國立中山大學八十七學年度碩博士班招生考試試題

科　目：工程數學（海下技術研究所碩士班，甲,乙,丙組）　　

1. (25%) Consider the following Initial Value Problem (IVP):

$$\begin{aligned}(x-1)y'' - xy' + y &= x,\ 0 < x < 1\\ y(0) &= 0\\ y'(0) &= 1\end{aligned}$$

Assume the solution of the above IVP is of the form

$$y(x) = c_1 e^x + c_2\phi(x) + \int_0^x \psi(x,\xi)\,d\xi$$

Find c_1, c_2, $\phi(x)$, and $\psi(x,\xi)$.

2. (25%) Consider a vector field: $\mathbf{F} = z^2\mathbf{r}/|\mathbf{r}|$, where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors in x, y, z coordinates, respectively.

(a) (5%) Sketch the vector field (use a right-handed Cartesian coordinate with z pointing upward.)

(b) (20%) Evaluate

$$\iiint_V \nabla\cdot\mathbf{F}\,dV$$

where V is the volume of a sphere centered at origin and with radius 2.

3. (25%) Solve the following non-homogenous PDE system:

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\delta^2 u}{\partial x^2} + e^{-t}\sin 5x,\ 0 < x < \pi\\ u(0,t) &= 0\\ u(\pi,t) &= 1\\ u(x,0) &= 0\end{aligned}$$

4. (25%) Apply contour integration to evaluate the following integral:

$$\int_0^\infty \frac{\cos x}{\sqrt{x}}dx.$$

You may need to express your answer in terms of Gamma function: $\Gamma(1/2) = \int_0^\infty e^{-y}y^{-\frac{1}{2}}\,dy$. Justify each step in your derivation.

1. A frame is shown in Figure 1, where the pulley at *D* has a mass of 200 kg. Neglecting the weights of the bars, find the force transmitted from one bar to another at *C*. (20%)

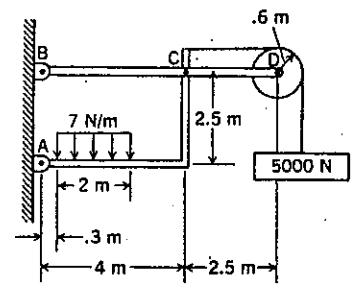

Figure 1 Loaded frame.

2. What is the resultant and the center of pressure for the pressure distribution as shown in Figure 2? Notice that the pressure varies linearly in the *x* and *y* directions. (20%)

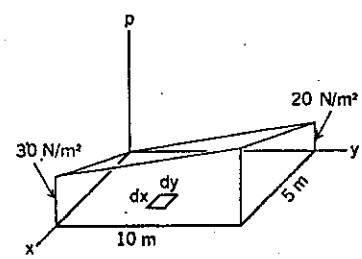

Figure 2 Pressure distribution.

3. Formulate the shear-force and bending-moment equations for the beam in Figure 3. Sketch the shear and moment diagrams. (20%)

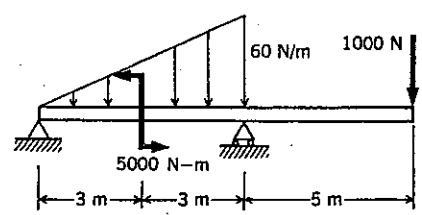

Figure 3 Loaded beam.

4. For target practice, a 9 N rock is thrown into the air and fired on by a pistol. The pistol bullet, of mass 57 g and moving with a speed of 312 m/sec, strikes the rock as it is descending vertically at a speed of 6.25 m/sec (Figure 4 (a)). Both the velocity of the bullet and the rock are parallel to the *xy* plane. After the bullet hits the rock, the rock breaks up into two pieces, *A* weighting 5.78 N and *B* weighting 3.22 N. (20%)
   a. What is the velocity of *B* after collision for the given post-collision velocities (all in the *xy* plane) of the bullet and piece *A* as shown in Figure 4 (b)?
   b. If wind resistance is neglected, how high up does the center of mass of the rock and bullet system rise after collision?

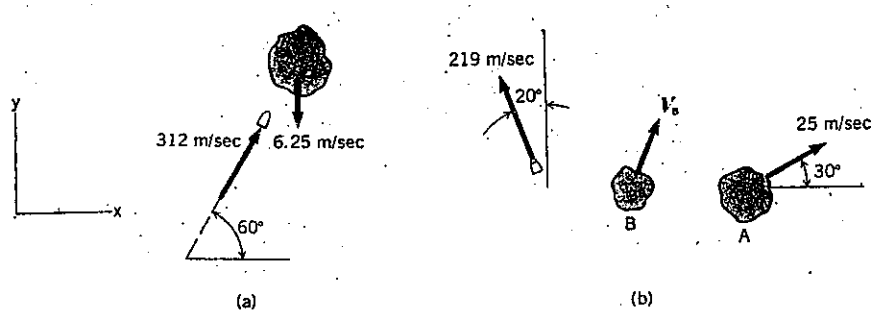

Figure 4 Bullet striking a rock.

5. A uniform cylinder of radius 400 mm and mass 100 kg is acted on at its center by a force of 500 N. Please draw the free-body diagram of the cylinder, and compute the friction force *f* (assuming no-slipping condition and friction coefficient μ_s = 0.2) (20%)

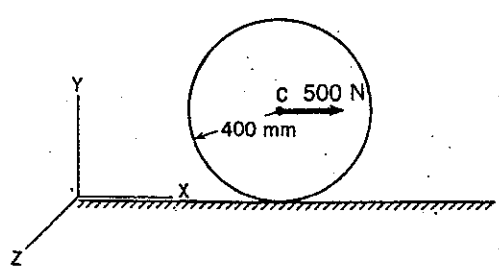

Figure 5 Rolling cylinder.

國立中山大學八十七學年度碩博士班招生考試試題

科 目：電子學（海下所甲、乙、丙組）

1.(a) For what voltage will the reverse current in a pn junction silicon diode reach 95 percent of its saturation value at room temperature? (4%)

(b) What is the ratio of the current for a forward bias of 0.2 V to the current for the same magnitude of reverse bias ? (4%)

(c) If the reverse saturation current is 10 pA, what are the forward currents for voltages of 0.5 and 0.7V, respectively? (4%)

2. In the circuit in Fig.1, a 5-V Zener diode is used which provides regulation for $50mA \leqq I_Z \leqq 1.0A$. Determine the range of load currents for which regulation is achieved if the unregulated voltage V_S varies between 7.5 and 10 V. The resistance $R_S = 4.75\,\Omega$. (10%)

3. Calculate the dc bias voltages and currents I_C, I_E and V_{CE} in the circuit in Fig.2. (12%)

4. An n-channel JFET has $V_P = -5V$ and $I_{DSS} = 12mA$ and is used in the circuit shown in Fig.3. The parameter values are $V_{DD} = 18V$, $R_S = 2K\,\Omega$, $R_D = 2K\,\Omega$, $R_1 = 400\,K\,\Omega$, and $R_2 = 90K\,\Omega$.

(a) Determine V_{DS} and I_D. (5%)

(b) Draw the small-signal equivalent of the circuit. (5%)

(c) Determine the output resistance seen between terminal 1 and ground. (6%)

5. It is desired to have a high-gain amplifier with high input resistance and high output resistance. If a three-stage cascade is used, what configuration should be used for each stage? Explain. (10%)

6. For the instrumentation amplifier shown in Fig.4, derive the relationship between V_O and (V_2-V_1). (10%)

7. (a) The Schmitt trigger in Fig.5 uses 6-V Zener diodes, with $V_D = 0.7V$. Assuming that the threshold voltage V_1 is zero and the hysteresis is $V_H = 0.2V$, calculate R_1/R_2 and V_R. (7%)

(b) This comparator converts a 4-kH_Z sine wave whose peak-to-peak value is 2V into a square wave. Calculate the time duration of the negative and of the positive portions of the output waveform. (7%)

8. In the inverted-ladder DAC shown in Fig.6, the switches are connected directly to the Op-Amp input.

(a) Show that the current I drawn from V_R is a constant independent of the digital word. (4%)

(b) What is the switch current and V_O if the MSB is 1 and all other bits are zero? (4%)

(c) Repeat (b), assuming that the next MSB is 1 and all other bits are zero. (4%)

(d) Calculate V_O for the LSB in the 4-bit D/A converter with all other bits zero. (4%)

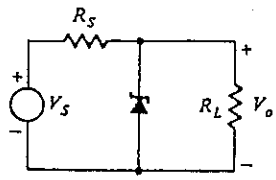

Fig.1

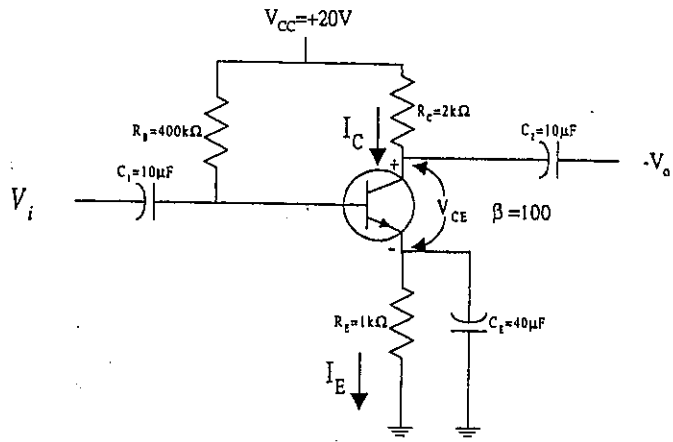

Fig.2

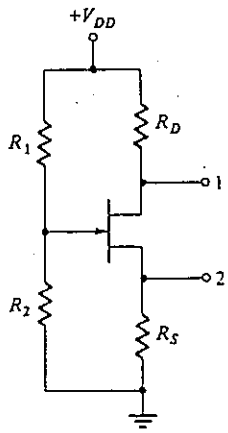

Fig.3

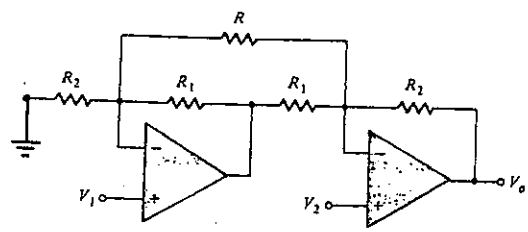

Fig.4

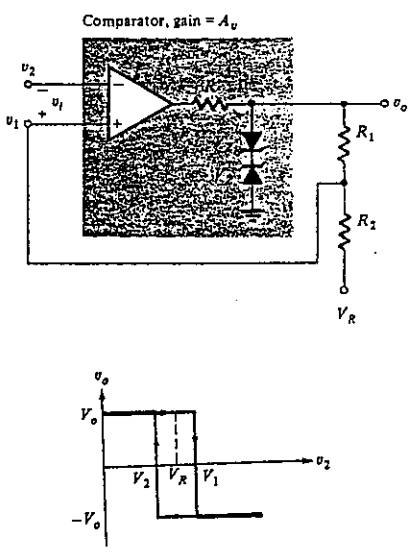

Fig.5

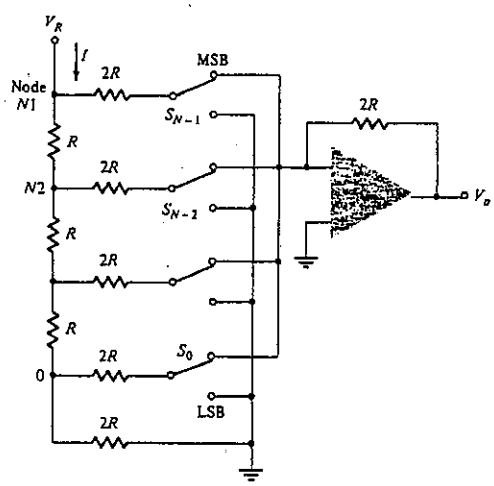

Fig.6

國立中山大學八十七學年度碩博士班招生考試試題

科　目：流體力學（海下技術研究所碩士班，甲丙選考）　

1. (15%)簡答題：

   (a) 台北與高雄之間大氣中所存在之溫度梯度約爲$5°C/350$ km（高雄溫度比台北高）。假如有一艘船以每小時25 km的速度由台北南下，行經台中地區時，當地溫度變化爲每小時$5°C$。試問行經台中時在該艘船上所感受到的溫度變化如何？

   (b) 試問什麼是波(wave)？什麼是流(fluid flow)？試以流體的質點運動說明之。另，波的相位速度(phase speed)是否就是介質質點速度(particle speed)？

   (c) 試問什麼是流場的散度(divergence)、旋度(curl)、質通量(mass flux)、環流量(circulation)的定義與物理含意。又，試問後兩者與前兩者之間有什麼關係？

2. (15%)試問在牽涉到自由液面（如海面）的流體力學問題中（如船隻在海面航行、結構物受海波的作用等）會牽涉到哪些無因次化參數(non-dimensional parameters)。試定義這些參數並討論這些參數與本問題之關連性。

3. (15%)有艘潛水艇位於海面下100 m，如圖1所示。試問要多少施力才能將圓形的艙口打開。（假設艙內壓力爲大氣壓）

4. (15%)潛水艇在第二次世界大戰期間都被稱作「鐵棺材(iron coffin)」。Werner艦長在他所著的書中(H.A. Werner, *Iron Coffins*, 1st edition, Holt, Rinehart & Winston, New York, 1969.)記載到：「我所指揮的潛艇假如以每小時9節的速度航行，則可續航1小時，不過，若是以每小時2節的速度航行，那麼將可續航3天。」請從技術觀點上探討Werner艦長所說的話是否合乎邏輯？爲什麼？

5. (15%) A cylinder tank in a rocket contains 10 gallons of water. The internal tank dimensions are 8 inches in diameter and 100 inches long. The tank is 3000 feet above sea level and is being accelerated parallel to its length at 20 feet per second at an angle 20 degrees above horizontal. The space not occupied by water is air at 100 psia. Determine the angle of the liquid surface exposed to the air.

6. (25%) An infinite plate overlying a semi-infinite viscous fluid with kinematic viscosity ν as shown in Figure 2. The plate oscillates with velocity $v_0 \cos \omega t$. Determine：

   (a) the velocity profile of the fluid as a function of depth; (15%)

   (b) the depth at where the maximum velocity of the fluid is $\frac{v_0}{e}$. This may be referred to as critical depth; (5%)

   (c) the depth at where the motion of the fluid is 180° out of phase with respect to the plate. (5%)

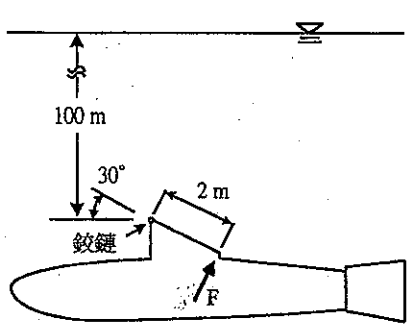

圖1

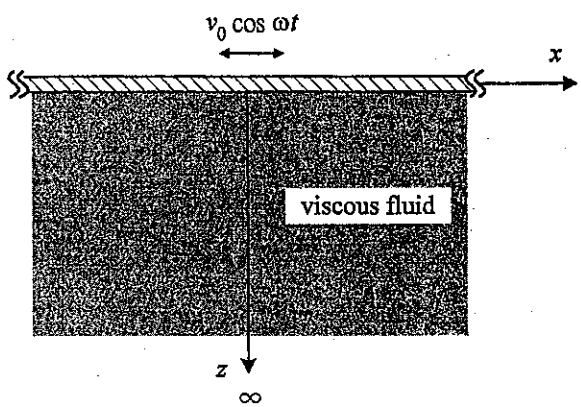

Figure 2