

國立中山大學九十三年度碩士班招生考試試題

科目：微積分（海洋物理研究所碩士班）

共 2 頁 第 1 頁

1) 【Quadratic equations】【10 marks】

Consider the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$.

- (a) Find the quadratic formula for x .
- (b) Use the *discriminant* (辨別式) of the quadratic equation, state when the equation will have two real and distinct roots, two real and equal roots, or two distinct imaginary numbers.

2) 【Quadratic equation】【5 marks】

For a rectangular with length $(6-x)$ and width x , express its area as quadratic function of x .

For what value of x will the area be a maximum?

3) 【Operation with complex numbers】【5 marks】

Given $f(z) = z^2 + 6z + \frac{1}{z}$, where z is a complex number. Find $f(3f)$.

4) 【Inverse variation】【10 marks】

If y varies inversely as the cube root of z , and $y = 2$ when $z = 2$, find y when $z = 8$.

5) 【Radical equation】【5 marks】

Solve radical equation $\sqrt{x+6} - \sqrt{2x+5} = -1$.

6) 【Radical equation】【10 marks】

Solve $\frac{1}{x^4} - \frac{8}{x^2} + 7 = 0$.

7) 【Logarithmic equation】【5 marks】

Solve $\log_4(2x+4) = 3$.

8) 【Limits】【10 marks】

(a) $\lim_{x \rightarrow 0^+} \frac{\sin(x)}{x}$

(b) $\lim_{x \rightarrow 2^+} \frac{\ln(2x-3)}{x^2-4}$

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共 2 頁 第 2 頁

9) 【Derivative by chain rule】【10 marks】

Apply the chain rule to find $\frac{du}{dt}$, given $u = x^2 + 2xy + y^2$ where $x = t \cos t$ and $y = t \sin t$.

10) 【Integration by substitution】【10 marks】

Find the indefinite integral of $\int (x^2 \sqrt{x^3 + 4}) dx$.

11) 【Integration of trigonometric functions】【5 marks】

Find the indefinite integral of $\int \sin^3 x \cos x dx$.

12) 【Application of differential calculus】【15 marks】

Sand is falling into a conical pile at the rate of $5 \text{ ft}^3/\text{min}$. If the height of the pile is always twice the radius of the base, how fast is the height increasing when the pile is 3 ft high?

【Hint: The volume of the cone (pile) at any time is expressed as $V = \frac{1}{3} \pi r^2 h$, where r is the radius and h is the height】

國立中山大學九十三年學年度碩士班招生考試試題

科目：流體力學【海洋物理研究所碩士班】(選考)

共 2 頁 第 1 頁

第一部份：定義/數學公式/簡要說明題【50分】

1. 【Definition/Equation/Brief Answer : 5% each】

- (1). How can we measure the shear stress in laboratory condition?
- (2). What is a Venturi tube?
- (3). Continuity equation in a two-dimensional incompressible flow.
- (4). Bernoulli equation in a two-dimensional incompressible flow.
- (5). Froude number.
- (6). Absolute pressure.
- (7). Path line of a fluid particle.
- (8). Control volume.
- (9). State the three laws of similarity in modeling a prototype condition.
- (10). Main difference between Eulerian and Lagrangian description of fluid motion.

第二部份：計算題【50分】

2. 【Streamline : 10%】

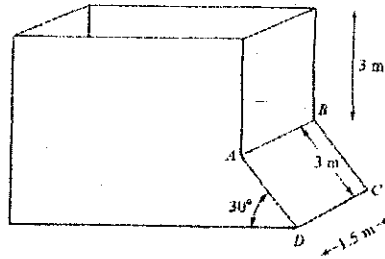
Given the streamline equation for a fluid particle in a two-dimensional flow field as

$$\frac{dy}{dx} = \frac{v}{u}. \text{ Find the streamline equation when a particle passes the point } (1, 1), \text{ if the velocity}$$

components are given by $u = x$ and $v = y$.

3. 【Hydrostatic force : 10%】

A water tank as shown is completely filled with water with density 1000 kg/m^3 . Calculate the water force on the slanted side wall $ABCD$ of the tank and the location of this force.



4. 【Energy equation : 10%】

Determine the power required for a pump to deliver $0.05 \text{ m}^3/\text{s}$ of water at atmospheric pressure through a 100-mm pipe to a building 100 m above sea level. Assume the density of water as 1000 kg/m^3 , and neglect heat transfer and all internal energy losses when the flow through the pipe.

【 Hint: The energy equation applicable may be written as

$$-\frac{dW}{dt} = \rho AV \left[\Delta \left(\frac{p}{\rho} + \frac{V^2}{2} + gz \right) \right], \text{ where } W \text{ is the work done on the fluid and } \Delta \text{ denotes the}$$

difference of total energy $\left(\frac{p}{\rho} + \frac{V^2}{2} + gz \right)$ between the discharge and the entrance points.】

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共 2 頁 第 2 頁

5. 【Dimensional analysis and similitude : 10%】

Air flows with an average velocity of 10 m/s through a circular pipe having diameter of 250 mm, under the condition at 1 atmospheric pressure and 20°C.

(1). What must be the average velocity in a model of this flow to be reproduced in a water pipe of 60-mm in diameter, if the flow is dynamically similar to the prototype?

(2). Find the pressure drop in the prototype, if the pressure drop in the model is 200 kPa.

【Hint 1: In (1), equate Reynolds number for the prototype and model,

and in (2) let $\Delta p \approx \rho V^2$.】

【Hint 2: Additional data: kinematic viscosities $\nu_{\text{air}} = 1.51 \times 10^{-5} \text{ m}^2/\text{s}$, $\nu_{\text{water}} = 1 \times 10^{-6} \text{ m}^2/\text{s}$; densities $\rho_{\text{air}} = 1.204 \text{ kg/m}^3$, $\rho_{\text{water}} = 998.3 \text{ kg/m}^3$ 】

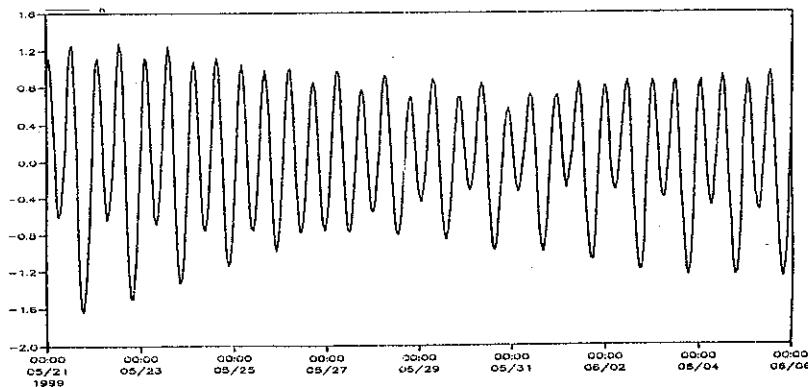
6. 【Velocity distribution in a typhoon : 10%】

The eye of a typhoon (tropical cyclone) has a radius R of 20 m and the maximum wind velocity at the edge of the eye is 50 m/s. Find the variation of tangential velocity in the flow field of the typhoon, for the radial distance $r \leq R$ (assume rigid body rotation, i.e., forced vortex) and $r \geq R$ (free vortex), respectively.

國立中山大學海洋物理研究所碩士班入學考試 海洋物理學試題 (總分 100 分)

1. 潮汐 (30%)

- (a) 何謂大潮？何謂小潮？何謂潮流橢圓？何謂引潮力？(10%)
- (b) 請申論模擬淺海（如澎湖地區）海水之潮汐運動時，是否需考慮引潮力？(10%)
- (c) 以下為澎湖馬公地區之潮位紀錄，請描述其特性（潮差、週期等）。(10%)



2. 波浪 (20%)

- (a) 波浪由深海傳至淺海，請問深水波之適用範圍為何？其相位速度為何？能量傳波之速度為何？(10%)
- (b) 在海邊看到的波浪幾乎都是與海岸垂直，請問這是水波的何種效應？在受障礙物遮蔽的區域也有波動，請問這是水波的何種效應？(10%)

3. 假設香港外海發生地震後產生海嘯，若震央距高雄外海測站 500 公里，當地水深 200 公尺，高雄外海測站水深 20 公尺，請估計發生地震多少時間後海嘯會抵達高雄外海測站。(10%)

4. 解釋名詞 (25%)

- (a) El Nino
- (b) 科氏力
- (c) 1 節 = ? m/sec
- (d) Ekman spiral
- (e) Internal Wave

5. 試繪出海水溫度之典型剖面圖與聲速分布圖。以上兩圖有何相關？何謂 Sound Channel？(15%)