國立中山大學八十八學年度碩博士班招生考試試題

科 目：工程數學（海下技術研究所碩士班）

1. (30%)

   (a) If the roots of the characteristic (auxiliary) equation corresponding to a 7th-order homogeneous ODE with constant coefficients are: $3,\ 2\pm 3i,\ 2\pm 3i,\ 2\pm 3i$, write down the general solution. (5%)

   (b) Determine the solution of the following Initial Value Problem (IVP) for $x > 0$: (5%)

   $$y'' + x\,y' + (\sin x)\,y = 0,\ y(0) = 0,\ y'(0) = 0$$

   (c) Consider the non-homogeneous Bessel's differential equation:

   $$x^2y'' + xy' - \left(x^2 - \frac{1}{4}\right)y = g(x),\ x > 0$$

   Knowing that $\frac{\sin x}{\sqrt{x}}$ is a homogeneous solution, solve the ODE. (you may express your answer in terms of integral.) (10%)

   (d) Solve the integral equation: (10%)

   $$y(t) + \int_0^t (t-\xi)y(\xi)\,d\xi = 2\sin 2t$$

   (Hint: Laplace transform)

2. (35%) Consider the following partial differential equation (heat/diffusion equation):

$$\begin{aligned}
\text{(PDE)}\quad \frac{\partial u}{\partial t} &= \frac{\partial}{\partial x}\left(e^x\frac{\partial u}{\partial x}\right) - 2xu,\ 0<x<1,\ t>0\\
\text{(BC1)}\quad u(0,t) &= 0\\
\text{(BC2)}\quad \frac{\partial u(1,t)}{\partial x} &= 0\\
\text{(IC)}\quad u(x,0) &= f(x),\ 0<x<1
\end{aligned}$$

Perform a separation of variables to arrive at the following eigenvalue problem:

$$\begin{aligned}
\frac{d}{dx}\left(e^x\frac{d\phi}{dx}\right) - 2x\phi + \lambda\phi &= 0,\ 0<x<1\\
\phi(0) &= 0\\
\frac{d\phi(1)}{dx} &= 0
\end{aligned}$$

   (a) Show that all the eigenvalues λ's are *real* and *positive* numbers. (15%)

   (b) Show that the eigenfunctions corresponding to the different eigenvalues are orthogonal. (10%)

   (c) It may be shown (you don't need to show this) that the collection of all the eigenfunctions $\phi_n(x)$ of the above eigenvalue problem forms a complete set, meaning that any piecewise continuous function $f(x)$ may be expressed as $f(x) = \sum_{n=1}^{\infty} a_n\phi_n(x)$. This is referred to as eigenfunction expansion. Now, solve $u(x,t)$ in terms of eigenfunctions and any other relevant functions. (10%)

3. (15%) Consider the matrix: $A = \begin{pmatrix} 9 & 1 & 1\\ 1 & 9 & 1\\ 1 & 1 & 9\end{pmatrix}$. Find a transformation matrix T so that $T^{-1}AT = D$, where D is a diagonal matrix. Also, find the matrices T^{-1} and D.

4. (20%) Evaluate the following integral by contour integration:

$$\int_0^\infty \frac{3x^2-2}{x^4+5x^2+4}\,dx$$

Justify each step in your evaluation, in particular, show that

$$\lim_{R\to\infty}\int_{C_R}\frac{3z^2-2}{z^4+5z^2+4}\,dz = 0$$

where C_R is a semi-circle of radius R on the upper-half complex z-plane.

(橫書式)　國立中山大學八十八學年度碩博士班招生考試試題

科　目：流體力學（海下技術研究所碩士班　選考）　　共1頁 第1頁

1. (20%) Answer the following questions: (4% for each problem)

   (a) What is a streamline? What is a streakline? What is a pathline?

   (b) What is a steady flow? What is an incompressible flow? What is a uniform flow?

   (c) What is the Eulerian description of flow? What is the Lagrangian description of flow?

   (d) Define the following dimensionless parameters and give their physical significance: Reynold number, Froude number, Weber number.

   (e) Consider a flow over a flat plate. State the boundary conditions for viscous and inviscid fluid, respectively.

2. (20%) A semi-spherical viewing window (半球體觀看窗) of radius 0.8 m is installed at depth 3 m in the side of a water tank filled with fresh water, as shown in Figure 1. Evaluate the magnitudes of the vertical and horizontal forces of the water acting on the viewing window.

3. (20%) Refer to Figure 2. Consider a pair of circular disks of radius R, separated by a small gap h. An inviscid, incompressible fluid of density ρ is injected into the gap at a volume flow rate Q from the center of the disks and is discharged to atmosphere. Assume that the fluid flowing away from the center has only radial motion, and the flow is uniform across any vertical section. Obtain an expression for the pressure variation as a function of radius r, i.e., $p(r)$.

4. (25%) For an inviscid and incompressible fluid, the fluid motion may be described by the Euler equation:

$$\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho}\nabla p - g\mathbf{k}$$

where $\mathbf{v}$ is the velocity vector, ρ is the density, p is the pressure, and g is the gravitational constant. $\frac{D}{Dt}$ stands for material derivative, and $\mathbf{k}$ is unit vector in z coordinate, which points upward.

   (a) Starting from the Euler equation, derive the Bernoulli's equation for irrotational flow:

$$-\frac{\partial\phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2\right] + \frac{p}{\rho} + gz = c(t)$$

   where ϕ is the velocity potential: $\mathbf{v} = -\nabla\phi$. (15%)

   (b) Show that, for an inviscid and incompressible fluid flow, if the flow is irrotational initially, then it remains irrotational all the time. (10%)

5. (15%) The phase speed v of a free-surface gravity wave in deep water is a function of the wavelength λ, depth d, density ρ, and the acceleration of gravity g. Use dimensional analysis to find the functional dependence of v on the other variables. Express v in the simplest form possible.

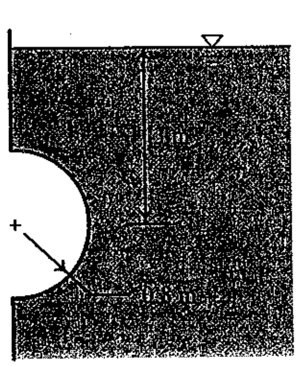


Figure 1

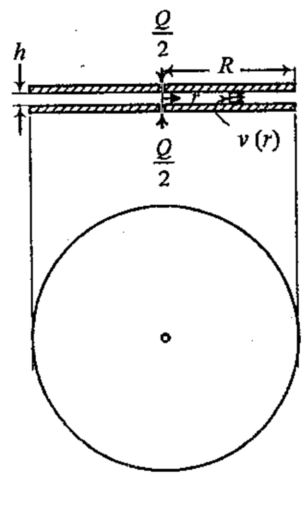


Figure 2

1. Find the maximum force P for which no slipping will occur anywhere for the system of blocks in Figure 1. (25%)

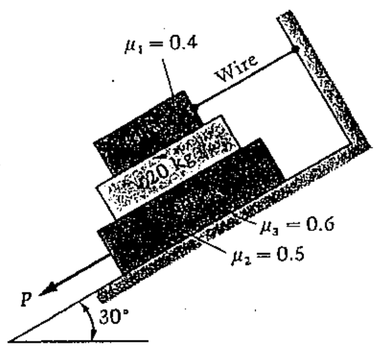


Figure 1 System of blocks.

2. At the instant shown in Figure 2, the velocity of point A is 0.2 m/s to the right. Find the angular velocity of rod C, and determine the velocity of its other end (point B), which is constrained to move in the circular slot. (25%)

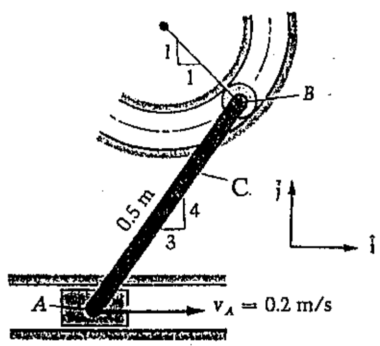
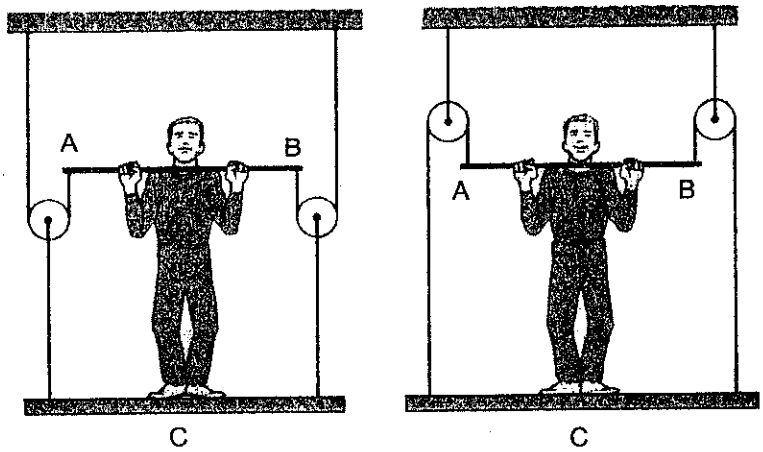


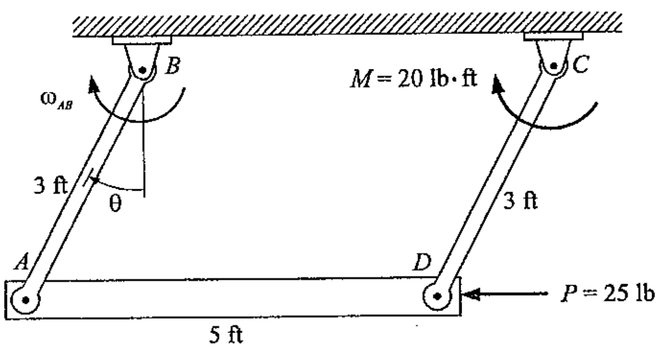
Figure 2 Moving rod.

3. A man having a weight of 200-lb attempts to lift himself using one of the two methods shown. The platform has a weight of 40-lb.(25%)
   (1) Sketch the free-body diagrams for each case.
   (2) Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C.



(a) (b)

4. The linkage consists of two 10-lb rods AB and CD and a 15-lb bar AD. When $\theta = 0^\circ$, rod AB is rotating with an angular velocity $\omega_{AB} = 2$ rad/sec. If rod CD is subjected to a couple moment $M = 20\ \text{lb}\cdot\text{ft}$ and bar AD is subjected to a horizontal force $P = 25\ \text{lb}$ as shown, determine ω_{AB} at the instant $\theta = 90^\circ$. (25%)



1. (a)Sketch the voltage transfer characteristic (V_0 versus V) for the circuit in Fig.1, assuming that the diode is ideal. (6%)
(b)Sketch the output voltage $V_0(t)$ in the circuit for $0 \leqq t \leqq 5$ ms. (6%)

2. The low-frequency small-signal parameters for the transistor in the common-collector circuit of Fig.2 are g_m=40 m℧, β_0=150, $r_0 \to \infty$, and r_b= 0.
(a)Draw the small-signal equivalent of this stage. (4%)
(b)Determine R_{in} and R_o. (4%)
(c)Evaluate the transfer function V_0/Vs. (4%)

3. (a)Sketch the cross section of an NMOS enhancement transistor. (4%)
(b)Sketch the output and transfer characteristics of an NMOS enhancement transistor. (4%)
(c)Qualitatively explain the shape of the characteristics in (b). (4%)

4. (a)For the circuit shown in Fig.3, verify that $Y=\overline{ABC}$. (4%)
(b)If β_F=25, what is the fan-out? (4%)
(c)What is the average power dissipated by the gate assuming Y= V(1) 50 percent of the time? (4%)

5. Transistor Q1 has r_{d1}=10 kΩ and g_{m1} =3 m℧; Q2 has r_{d2}=15kΩ and g_{m2}= 2m℧. (Fig.4)
(a)Find the gain V_0/V_2 for V_1=0. (7%)
(b)Find the gain V_0/V_1 for V_2=0. (7%)

6. Consider the voltage-to-current converter shown in Fig.5. Let Z_L=100Ω, R_1=10kΩ, R_2=1kΩ, and R_F=10kΩ. If V_I= −5V, determine the load current i_L and the output voltage V_0. (14%)

7. Consider a power MOSFET for which the thermal resistance parameters are: $\theta_{dev\text{-}case}$= 1.75 ℃/W, $\theta_{case\text{-}sink}$=1 ℃/W, $\theta_{sink\text{-}amb}$= 5 ℃/W, and $\theta_{case\text{-}amb}$ =50 ℃/W. The ambient temperature is T_{amb}=30℃, and the maximum junction or device temperature is $T_{j,max}$ =T_{dev}=150℃. Determine the maximum power dissipation in a transistor with and without a heat sink. (12%)

8. Consider the two-pole low-pass Butterworth filter in Fig.6. Design the circuit such that f_{3db}=20 kHz. (12%)

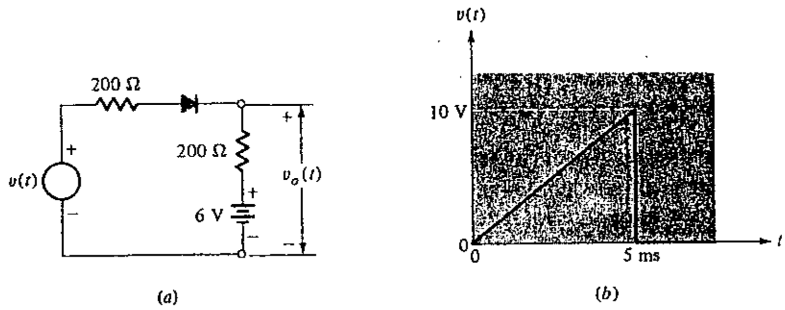


Fig.1

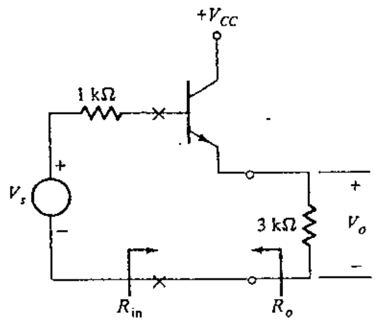


Fig.2

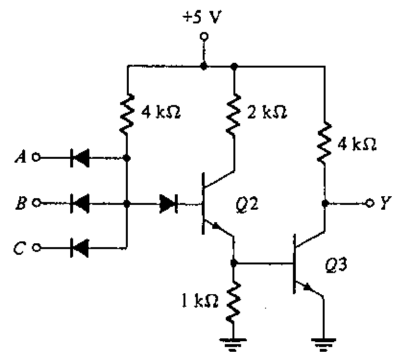


Fig.3

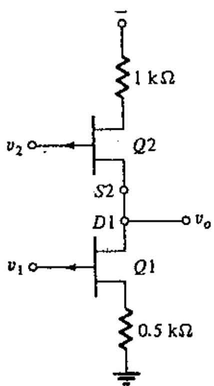


Fig.4

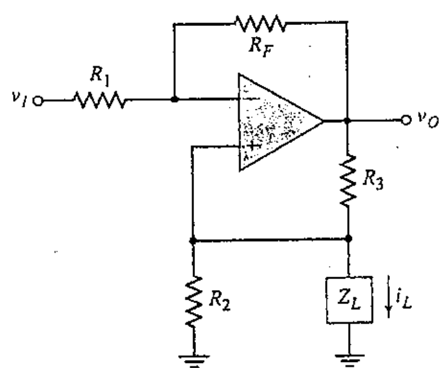


Fig.5

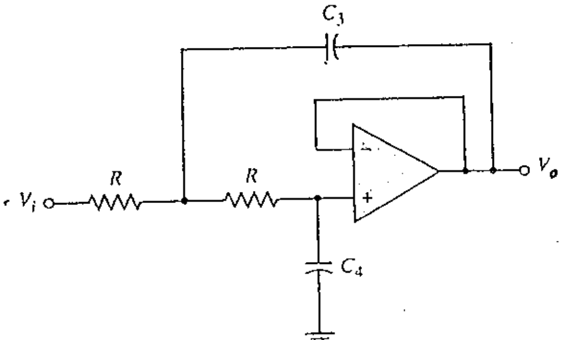


Fig.6