

# 國立中山大學九十學年度碩博士班招生考試試題

科目：工程數學【海下所碩士班】

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1. (10%) Derive the general solution of the following non-homogeneous ODE:

$$y'' + 4y = g(x)$$

Express the answer in terms of integral.

2. (10%) Consider the following ODE:

$$y'' - 4y' + 4y = 3t^2 + 5te^{2t} + t \cos t$$

Determine a *suitable form* for the particular solution, if the method of undetermined coefficients is to be used. (Note: You don't need to find the unknown coefficients)

3. (15%) Find the general solution of the following ODE:

$$x^2 y'' - 3xy' + 4y = 2 \ln x$$

4. (15%) Find the value of  $c$  so that the solution of the following initial value problem remains finite as  $t \rightarrow \infty$ :

$$\frac{dy}{dt} - y = 1 + 3 \sin t, \quad y(0) = c$$

5. (15%) Find the closest point on the plane  $Ax + By + Cz + D = 0$  to the point  $(x_0, y_0, z_0)$  in space. Formulate this problem as a constrained minimization problem.

6. (15%) Apply contour integration to evaluate the following improper integral:

$$\int_0^{\infty} \frac{3}{x^3 + 1} dx$$

Justify each step in the evaluation process.

7. (20%) Solve the following diffusion equation:

$$\begin{aligned} \frac{\partial T}{\partial t} &= \frac{\partial^2 T}{\partial x^2} + 3e^{-t} \sin(3x), \quad 0 < x < \pi, \quad t > 0 \\ T(0, t) &= 0, \quad t > 0 \\ T(\pi, t) &= 1, \quad t > 0 \\ T(x, 0) &= x, \quad 0 < x < \pi \end{aligned}$$

# 國立中山大學九十學年度碩博士班招生考試試題

科目：流體力學【海下所碩士班】選考

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1. (20%) A gate of width 35 m is used to control water flow from a dam as shown in Figure 1. Determine the magnitude, direction, and line of action of the force from the water acting on the gate.
2. (20%) Figure 2 shows that water flows at low speed through a circular tube with inside diameter of 50 mm, and a smoothly contoured plug of 40 mm diameter is held in the end of the tube where the water discharges to atmosphere. Neglecting frictional effects and assuming uniform velocity profiles at each section, determine the pressure measured by the gate and the force required to hold the plug.
3. (20%) The power  $\mathcal{P}$  required to drive a fan is believed to depend on fluid density  $\rho$ , volume flow rate  $Q$ , impeller diameter  $D$ , and angular velocity  $\omega$ . Use the dimensional analysis to determine the dependence of  $\mathcal{P}$  on the other parameters.
4. (20%) A viscous-shear pump is made from a stationary housing with a close-fitting rotating drum inside as shown in Figure 3. The clearance is small compared to the diameter of the drum, so flow in the annulus space may be treated as flow between parallel plates. Fluid is dragged around the annulus by viscous forces. Evaluate the performance characteristics of the shear pump, including pressure differential, input power, and efficiency, as functions of the volume flow rate. Assume the depth normal to the diagram is  $b$ .
5. (20%) Consider a flow field with mass generation  $Q(r)$  per unit volume per unit time, derive the general governing equation of mass conservation (i.e., continuity equation) for this flow field, in terms of density  $\rho(r, t)$ ,  $v(r, t)$ , and  $Q(r)$ . And then reduce the equation to one that satisfies the following conditions, respectively:
  - (a) if the flow is steady state.
  - (b) if the flow is incompressible.
  - (c) if the flow is spatially uniform.

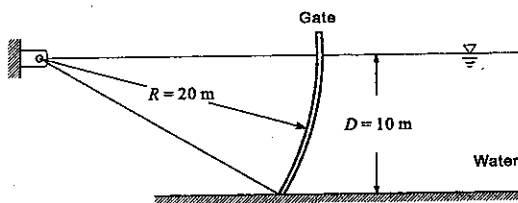


Figure 1

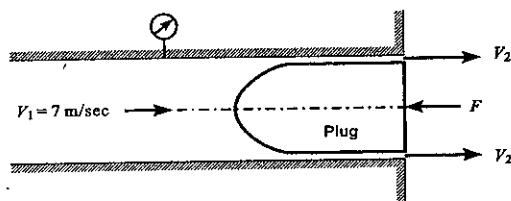


Figure 2

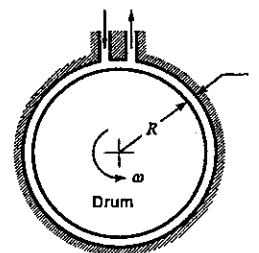


Figure 3

# 國立中山大學九十學年度碩博士班招生考試試題

科目：應用力學【海下所碩士班】選考

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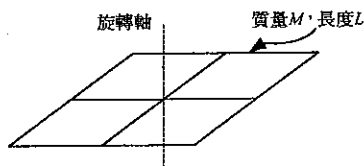
說明：本試卷共六題，六題之中請任選四題作答，至多不得超過四題。若答題超過四題，閱卷時將依答題次序僅評閱前四題，超過四題部分不予計分。每題 25 分，總分 100 分。

1. (a) (5%) 一支材質均勻，長度  $L$ ，質量  $M$  的細棒，若其旋轉軸選在質心  $O$  點上，由旋轉慣量的定義為

$$I_o = \int_M x^2 dm$$

請證明此細棒的旋轉慣量  $I_o = \frac{1}{12} L^2 M$ 。

- (b) (10%) 平行軸定理：當旋轉軸從質心平移至另一點  $P$  時，則旋轉慣量可表示為  $I_p = I_o + Md^2$ ，其中  $d$  為  $P$  點到質心的距離。藉由此定理，請計算圖中由 6 根長度  $L$ 、質量  $M$  的細棒組成的“田”字形狀，以中心為旋轉軸的旋轉慣量。



- (c) (10%) 如果此“田”字從靜止起，受到一個施在旋轉軸的力偶  $T$  持續  $t$  秒鐘，請問系統的動能為多少？
2. (25%) 下列何者為保守力場，請檢證。如為保守力場其位能場又為何？
- (a)  $F_x = y \cos(xy) + y^3, F_y = x \cos(xy) + 3xy^2$
- (b)  $F_x = e^y, F_y = e^x$
- (c)  $F_x = e^x + y, F_y = e^y + x$

3. (25%) 請計算出下列外力及力偶在原點的等效合力及合力矩。

$F_1 = (50i + 40j - 30k)N$  施在  $A = (-2i + 3j + 4k)m$

$F_2 = (20i - 25j + 120k)N$  施在  $B = (3i + 1j + 2k)m$

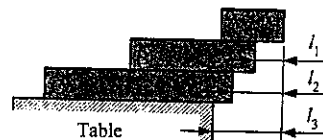
$F_3 = (-15i + 15j + 50k)N$  施在  $D = (3i + 1j + 2k)m$

$C_1 = (-30i + 35j + 20k)N \cdot m$

$C_1 = (-30i + 35j + 20k)N \cdot m$

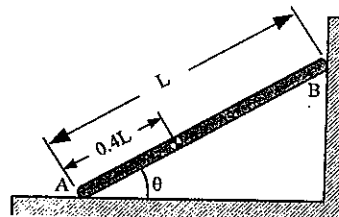
4. 有三塊編號分別為 A、B、C 的木板依照圖右的上下次序疊在桌上，這三塊木板的長度分別為  $L$ 、 $2L$ 、 $3L$ ，其重量則分別為  $W$ 、 $2W$ 、 $3W$ ，請問：

- (a) (6%) 若木板 B 與 C 固定於桌面，木板 A 在不發生翻落的情況下所能伸出木板 B 邊緣的最遠距離為何？  
 (b) (9%) 若木板 C 固定於桌面，在木板 A 與 B 皆不會翻落的情況下，木板 A 能伸出木板 C 邊緣的最遠距離為何？  
 (c) (10%) 若三塊木板皆不與桌面固定，則在這三塊木板皆不翻落的情況下，木板 A 能伸出桌緣的最遠距離是多少？



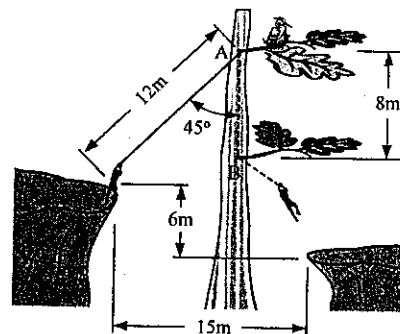
5. 有一長度  $L$  的梯子斜倚在牆上，如右圖所示，其中梯子與牆壁之間的摩擦係數在 A 與 B 處分別為  $\mu_A$  與  $\mu_B$ 。梯子的重量為  $W$ ，梯子重心位於距離梯子底部  $0.4L$  處。

- (a) (15%) 利用上述參數求出梯子不會滑落的最小角度  $\theta$ 。  
 (b) (10%) 當  $W=10\text{kg}$ 、 $L=3\text{m}$ 、 $\mu_A=0.2$ 、 $\mu_B=0.4$  時，梯子不會滑落的最小角度  $\theta$  為多少？



6. 泰山體重  $100\text{kg}$ ，想藉由藤蔓盪到另一邊懸崖上，如右圖所示，藤蔓的支撐點 A 到泰山質量中心的距離為  $12\text{m}$ 。請問：

- (a) (5%) 當藤蔓碰到 B 點時，根據能量守恆原理，計算泰山的速度為何？  
 (b) (5%) 藤蔓幾乎碰到 B 點的瞬間，藤蔓所承受的張力是多少？（同時考慮重力與離心力的作用，曲率半徑  $12\text{m}$ 。）  
 (c) (5%) 藤蔓碰到 B 點後的瞬間，藤蔓所承受的張力又是多少？  
 (d) (10%) 若藤蔓能承受的最大張力為  $2500\text{N}$ ，泰山有沒有辦法盪到另一邊懸崖上？請說明理由。



國立中山大學九十學年度碩博士班招生考試試題

科目：電子學【海下所碩士班】選考

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1. Assume each diode in the circuit shown in Figure 1 has a cut-in voltage of  $V_r = 0.65\text{V}$ . The input voltage is  $V_i = 5\text{V}$ . Determine the value of  $R_1$  required such that  $I_{D1}$  is one-half the value of  $I_{D2}$ . What are the values of  $I_{D1}$  and  $I_{D2}$ ? (20%)
2. (a) For the circuit shown in Figure 2, assume  $\beta = 75$ . Determine  $I_{BQ}$ ,  $I_{CQ}$ , and  $V_{ECQ}$ . (b) Determine the value of  $I_{BQ}$ ,  $I_{CQ}$ , and  $V_{ECQ}$  if  $\beta = 100$ . (20%)
3. For the common-emitter circuit shown in Figure 3, assume the emitter bypass capacitor  $C_E$  is very large, and the transistor parameters are:  $\beta_o = 100$ ,  $V_{BE(\text{on})} = 0.7\text{V}$ ,  $V_A = \infty$ ,  $C_\mu = 2\text{pF}$ , and  $f_T = 400\text{MHz}$ . Determine the lower and upper 3dB frequencies for the small-signal voltage gain. Use the simplified hybrid- $\pi$  model for the transistor. (20%)
4. For the Widlar current source shown in (Figure 4), the parameters are:  $R_1 = 7\text{k}\Omega$ ,  $R_{E2} = 5\text{k}\Omega$ , and  $R_{E3} = 2\text{k}\Omega$ . Assume  $V_{BE1(\text{on})} = 0.7\text{V}$ . Determine  $I_{REF}$ ,  $I_{O2}$ , and  $I_{O3}$ . (20%)
5. The differential amplifier shown in (Figure 5) has a pair of pnp bipolars as input devices and a pair of npn bipolars connected as an active load. The circuit bias is  $I_{Q_0} = 0.2\text{mA}$ , and the transistor parameters are  $\beta = 100$  and  $V_A = 100\text{V}$ .
  - (a) Determine  $I_0$  such that the dc currents in the diff-amp are balanced. (7%)
  - (b) Find the open-circuit differential-mode voltage gain. (7%)
  - (c) Determine the differential-mode voltage gain if a load resistance  $R_L = 250\text{k}\Omega$  is connected to the output. (6%)

國立中山大學九十學年度碩博士班招生考試試題

科目：電子學【海下所碩士班】選考

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Fig. 1

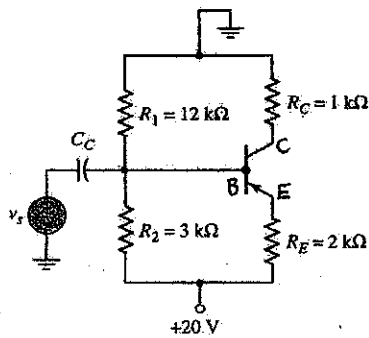
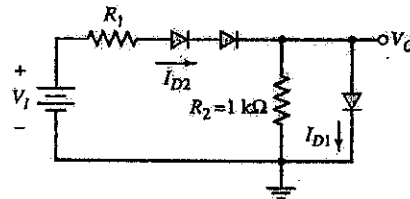


Fig. 2

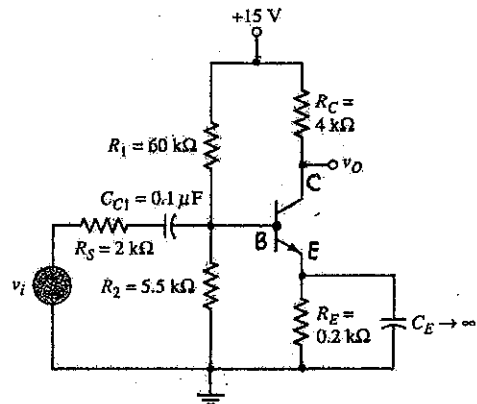


Fig. 3

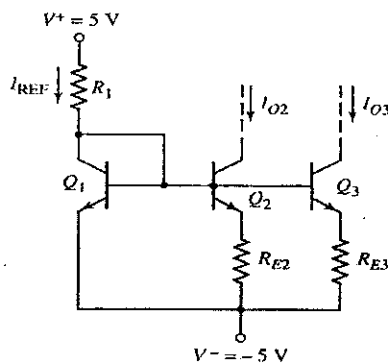


Fig. 4

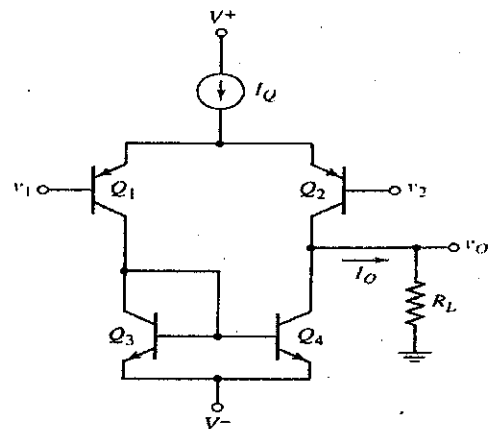


Fig. 5