i}} + 2(x^2+y^2)\,\hat{\mathbf{j}}$$

   Is this flow compressible? Why? Is this flow irrotational? Why?

3. (10%) Use the Bernoulli's equation to explain why the cavitation is formed at the high-speed rotating propeller of a ship.

4. (20%) A 20 m wide spillway for a dam is designed to carry 125 m^3/s at flood stage. A 1:15 model is constructed to study the flow characteristics through the spillway. Determine the required width and flow rate of the model. What operating time for the model corresponds to a 24-hr period in the prototype? The effects of surface tension and viscosity are negligible.

5. (20%) In ancient time, circular containers filled with water sometimes were used as a crude clock. The containers were shaped in such a way that, as water drains from bottom, the surface level dropped at a constant rate, s. Assuming that water drains from a small hole of area A, find an expression for the radius of the container, r, as a function of the water level, h. Determine the volume of water needed so that the clock willl operate for n hours.

6. (20%) For a fluid particle on a moving boundary such as ocean surface, the fluid motion requires that the particle remain on the surface during the movement of the boundary. According to this principle, determine the boundary condition for the fluid motion, if the moving surface is given by:

$$z(x,y,t) = \frac{1}{2}\left(e^{-x^2} + e^{-y^2}\right)\cos(\omega t)$$

   where ω is a constant. Also, find the normal velocity of the fluid particle at $(x,y) = (1,-1)$.

國立中山大學九十二學年度碩士班招生考試試題

科目：應用力學【海下技術研究所碩士班（選考）】

說明：本試卷共六題，六題之中請任選四題作答，至多不得超過四題。若答題超過四題，閱卷時將依答題次序僅評閱前四題，超過四題部分不予計分。每題 25 分，總分 100 分。

1. (25%) 將一個楔形塊置於圓柱與牆面之間，如圖 1 所示，圓柱重量爲 10 kg，A 和 C 的摩擦係數爲 $\mu = 0.3$，B 的摩擦係數爲 $\mu' = 0.5$。請問在不破壞平衡狀態下，楔形塊的最大重量爲何？

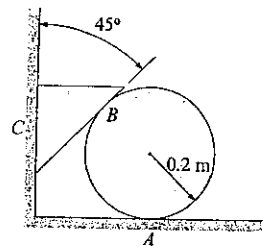


圖 1

2. (25%) 如圖 2 所示，AD 隔牆受到水壓以及填土壓力的作用，隔牆重量爲 1000 kg。假設隔牆 AD 以迴轉支撐（Pin support）於 A 點，請求出 A 點的水平作用力和垂直作用力，同時求出平衡狀態下土壤固定支撐 BC 所需的張力大小。

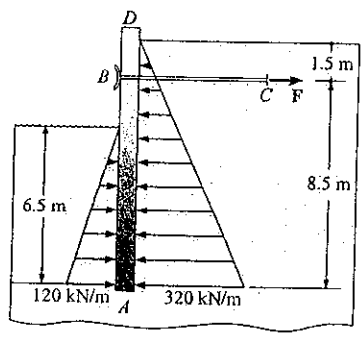


圖 2

3. (25%) 如圖 3 所示，輸送帶 B 的輸送速度爲 5 m/s，箱子 A 的重量爲 12 kg，如果輸送帶與箱子 A 之間的靜摩擦係數爲 $\mu = 0.25$，爲了不讓箱子在輸送帶停止時發生滑動，請問輸送帶最短的停止時間爲何？

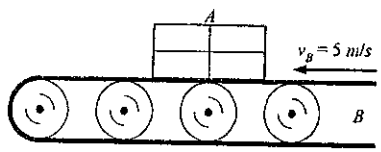


圖 3

4. (25%) 如圖 4 所示，一根端點爲轉軸的懸臂樑，質量 m，用一根纜索支撐著。另有一個質量也爲 m 的質點由左向右慢慢地滾。如果懸臂樑的材質均勻，纜索的最大抗張力爲 $1.5mg$，請問質點到達哪一個位置時，纜索會斷裂。

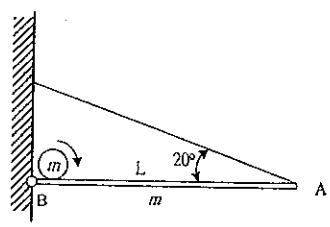


圖 4

5. (25%) 一個密度爲 200 kg/m^3，體積爲 V=1m^3，底面積爲 1 m^2 的浮體漂在水面上，如圖 5 所示。請問以外力 F=400N 向下壓後，浮體會向下沒入多少深度？外力瞬間移除後，浮體上升至原先位置時，速度爲多少？

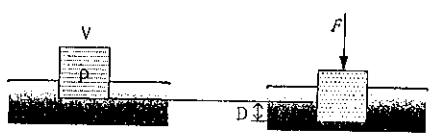


圖 5

6. (25%) 露營時搭帳棚會使用到營杖。如圖 6 所示，這是一個鋁管，在末段加上一塊塑膠後，穿入一根鋼釘作爲固定帆布之用。不小心營釘滑入塑膠中，手邊又無任何工具可以將之夾出或推入。請問你是否可以用一個學過的應用力學概念，說出一個可行的解決方法。

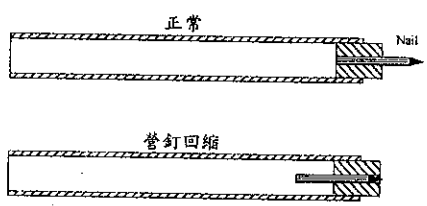
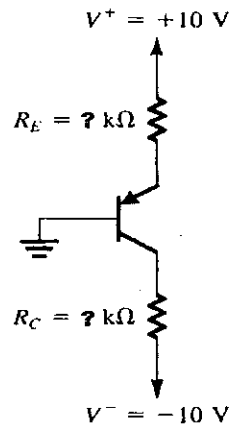


圖 6

(1). Design the circuit shown in the right (i.e. find the values for R_C and R_E) to establish a collector current of 1mA and a reverse bias on the collector-base junction of 4V. Assume $\alpha \doteqdot 1$ and $V_E \doteqdot 0.7$V. 20%



(2). (a) What is the circuit as shown in the below? 5%

(b) Derive the transfer function of v_o over v_i. 5%

(c) Use the data of Table 1 shown in below to design the circuit in (a) to realize an all-pass filter with ω_0 rad/s, $Q = 5$, and flat gain = 1. Use $C = 10$nF and $r = 10$kΩ. (i.e. to find the values for R, QR, C_1, R_1, R_2 *and* R_3). 10%

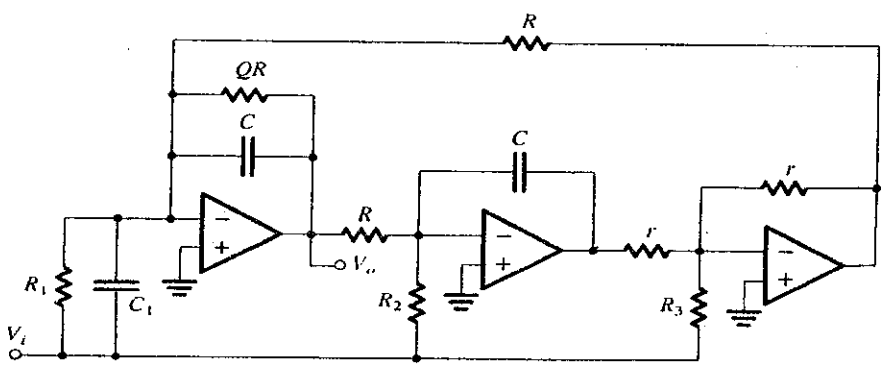


Table 1 DESIGN DATA FOR THE CIRCUIT

All cases	C = arbitrary, $R = 1/\omega_0 C$, r = arbitrary
LP	$C_1 = 0$, $R_1 = \infty$, $R_2 = R$/dc gain, $R_3 = \infty$
Positive BP	$C_1 = 0$, $R_1 = \infty$, $R_2 = \infty$, $R_3 = Qr$/center-frequency gain
Negative BP	$C_1 = 0$, $R_1 = QR$/center-frequency gain, $R_2 = \infty$, $R_3 = \infty$
HP	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = \infty$, $R_3 = \infty$
Notch (all types)	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = R(\omega_0/\omega_n)^2$/high-frequency gain, $R_3 = \infty$
AP	$C_1 = C \times$ flat gain, $R_1 = \infty$, $R_2 = R$/gain, $R_3 = Qr$/gain

(3).

(a) Draw and show the configuration of the four types of feedback design known to be used in an amplifier. 10%

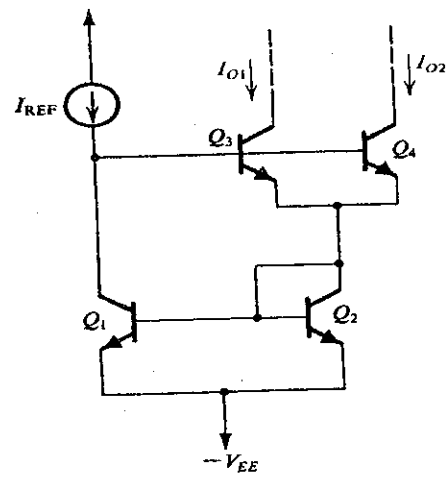
(b) To increase the input resistance R_i of an amplifier, what type of feedback design should be used? Why? 5%

(c) To decrease the output resistance R_o of an amplifier, what type of feedback design should be used? Why? 5%

(4).

(a) For the circuit shown in the right, find I_{O1} and I_{O2} in terms of I_{REF} and β. Assume all transistors to be matched with current gain β. 10%

(b) Use this idea to design a circuit that generates currents of 1, 2, and 4 mA using a reference current source of 7mA. What are the actual values of the currents generated for $\beta = 50$? 10%



(5). For the basic gate circuit shown in the below, please answer the following questions.

(a) What is the logic family of the circuit shown? 4%

(b) What are the logical functions of $Output_1$ and $Output_2$? 4%

(c) Assuming that the voltage drop across each of D_1, D_2, and the base-emitter junction of Q_1 is 0.75V, calculate the value of V_R. Neglect the base current of Q_1. 4%

(d) As in (c) with the input terminals A and B left open, find the current I_E through R_E. 4%

(e) As in (d) please also find the voltages at the $Output_1$ and $Output_2$. 4%