1. (10%) Solve for the ODE: $y' - 2xy = 1$ with initial condition: $y(0) = 1$, and express your answer in terms of error function defined as $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.

2. (10%) Knowing that x is a solution of the *nonlinear* ODE: $y' = 1 + x^2 - 2xy + y^2$, find a more general solution of the problem.

3. (25%) Consider the spring-mass-dashpot mechanical vibration system shown in Figure 1:

   (a) (5%) Applying the fundamental principles of mechanics, show that the ODE describing the displacement y as function of time t may be expressed as $my'' + by' + Ky = F_0(t)$.

   (b) (5%) Let the external force $F_0(t) = 0$, find the solution of the displacement $y(t)$ for $m = 2$, $b = 1$, $K = 1$. What is the angular frequency of the natural vibration?

   (c) (5%) If now $m = 1$, $b = 0$, $K = 4$, $F_0 = \sin 2t$, find the general solution.

   (d) (10%) Show that if $F_0 = 0$, and $b \neq 0$, then $\lim_{t\to\infty} y(t) = 0$. What is the implication for this case in terms of physics?

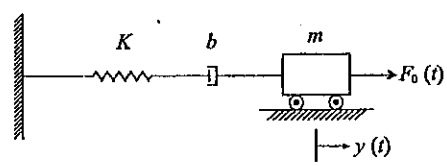


Figure 1: Spring-mass-dashpot mechanical system

4. (40%) According to physical principles, a one-dimensional sound wave described by the sound pressure p in an infinite domain is governed by the wave equation:

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}$$

   where c is the sound speed.

   (a) (10%) Show that *any* function with argument $x - ct$ or $x + ct$ is the solutions of the above PDE. What is the significance of these functions?

   (b) (15%) If the above PDE is subject to the following initial and boundary conditions:

$$\begin{aligned} \text{BC's} \quad & p(0,t) = 0, \quad \frac{\partial p}{\partial x}(1,t) = 0 \\ \text{IC's} \quad & p(x,0) = 0, \quad \frac{\partial p}{\partial t}(x,0) = x \end{aligned}$$

   determine the solution for $p(x,t)$.

   (c) (15%) If the initial and boundary conditions are changed to:

$$\begin{aligned} \text{BC's} \quad & p(0,t) = t, \quad \frac{\partial p}{\partial x}(1,t) = 0 \\ \text{IC's} \quad & p(x,0) = 0, \quad \frac{\partial p}{\partial t}(x,0) = 0 \end{aligned}$$

   determine the solution for $p(x,t)$.

5. (15%) Consider the following two functions:

$$\begin{aligned} y_1 &= f_1(x_1, x_2) = \cos x_1 + \cos(x_1 + x_2) \\ y_2 &= f_2(x_1, x_2) = \sin x_1 + \sin(x_1 + x_2) \end{aligned}$$

   If small perturbation at $x_1 = \pi/4$, $x_2 = \pi/6$ is made, so that the values of the functions varies by $\Delta y_1 = \Delta y_2 = 0.01$, estimate the new values of x_1 and x_2, (Hint: $\Delta y \simeq f'(x)\Delta x$)

1. (20%; 4% each) Answer the following questions as precisely as possible.
   (a) What is Lagragian method of description? What is Eulerian method of description?
   (b) Give the definitions of steady flow, uniform flow, incompressible flow, respectively.
   (c) Give the definitions and state the physical significance of the following dimensionless numbers: Reynolds number, Froude number, Weber number, Strouhal number.
   (d) State the Buckingham Pi theorem.
   (e) What is a fully-developed flow? What is the Reynolds stress?

2. (20%; 10% each) Make necessary assumptions and derive the Bernoulli's equations for (a) steady flow, (b) unsteady flow.

3. (15%) A small grain of sand, with diameter $D = 0.1$ mm and specific gravity $SG = 2.3$, settles to the bottom of a lake after having been stirred up by a passing boat. Determine how fast it falls through the still water, assuming the drag coefficient is $C_D = 24/\mathrm{Re}$, where Re is the Reynolds number. (Dynamic viscosity of water is taken to be 1.12×10^{-3} N-sec/m^2.)

4. (15%) The water level is controlled by a plane gate of uniform thickness as shown in Figure 1. The width of the gate normal to the diagram is $w = 10$ ft. Determine the mass, M, needed to maintain the water level at depth $H = 4$ ft or less, if the mass of the gate is negligible. Other related dimensions are: $r = 2.5$ ft, $R = 7.5$ ft.

5. (15%) A steady jet of water is employed to propel a small cart along a horizontal track as shown in Figure 2. Total resistance to motion of the cart assembly is given by $F_D = kU^2$, where $k = 0.92$ N-sec^2/m^2. Evaluate the acceleration of the cart at the instant when its speed is $U = 10$ m/sec. Related dimensions are: $D = 25$ mm, $V = 30$ m/sec, $\theta = 30°$, $M = 15$ kg.

6. (15%) Consider the problem of air at standard condition flowing over a flat plate as shown in Figure 3. The plate is 1 m long and 0.5 m wide. The flow is uniform at leading edge of the plate. Assume that the velocity profile at the trailing edge bc is parabolic:

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

where $U = 3$ m/s is the free stream, and $\delta = 13$ mm is the boundary-layer thickness at bc. Assume that the flow is two-dimensional, and independent of z, compute the mass flow rate across surface ab, and also determine the magnitude and direction of the x component of force required to hold the plate stationary. (CV stands for Control Volume.)

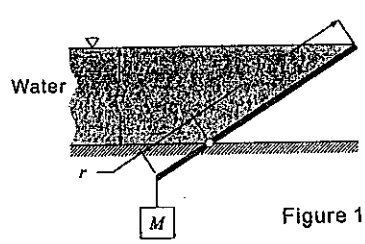


Figure 1

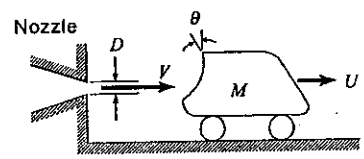


Figure 2

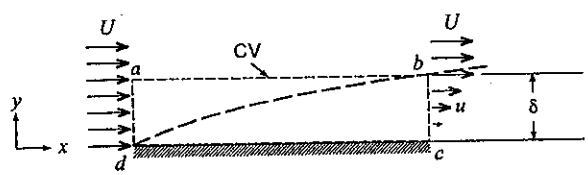


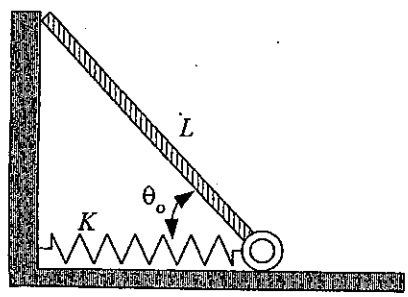
Figure 3

國立中山大學八十九學年度碩博士班招生考試試題

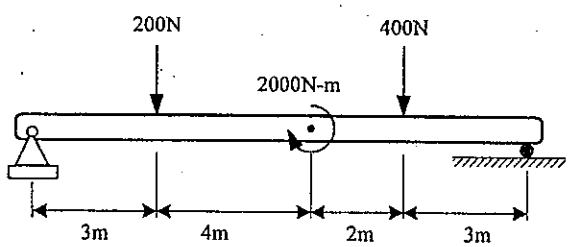
(選考)科　目：　海下技術研究所碩士班　　應用力學　　

說明：本試卷共六題，六題之中請任選四題作答，至多不得超過四題。若答題超過四題，閱卷時將依答題次序僅評閱前四題，超過四題部分不予計分。每題 25 分，總分 100 分。

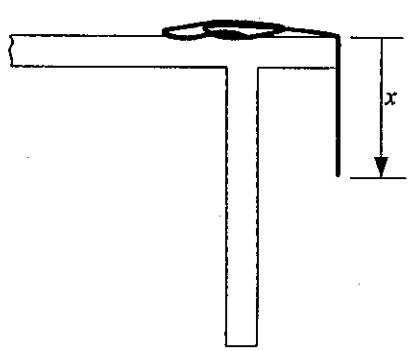
1. (25%) 質量均勻的長桿斜靠光滑壁面及光滑地板後釋放。圖中θ_0為起始角度，此時亦為彈簧的自然長度。（1）請寫出系統的總機械能，（2）列出靜力平衡時，m與θ的關係式為何。（彈簧彈性係數K，長桿質量m）



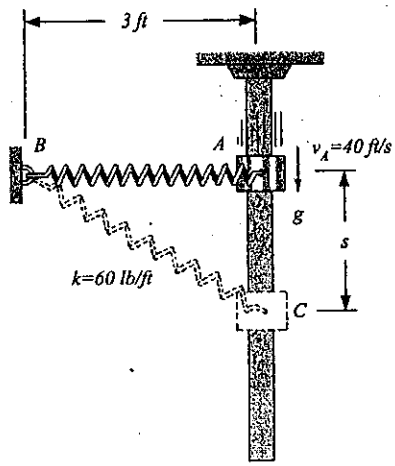
2. (25%) 請畫出下列結構的shear diagram和bending moment diagram.



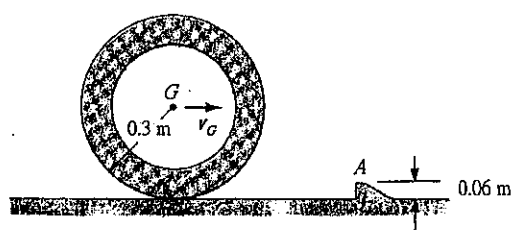
3. (25%) 有一條粗細不均勻的繩子慢慢地從桌緣垂下。繩子的線密度為$\rho(x)=(1.1-e^{-x})$ kg/m，繩子的最大抗張為500N。求此繩在撐斷前之最大之下垂長度。



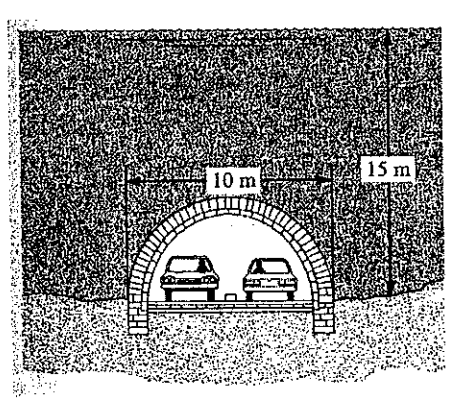
4. (25%) 一重量 5-lb 的套環在位置A的初始速率為 40 ft/s，假設彈簧未受力之原長度為 2 ft，彈簧之彈性係數 k = 60 lb/ft，試求套環在 s = 2 ft 時之速率。不計由 A 至 C 之摩擦力。



5. (25%) 一個 15-kg 的輪子如圖所示，其慣性矩（Moment of inertia）$I_G = 0.2$ kg-m^2， 假設輪子與地面之間的運動為純滾動，試求輪子恰好越過障礙物 A 所需的最小速率 v_G。



6. (25%) 具 10 m 寬之半圓形海底隧道位於水深 15 m 處，如圖所示。假設海水密度為 ρ_w = 1000 kg/m^3，試求沿隧道長度方向每 1 公尺長隧道所承受的海水靜力（Hydrostatic force）合力大小。



15%

1. A full-wave rectifier and an input voltage are shown in (Figure 1). The cut-in voltage of each diode is $V_\gamma = 0.6$ V. (a) Sketch the output voltage versus time. (b) The average power dissipated in R_L is to be $\overline{P}_L = 100$ mW. Determine the correct value of R_L.

15%

2. For the circuit shown in (Figure 2), if $\beta = 200$ for each transistor, determine : (a) I_{E1} (b) I_{E2} (c) V_{C1} ,and (d) V_{C2}.

20%

3. The transistor parameters for the circuit shown in (Figure 3) are $\beta = 180$ and $V_A = \infty$. (a) Find I_{CQ} and V_{CEQ}. (b) Calculate the small-signal voltage gain. (c) Determine the input and output resistances R_{ib} and R_o.

15%

4. For the op-amp circuit shown in (Figure 4), determine the gain $A_v = v_O/v_I$. Compare this result to the gain of the circuit shown in (Figure 5), assuming all resistor values are equal.

15%

5. For the Widlar current source shown in (Figure 6), the parameters are : $R_1 = 7$ kΩ, $R_{E2} = 5$ kΩ, and $R_{E3} = 2$ kΩ. Assume $V_{BE1}(\text{on}) = 0.7$ V. Determine I_{REF}, I_{O2}, and I_{O3}.

20%

6. The differential amplifier shown in (Figure 7) has a pair of pnp bipolars as input devices and a pair of npn bipolars connected as an active load. The circuit bias is $I_Q = 0.2$ mA, and the transistor parameters are $\beta = 100$ and $V_A = 100$ V.

(a) Determine I_0 such that the dc currents in the diff-amp are balanced.
(b) Find the open-circuit differential-mode voltage gain.
(c) Determine the differential-mode voltage gain if a load resistance $R_L = 250$ kΩ is connected to the output.

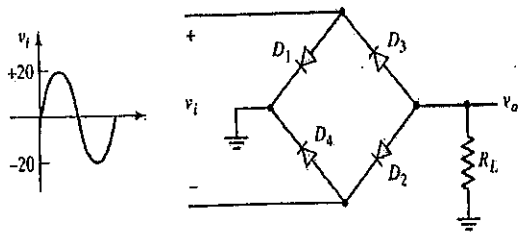


Figure 1

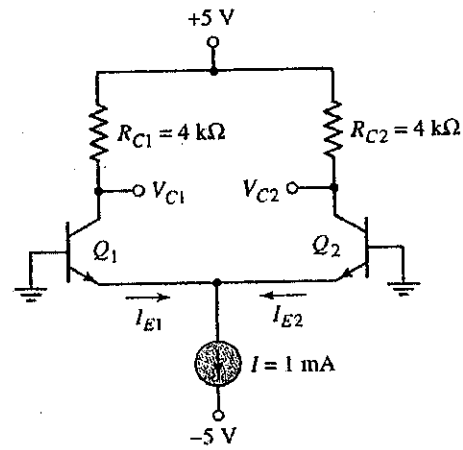


Figure 2

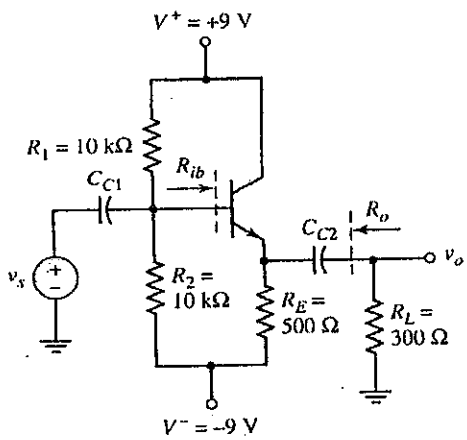


Figure 3

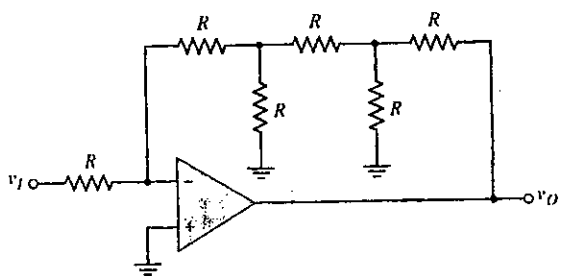


Figure 4

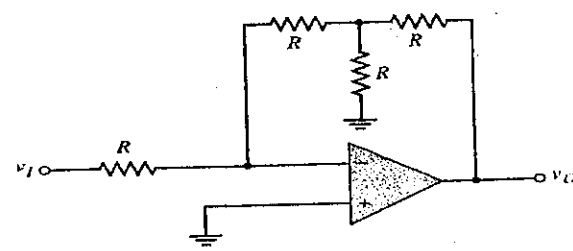


Figure 5

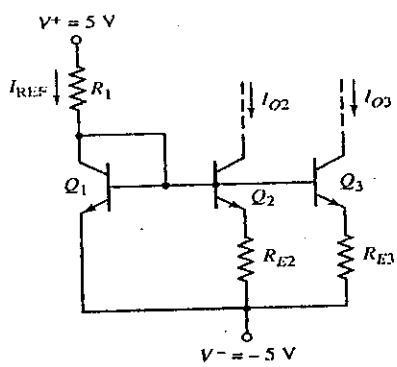


Figure 6

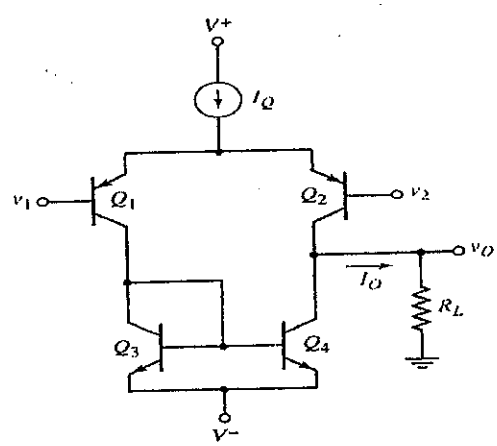


Figure 7