1. About Basic Concepts. (15%)

   (a) Why is Calculus practically important? (5%)

   (b) What does a derivative mean? (5%)

   (c) Can we find a differentiation or integral for all functions? Explain your answer. (5%)

2. About Limit. (10%)

   (a) Verify the continuity of the function $F(x) = \begin{cases} |x-3| & \text{if } x \neq 3 \\ 2 & \text{if } x = 3 \end{cases}$, and plot the function's diagram. (5%)

   (b) Find the limit of $\lim_{x \to 0}(x\sqrt{x-3})$. (5%)

3. About Derivative and Integral. (Be careful, no partial credits) (50%)

   (a) Given $\begin{cases} x = 2\sec\theta - 3 \\ y = 4\tan\theta + 2 \end{cases}$, find d^2y/dx^2. (10%)

   (b) Apply the Chain Rule to find $\dfrac{du}{dt}$, given $u = x^2 + 2xy + y^2$ and $x = t\cos t$, $y = t\sin t$. (10%)

   (c) Find $\displaystyle\int \frac{-x^2 + 11x + 18}{(x^2-1)(x^2+3x+3)}$. (10%)

   (d) Find $\displaystyle\int \sqrt{\frac{x}{1-x}}\, dx$ (10%)

   (e) Find $\displaystyle\int \sin(\ln x)\, dx$ (10%)

4. Find the area between two functions $y = x^2$ and $x + y = 2$. (10%)

5. A cone type water tank as shown in the following figure has a fixed volume at 120 ft^3. If we want to build the tank with the minimum material for the side surface, what's the radius (r) and height (h) should be? (15%) (hint: side surface = $\pi r L$, and volume = $(1/3)\,\pi r^2 h$)
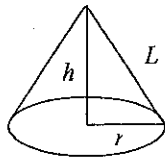
國立中山大學海洋物理研究所碩士班入學考試 海洋物理試題 (總分 100 分)

1. 以下是中山大學西子灣海濱 2003 年 3 月 1 日至 30 日的海水溫度記錄曲線，請描述溫度變化情形，並以海洋物理的觀點推論其溫度變化之原因。(20 分)

March 2003 Temperature records at Kaohsiung
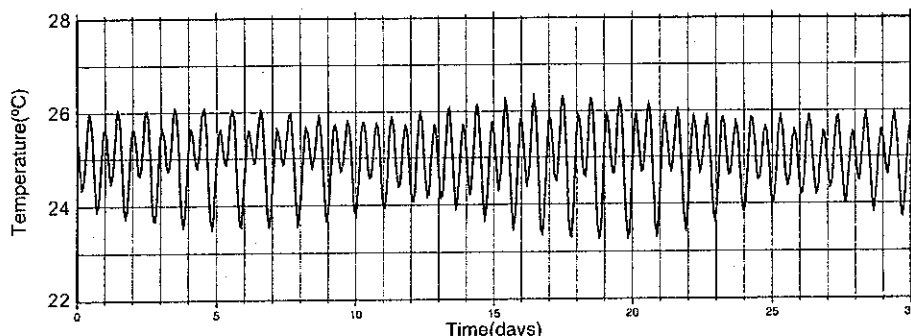
2. 南半球密度均一的海洋中有一支海流的動量平衡為：

$$fv = g\frac{\partial \eta}{\partial x}$$

式中 f 為科氏參數(Coriolis parameter)，v 為南北向(y 方向)速度分量，g 為重力加速度(約 10 m/s^2)，η 為海面高度。若海面高度由西往東 100 公里間隨著距離增加以線性關係上升 100 公分，則這支海流向哪個方向流？(5 分) 若地轉角頻率 Ω 近似 10^{-4} s^{-1}，那麼在南緯 30° 上這支海流速度多大？(10 分) 其 Rossby number 為多少？(5 分)

3. (a) 海洋中正壓(Barotropic)與斜壓(Baroclinic)流體之定義為何？(10 分)
   (b) 解釋位渦度守恆(Potential vorticity conservation)。(10 分)
   (c) 何謂減重力加速度(Reduced gravity)？(10 分)

4. 何謂內波(Internal wave)？(5 分) 分層頻率 N(Stratification frequency 或 Brunt-Vaisala frequency)之定義為何？(5 分)

5. 近岸海洋(Coastal ocean)在沿岸風的持續吹拂作用下，可能會發生怎樣的環流過程及現象？(10 分)

6. 熱力風關係(Thermal wind relation)為 $\dfrac{\partial u}{\partial z} = \dfrac{g}{\rho_0 f}\dfrac{\partial \rho}{\partial y}$ 式中 u 是東西向流速分量，z 是垂直座標軸，y 是水平南北向座標軸，g 是重力加速度，ρ_0 是海水平均密度，ρ 是隨空間變化之海水密度。請說明上式物理意義。(10 分)

第一部份：簡要說明/數學公式題　【40 分】

1. 【Equation/Brief Answer：5% each】

　　1). Venturi meter.

　　2). Continuity equation in a two-dimensional incompressible flow.

　　3). Bernoulli equation in a two-dimensional incompressible flow.

　　4). Navier-Stokes equation in a three-dimensional flow for a viscous fluid.

　　5). Similitude and modeling.

　　6). Terminal velocity.

　　7). Measurement of shear stress.

　　8). Reynolds number.

第二部份：計算題　【60 分】

2. 【Hydrostatic forces：10%】

An open tank 3 m x 1 m in cross section (tank elevation shown) holds water to a depth of 3 m. Determine the magnitude and line of action of all forces exerted upon the curved surface BC.
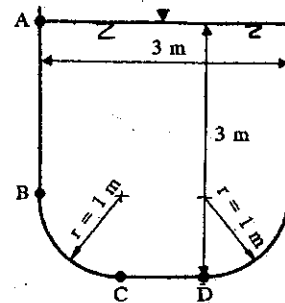
3. 【Pipe flow：20%】

A siphon pipeline with inner diameter 500 mm discharges water from a large tank. Determine (1) the maximum possible elevation of its summit for a discharge of 2.15 m^3/s without the pressure becoming less than 20 kN/m^2 absolute, and (2) the corresponding elevation of its discharge end. Assume atmospheric pressure as 1 bar (10^5 N/m^2) and neglect all losses.
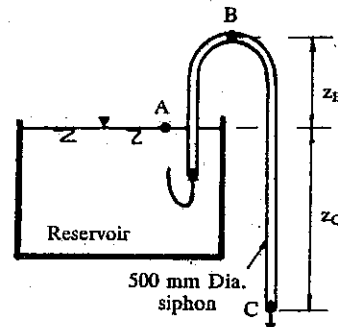
4. 【Dimensional analysis and similitude：10%】

An orifice on the side of an open water tank discharges sea water into the atmosphere. A small model of the tank is to be built, also using sea water as flow medium. If the scale ratio of prototype to model is 10：1, what are the ratios of volume discharge and force exerted on the tanks at dynamically similar conditions?

5. 【Momentum：20%】

Water flows through a horizontal Y-branch section as shown. Determine the force components required to hold the Y section in place, for a one-dimensional steady flow without internal friction losses. The gage pressure at section 1 is 30 kPa, with volumetric inflow 15 liters/s, outflow at section 2 is 10 liters/s, and water density is 1000 kg/m^3.