# 國立中山大學九十一學年度碩士班招生考試試題

科目：工程數學【海下技術研究所碩士班（必考）】



1. (15%) Derive the solution of the following initial-valued problem (IVP):

$$\begin{aligned} \frac{dy}{dx} + f(x)y &= g(x), \quad x > 0 \\ y(0) &= y_0 \end{aligned}$$

where $f(x)$ and $g(x)$ are continuous functions, and $y_0$ is a constant. Express your answer in integral form.

2. (15%) Find the general solution of the following nonhomogeneous ODE:

$$\frac{d^2y}{dx^2} + 4y = 5\csc x$$

3. (15%) Solve the following IVP:

$$\begin{aligned} \frac{d^2y}{dt^2} + 4y &= \begin{cases} 0, & 0 \le t < 5 \\ \frac{t-5}{5}, & 5 \le t < 10 \\ 1, & t \ge 10 \end{cases} \\ y(0) &= 0 \\ y'(0) &= 0 \end{aligned}$$

4. (15%) Solve/answer the following problems:

   (a) Define the surface (flux) integral of a vector function $\mathbf{F}(\mathbf{r})$ over a piecewise smooth surface $S$ with a unit normal vector $\mathbf{n}$. [5%]

   (b) Compute the flux of water passing through the parabolic cylinder $\{S : y = x^2, 0 \le x \le 2,\ 0 \le z \le 3\}$, if the velocity field $\mathbf{F}$ is given by: $\mathbf{F} = y\,\mathbf{i} + 2\,\mathbf{j} + xz\,\mathbf{k}$. [10%]

5. (20%) Consider the problem of vibrations in a circular membrane of radius $a$. Let $u(r,t)$ denote the vertical displacement of the membrane, and if the initial conditions are circularly symmetric, then the mathematical formulation of the problem is as follows:

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= \frac{c^2}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \ t > 0, \ r < a \\ u(a,t) &= 0 \\ u(r,0) &= f(r) \\ \frac{\partial u}{\partial t}(r,0) &= g(r) \end{aligned}$$

Solve $u(r,t)$ in terms of $f(r)$, $g(r)$, where $c$ is a constant.

6. (20%) Answer/solve the following problems:

   (a) In complex analysis, what is the Cauchy-Goursat theorem? what is the residue theorem? [6%]

   (b) For a function of complex variable $f(z)$, what is a pole of order $m$ for $f(z)$? What is a removable pole? What is an essential singularity? Give examples. [6%]

   (c) Use the residue theorem to evaluate the following integral:

$$I = \int_C z^3 e^{1/z} dz$$

where $C$ is a contour of a circle of radius 3 centered at the origin. [8%]

1. (10%) Answer the following questions:
   (a) What is a fluid flow? What is a wave? What are the differences between them? (5%)
   (b) What is the Reynolds stress? What is its expression in terms of velocity components? (5%)

2. (15%) A curved surface is formed as a circular arc with radius $R = 0.75$ m as shown in Figure 1. The width of the surface is $w = 3.55$ m. Water stands to the right of the curved surface to depth $H = 0.65$ m. Calculate the vertical as well as the horizontal hydrostatic forces on the curved surface; Also, determine their corresponding line of action of these forces.

3. (15%) Show that, for an inviscid and incompressible fluid flow, if an initial vorticity exists in the flow, then the vorticity remains constant. Explain why. [Hint: consider Euler equation for inviscid and incompressible fluid]

4. (15%) The velocity potential for a particular two-dimensional flow, in which the density is uniform, is given by $\phi = (-3x+5z)\cos(2\pi t/T)$, where $z$ axis is oriented vertically upward, and $T$ is a constant. Is the flow irrotational? Why? Is the flow nondivergent? Why? If so, derive the stream function.

5. (15%) For steady, incompressible, and two-dimensional flow, neglecting the gravity, the momentum equation for the Prandtle boundary-layer is:
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$
where $u$, $v$, $\rho$, $p$, $\nu$ are, respectively, $x$-component velocity, $y$-component velocity, fluid density, pressure, and kinematic viscosity. Using $L$ and $V_0$ as characteristic length and velocity, respectively, non-dimensionalize the equation, identify the similarity parameters, and give physical interpretation of these parameters.

6. (15%) The tank, with initial mass $M_0$ as shown in Figure 2, rolls along a level track. Water received from a jet is retained in the tank. The tank is to accelerate from rest toward the right with constant acceleration $a$. Neglect wind and rolling resistance, find an algebraic expression for the force (as a function of time) required to maintain the tank acceleration at constant $a$.

7. (15%) Refer to Figure 3. To model the velocity distribution in the curved inlet section of a wind tunnel, the radius of curvature of the streamlines is expressed as $R = LR_0/(2y)$. As a first approximation, assume the air speed along each streamline is $V = 20$ m/sec. Evaluate the pressure change from $y = 0$ to the tunnel wall at $y = L/2$, if $L = 150$ mm, and $R_0 = 0.6$ m.

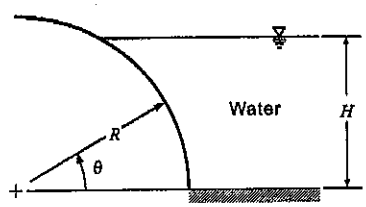



Figure 1

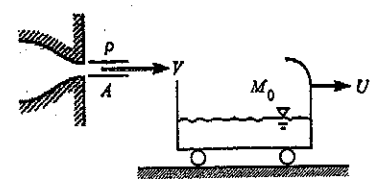



Figure 2

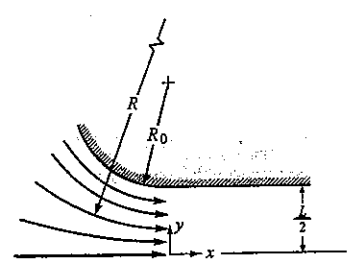



Figure 3

# 國立中山大學九十一學年度碩士班招生考試試題

科目：應用力學【海下技術研究所碩士班（選考）】



注意：本試卷一共有六題，請挑選其中四題作答。每題25分，滿分100分。答案卷上若出現四題以上的作答，批改時將以前四題計分。

1. 如圖1有三艘材質均勻的竹筏靠在岸邊，重量分別爲A, 130Kg, B,100Kg 及C, 70Kg。假設某人體重爲70Kg，從岸上P點登上竹筏A的一端之後，往另尾端走 至盡頭後，登上竹筏B。在竹筏B上向尾端行進至盡頭後在登上竹筏C向尾端行進。請問最後他的位置距離P點多遠(以橫方向計算即可)?。

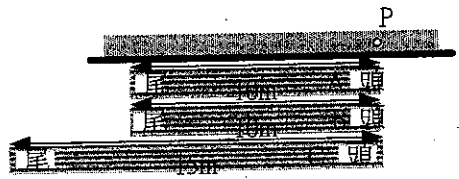


圖 1

2. 有一彈簧$K$如圖2.1所示，如果一質量爲$m$的法碼從平衡點處釋放，請問不計彈簧重量時法碼下降最低點爲何（依自然長度位置爲準）？
有兩彈簧$K_1$及$K_2$如下圖2.2所示，兩彈簧相連時是自然長度的狀態。請問從自然長度位置將質量爲$m$之砝碼向右位移$d$後釋放，請問法碼的最大速率爲多少。（接觸面光滑，無摩擦力）

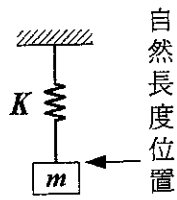


圖 2.1

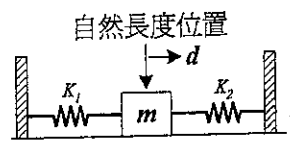


圖 2.2

3. 有一個質量遊樂設施，讓遊客坐在原盤上轉，如圖3所示。請問在這一瞬間，遊客A與遊客B的相對速度及相對加速度各爲多少？

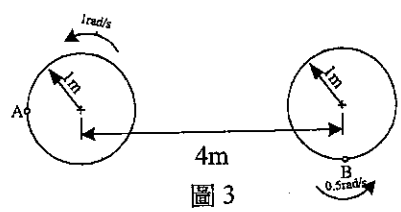


圖 3

4.兩塊堆疊的楔形塊，其中 $B_1$ 的重量爲 50 kg，$B_2$ 則是 200 kg，而其邊界的摩擦係數如圖 4 所示。請求出最小的推力 **P**，其大小剛好可推動 $B_2$ 開始往上滑動。

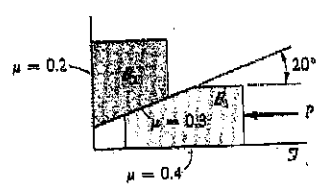


圖 4

5.請求解圖 5 桁架中，每一段構件的受力。

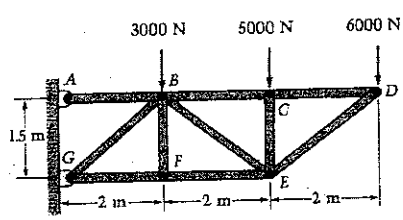


圖 5

6.在某一瞬間，圖 6 中的滾動圓柱具有順時鐘方向的角速度 $\omega = 2$ rad/s，且其逆時鐘方向的角加速度則爲 $\alpha = 1.5$ rad/s$^2$。請計算點 N 及 E 的速度與加速度。

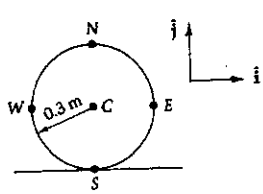


圖 6

# 國立中山大學九十一學年度碩士班招生考試試題

科目：電子學【海下所碩士班】選考

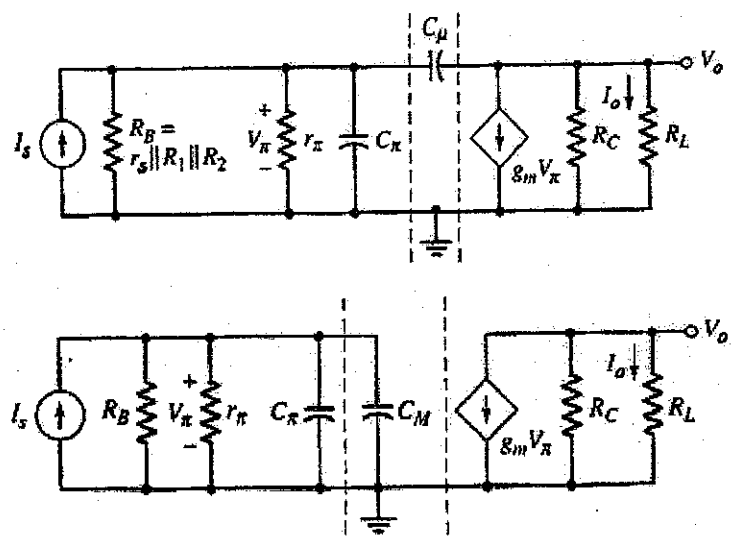


(1).

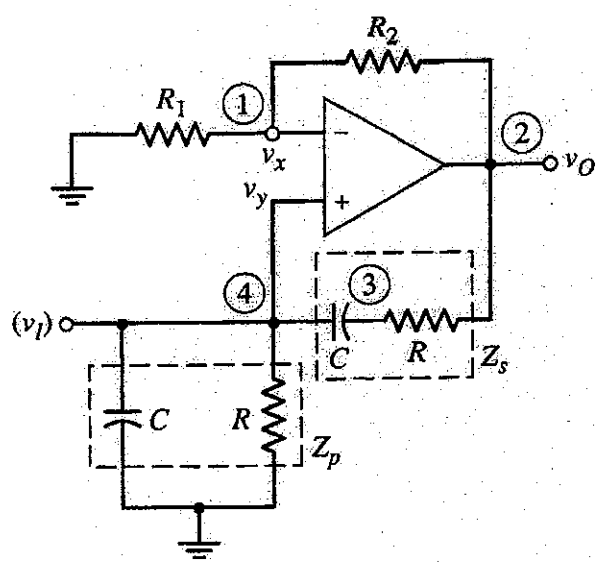
(a) Draw the current flow in a *pnp* BJT transistor biased to operate in the active mode and explain the Early effect with respect to $\alpha$ and $\beta$. (10%)

(b) Draw the current flow in a PMOS transistor biased to operated in saturation region and explain the channel length modulation effect. (10%)

(2). In Figure 2 the small signal equivalent of a bipolar junction transistor with simplified hybrid – $\pi$ model and including the equivalent Miller capacitance are shown respectively, where the circuit parameters are $R_C = R_L = 4\ \text{k}\Omega$, $r_\pi = 2.6\ \text{k}\Omega$, $R_B = 200\ \text{k}\Omega$, $C_\pi = 4$ pF, $C_\mu = 0.2$ pF, and $g_m = 38.5$ mA/V. Please determine the 3 dB frequency of the current gain for the circuit both with and without the effect of $C_M$. (20%)




(3). Design a Wien-bridge circuit as shown on the right to oscillate at a specified frequency $f_o = 20$ kHz. please analyze it (10%), then decide the values of capacitors and resistors (10%).

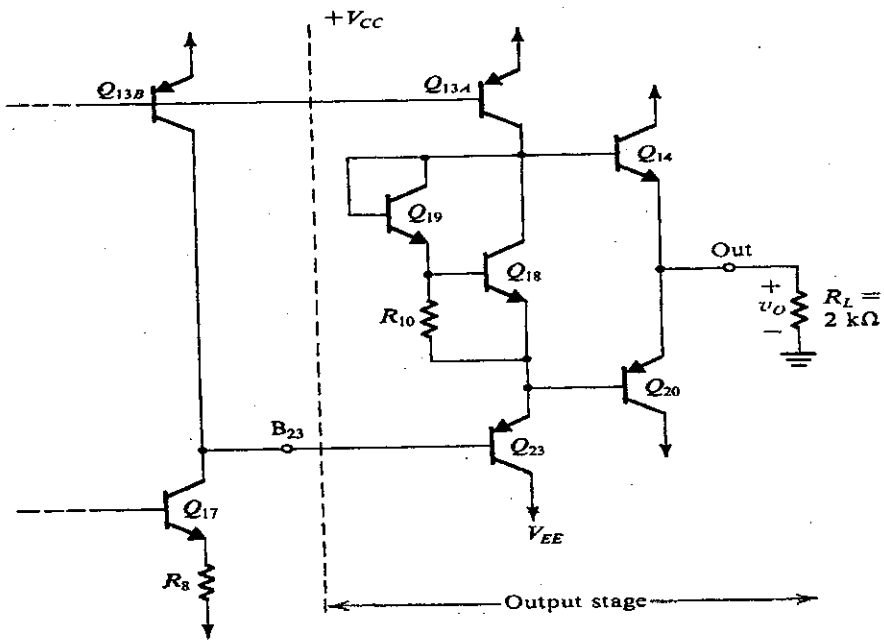


(4).

(a) What is the circuit shown in Figure 4? (4%)

(b) What is the maximum positive output voltage $v_{omax}$ of the circuit and what situation will limit this value? (8%)

(c) What is the minimum output voltage (i.e. maximum negative amplitude) $v_{omin}$ of the circuit and what situation will limit this value? (8%)




(5).

(a) For the circuit in Figure 5, find $I_{O1}$ and $I_{O2}$ in terms of $I_{REF}$. Assume all transistors to be matched with current gain $\beta$. (10%)

(b) Use this idea to design a circuit that generates currents of 1 and 4 mA using a reference current source of 7mA. What are the actual values of the currents generated for $\beta = 50$? (10%)