

國立中山大學99學年度碩士班招生考試試題

科目：工程數學【海工系碩士班甲組】

1. 【Ordinary Differential Equations】 20%

(a) Solve $y^{(4)} + 10y'' + 9y = 13 \cosh 2x$ (10%)

(b) Solve the IVP $(\cos \omega x + \omega \sin \omega x)dx + e^x dy = 0, y(0) = 1$ (10%)

2. 【Laplace Transform】 10%

Solve the integral equation $y(t) + 2e^t \int_0^t e^{-\tau} y(\tau) d\tau = te^t$

3. 【Linear Algebra】 10%

Given two matrices: $A = \begin{bmatrix} 4 & 2 \\ -4 & -2 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 3 \\ 3 & 6 \end{bmatrix}$. $\hat{A} = P^{-1}AP$

(a) Find eigenvectors \mathbf{y} of \hat{A} . (5%)

(b) Show that $\mathbf{x} = P\mathbf{y}$ are eigenvectors of A . (5%)

4. 【Vector Calculus】 10%

Given $\mathbf{F} = [y^3 \quad x^3 \quad z^3]$, the surface $S: x^2 + 4y^2 = 4, x \geq 0, y \geq 0, 0 \leq z \leq h$, \mathbf{n} is the unit surface normal vector of S . Calculate $\iint_S \mathbf{F} \cdot \mathbf{n} dA$

5. 【Partial differential equation】 20%

(a) Write down the general expression of a linear partial differential equation. (5%)

(b) Partial differential equations have been applied to many natural phenomena in our daily life.

They are classified as elliptic, parabolic or hyperbolic, based on the mathematical form of the equation. Please give (1) the criterion, (2) mathematical expression, and (3) one example for each type of the equations mentioned in (b). (15%)

6. 【Method of separation of Variables】 10%

Solve the following partial differential equation,

$$\frac{\partial^2 u(x, y)}{\partial x^2} = \frac{\partial u(x, y)}{\partial y}$$

based on the solution in the form of $u(x, y) = X(x)Y(y)$.

[Please note that, without the initial and boundary conditions, you may leave the undetermined coefficients in the final solutions.]

7. 【Fourier analysis】 20%

Determine the Fourier series for a periodic square wave given by the function $f(x)$:

$$f(x) = \begin{cases} 0, \dots \text{for } -2 < x < -1 \\ k, \dots \text{for } -1 < x < 1; p = 2L = 4 \\ 0, \dots \text{for } 1 < x < 2 \end{cases}$$

And show the answer in (a) series approximation, and (b) expansion of the series to the third term of $n=3$.

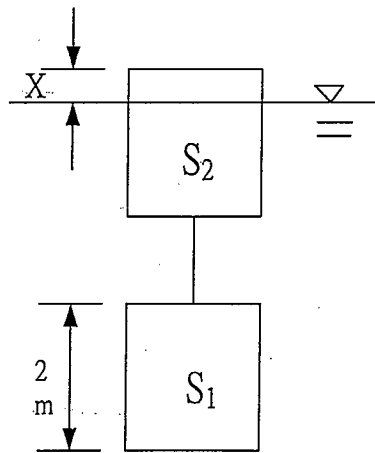
國立中山大學99學年度碩士班招生考試試題

科目：流體力學【海工系碩士班甲組選考】

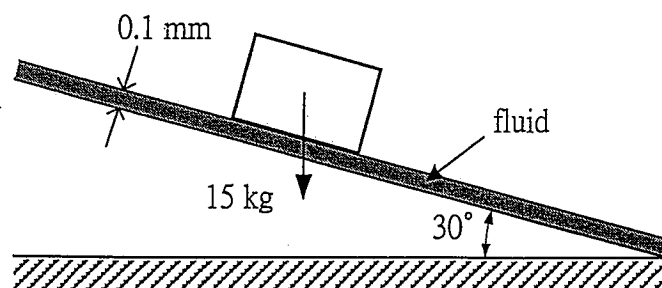
1. 名詞解釋 (20%, 每小題 4%)

- (1) vapor pressure
- (2) cavitation
- (3) Froude number
- (4) Perfect gas law
- (5) Stream function

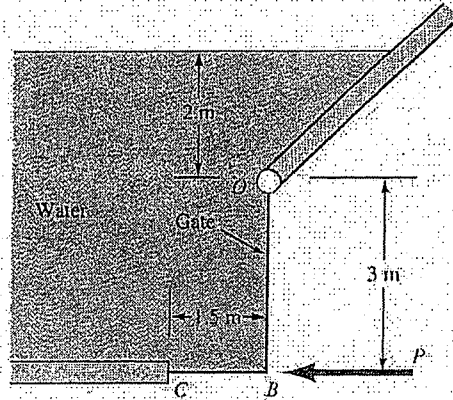
2. Two cubes of the same size, 8.0 m^3 , one of specific gravity $S_1 = 1.2$ and the other of $S_2 = 0.6$, are connected by a short wire and placed in water. What portion of the lighter cube is above the water surface, and what is the tension in the wire? Neglect the wire's volume and weight (20%)



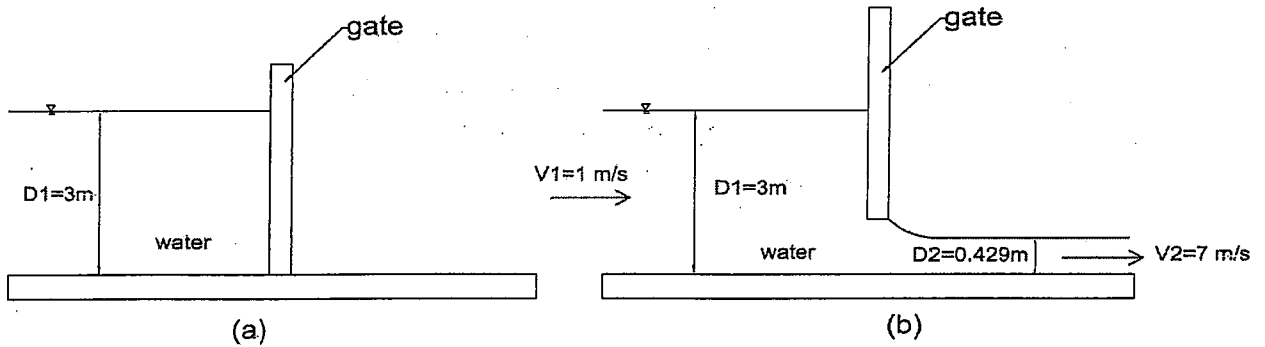
3. A 15 kg cube slides down an inclined plane making an angle of 30° with the horizontal, shown in the figure. A fluid film 0.1 mm thick separates solid and surface. The fluid viscosity is $0.04 \text{ N}\cdot\text{sec}/\text{m}^2$. Assuming that the velocity distribution in the film is linear, find (1) the terminal velocity of the block, (2) the velocity function of time before it reaches the terminal velocity. The area of the cube is constant with the film is 0.25 m^2 . Neglect the air drag force. (20%)



4. The gate, OBC, is 4m wide and is rigid. Neglect the weight of the gate, and assume the hinge friction to be negligible. What force P is necessary to hold the gate closed? (20%)



5. Water in an open channel is held in by a sluice gate. Compare the horizontal force of the water on the gate (a) when the gate is closed and (b) when it is open. Neglect bottom friction. (20%)



國立中山大學 99 學年度 碩士班 招生考試 試題

科目：工程力學 (含靜力與材力) 【海工系碩士班甲組選考】

1. According to Fig.1, the weight $W=2$ kN. Please determine the tension in the cable and the reactions at A. (15%)

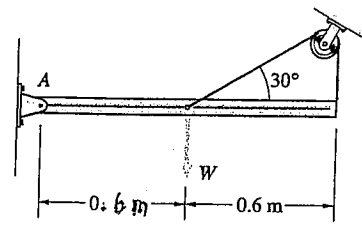


Fig.1.

2. There is a wall shown in Fig.2, where the height of the wall is y . If the material is rigid, to estimate the wind loads, what factors of wall are important and must be found by an engineer? Please calculate the important factors for this wall. (20%)

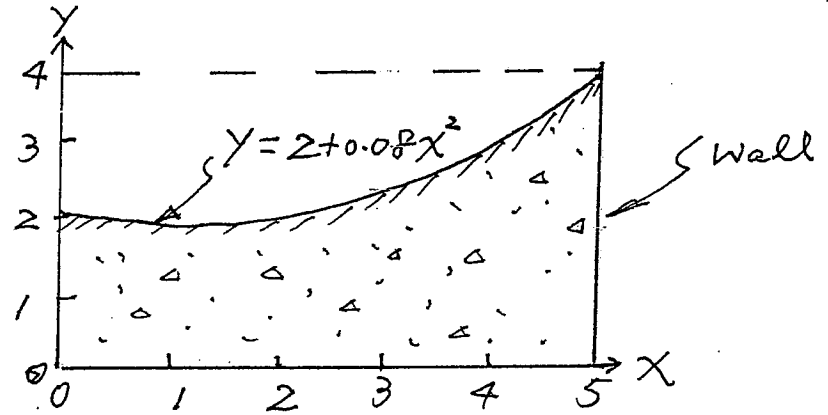


Fig.2.

3. Please determine the reactions on member AB at A and B when BC is a two-force member. (15%) (15%)

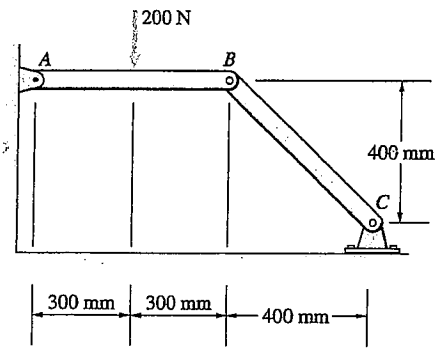
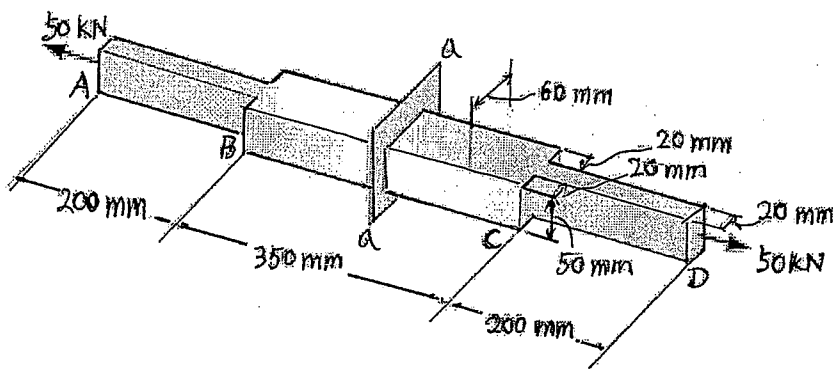


Fig.3

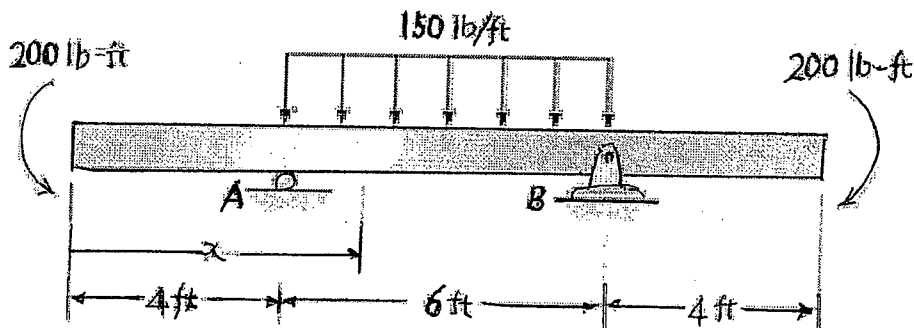
國立中山大學 99 學年度 碩士班 招生考試 試題

科目：工程力學 (含靜力與材力) 【海工系碩士班甲組選考】

4. The size of the steel bar is shown in Fig. P4., the axial load is 50 kN. Please calculate the variation of the length of the bar and the cross-sectional area of section a-a. $E_{st} = 200 \text{ Gpa}$, $\nu_{st} = 0.29$. (20%)



5. Please determine the expression of shear force and bending moment in terms of x and plot the corresponding diagrams. (20%)



6. Please answer the following terms (in English) (10%)
- (a) prismatic bar
 - (b) Stiffness and flexibility

一、解釋名詞：(40%，每小題4分)

1. ecological engineering
2. ecological corridor
3. ecological footprint
4. cleaner product
5. symbiotic
6. population momentum
7. El Niño and La Niña
8. sustainability of resources
9. primary productivity
10. nonpoint sources pollution

二、問答題：(60%)

1. 何謂「biofuel energy」？何謂「clean energy」？何謂「green energy」？試分別續述三種能源替代化石燃料能源之優缺點。(15%)
2. 何謂「臭氧層破洞」？其原理為何？有何影響性？(15%)
3. 何謂「Environmental Hormone」？何謂「emerging contaminants」？何謂「PPCPs」？有何關聯性？(15%)
4. 何謂「京都議定書」？其主要內容為何？近日就全球暖化議題所召開的「哥本哈根會議」各國無法達成協議的關鍵點為何？你認為該如何解決此一問題？(15%)

國立中山大學99學年度碩士班招生考試試題

科目：環境微生物學與環境化學【海工系碩士班乙組】

1. 試列舉三個與微生物相關之環境污染議題，並敘述由何種類微生物所主導，以及說明為何該種微生物會引發此一環境污染問題。(10%)
2. 何謂活性污泥(activated sludge)? 活性污泥中包含哪些微生物物種? 這些物種在廢水處理中所扮演的角色各為何?(10%)。
3. 檢測水中大腸桿菌的方法有幾種? 試分別說明之，並比較其優缺點。(10%)
4. 有一個地下水監測井遭不法傾倒廢有機溶劑，請描述後果。影響範圍將受哪些因子掌控? 如何整治?(10%)
5. 一醋酸(acetic acid)溶液 100 mg/L，請估計理論需氧量(theoretical oxygen demand)。再請估計 BOD₅ 值為何?(假設：可完全生物分解，反應速率常數 $k=0.20/\text{day}$) (需要的話可參考 atomic weight: C=12, H=1, O=16) (10%)
6. 何謂鹼度? 某水樣 pH 值為 9.1，體積 100ml。以 0.02N 硫酸溶液滴定至 pH 值 8.3，需 0.9 ml；滴定至 pH 值 4.3，需另加 12.1 ml。試計算總鹼度。(10%)
7. 檢測水中溶氧的方法主要有哪兩種? 試分別說明之，並比較其優缺點。(8%)

國立中山大學99學年度碩士班招生考試試題

科目：環境微生物學與環境化學【海工系碩士班乙組】

8. 某一污染物在環境中半衰期為 $t_{1/2}$ 年(分解動力學假設為一階動力反應)，這個污染物在進入環境後由原先濃度 C_0 衰減 99.9% (也就是只剩 0.1% C_0 在環境中) 的時間需要多長？ (列出式子說明可解即可)(8%)
9. 請說明何謂(A)新興污染物(emerging contaminants)(B) Red tide (赤潮) 或 Harmful algal blooms (HABs). (8%)
10. 請說明何謂界面活性劑(surfactant)與其增溶作用(solubilization)? (8%)
11. 試舉兩個環境中發生之氧化還原例子，並且說明。(8%)

國立中山大學99學年度碩士班招生考試試題

科目：海洋及海岸管理概論【海工系碩士班丙組選考】

一、解釋名詞 20% (每小題 4 分，請解釋以下之詞句，非英翻中!!)

1. Integrated Coastal Zone Management (ICZM)
2. Wise Use (in Ramsar Convention)
3. International Union for Conservation of Nature (IUCN)
4. No-Net-Loss (for Wetland Policy)
5. Intergovernmental Panel on Climate Change (IPCC)

二、問答題 80%

1. 「海岸地區」的範圍劃設複雜，請問劃設的基本原則應為何？(5%) 如以目前立法院審議中之「海岸法」草案為依據，海岸地區之定義及劃設範圍為何？(15%)
2. 假設台灣西部海岸正規劃一填海造陸之工業區塊，以發展石化相關產業，已知該區域位於中部重要河川出海口之北岸，且為台灣僅存大面積之泥質灘地之一，外海為保育類海豚(中華白海豚)分佈棲地，請問，該工業區之興建可能會造成什麼衝擊及影響？(10%) 在環境影響評估時之重要考量為何？(10%)
3. 請描述台灣東、西、南、北、離島之海洋與海岸特性為何？(10%) 及各自面臨之議題為何？(10%)
4. 請問台灣第一座以海洋為主體之國家公園為何？(2%) 「海洋保護區」與陸域保護區之異同為何？(8%) 如以「海洋保護區」(Marine Protected Areas) 及「海洋保護區網絡」(Marine Protected Area Networks) 之概念出發，台灣該如何達成海洋資源之永續發展策略？(10%)

1. 請說明下列段落所談內容，與 Information System 之功能及可能應用。(25分)

Information System (or IS) is historically defined as a 'bridge' anchored between the business world and computer science, but this discipline is slowly evolving towards a well-defined science. In a very broad sense, the term *information system* is frequently used to refer to the interaction between people, processes, data and technology. In this sense, the term is used to refer not only to the information and communication technology (ICT) an organization uses, but also to the way in which people interact with this technology in support of business processes.

Some make a clear distinction between information systems, ICT and business processes. Information systems are distinct from information technology in that an information system is typically seen as having an ICT component. Information systems are also different from business processes. Information systems help to control the performance of business processes.

Alter argues for an information system as a special type of work system. A work system is a system in which humans and/or machines perform work using resources (including ICT) to produce specific products and/or services for customers. An information system is a work system whose activities are devoted to processing (capturing, transmitting, storing, retrieving, manipulating and displaying) information.

Part of the difficulty in defining the term *information system* is due to vagueness in the definition of related terms such as system and information. Beynon-Davies argues for a clearer terminology based in systemics and semiotics. He defines an information system as an example of a system concerned with the manipulation of signs. An information system is a type of socio-technical system. An information system is a mediating construct between actions and technology.

As such, information systems inter-relate with data systems on the one hand and activity systems on the other. An information system is a form of communication system in which data represent and are processed as a form of social memory. An information system can also be considered a semi-formal language which supports human decision making and action.

Information systems are the primary focus of study for the information systems discipline and for organisational informatics.

2. 請說明下列段落所談內容，與 **computer graphics** 之功能及可能應用。(25 分)

Computer graphics are graphics created using computers and, more generally, the representation and manipulation of image data by a computer.

Today, computers and computer-generated images touch many aspects of our daily life. Computer imagery is found on television, in newspapers, for example in their weather reports, or for example in all kinds of medical investigation and surgical procedures. A well-constructed graph can present complex statistics in a form that is easier to understand and interpret. In the media "such graphs are used to illustrate papers, reports, theses", and other presentation material.

Many powerful tools have been developed to visualize data. Computer generated imagery can be categorized into several different types: 2D, 3D, and animated graphics. As technology has improved, 3D computer graphics have become more common, but 2D computer graphics are still widely used. Computer graphics has emerged as a sub-field of computer science which studies methods for digitally synthesizing and manipulating visual content. Over the past decade, other specialized fields have been developed like information visualization, and scientific visualization more concerned with "the visualization of three dimensional phenomena (architectural, meteorological, medical, biological, etc.), where the emphasis is on realistic renderings of volumes, surfaces, illumination sources, and so forth, perhaps with a dynamic (time) component

3D projection is a method of mapping three dimensional points to a two dimensional plane. As most current methods for displaying graphical data are based on planar two dimensional media, the use of this type of projection is widespread, especially in computer graphics, engineering and drafting.

Ray tracing is a technique for generating an image by tracing the path of light through pixels (picture elements) in an image plane. The technique is capable of producing a very high degree of photorealism; usually higher than that of typical scanline rendering methods, but at a greater computational cost.

Volume rendering is a technique used to display a 2D projection of a 3D discretely sampled data set. A typical 3D data set is a group of 2D slice images acquired by a CT or MRI scanner.

Usually these are acquired in a regular pattern (e.g., one slice every millimeter) and usually have a regular number of image pixels in a regular pattern. This is an example of a regular volumetric grid, with each volume element, or voxel (volumetric pixel)

represented by a single value that is obtained by sampling the immediate area surrounding the voxel.

3. XML 為 W3C 的標準之一，XML 為何？XML 之特色為何？XML 之可能應用？(15 分)

4. 在維基百科中宣稱，Adobe Flash 之特色有：1). 被大量應用於網際網路網頁的向量動畫檔案格式，2). 使用向量運算 (Vector Graphics) 的方式，產生出來的影片佔用儲存空間較小，3). 使用 Flash 創作出的影片有自己的特殊檔案格式 (swf)，4). 該公司聲稱全世界 97% 的網路瀏覽器都內建 Flash 播放器，5). 是 Adobe 提出的「富網際網路應用」(RIA) 概念的實作平台，6). Flash6 之後版本納入物件導向程式概念。與其他語言比較，不論是在資料庫、XML、PHP 等各種平台上，都能更進一步的相互結合應用。而對 Adobe Flash 之批評，包括：1). Flash cookies，2). 因一些網頁上的 flash 是要使用高性能的電腦才能滿足其需求，flash 的影音是無法利用電腦上的硬體加速被認為浪費電腦資源，3). 對 64 位元的支援不完整。而目前 Apple 公司之 iPhone/iPad 並必不支援 Flash，基於技術的角度，請敘述你的看法為何？(15 分)

5. 試以任一種程式語言(例如：Fortran、Visual Basic、Visual C++等)計算一數列 $1 + (1/3) + (1/5) + (1/7) + (1/9) + \dots$ 之值(精度達到小數點第六位)，程式中須附註說明所使用各參數之意義。(20 分)

國立中山大學99學年度碩士班招生考試試題

科目：線性代數【海工系碩士班丙組選考】

1. (20%) Given a linear system $Ax=b$ where A is an m by n matrix.
- (a) (10%) Under what condition will the linear system have exact one solution?
- (b) (10%) If $b=0$. Under what condition will the linear system have a non-trivial solution?

2. (20%) Solve the following linear system by Cramer's rule and check by Gauss elimination.

$$\begin{cases} w + 2x - 3z = 30 \\ 4x - 5y + 2z = 13 \\ 2w + 8x - 4y + z = 42 \\ 3w + y - 5z = 35 \end{cases}$$

3. (20%) Find limit states (or steady state) of the Markov processes modeled by the following stochastic matrix.

	To State I	To State II	To State III
From State I	0.6	0.4	0.0
From State II	0.1	0.1	0.8
From State III	0.2	0.4	0.4

4. (20%) Find an eigenbasis (a basis of eigenvectors) and diagonalize the following matrix.

$$\begin{bmatrix} -6 & -6 & 10 \\ -5 & -5 & 5 \\ -9 & -9 & 13 \end{bmatrix}$$

5. (20%) Prove the following theorems.
- (a) (10%) Let A be a unitary matrix. Prove its determinant has absolute value one, that is $|\det A| = 1$
- (b) (10%) Let A be a Hermitian matrix. Prove its eigenvalues are real numbers.

國立中山大學 99 學年度碩士班招生考試試題

科目：統計學【海工系碩士班丙組選考】

Part 1：簡答題（定義、名詞）【50%】

1. (10%)【統計學的定義】

簡扼說明下列名詞的定義：

(a) 統計學 (statistics) (限 40 字內)；(b) 數據變化的種類 (types of data variations)。

以關鍵字或片語表示：

(c) 集中型態的各種統計量名稱；(d) 分散型態的各種統計量名稱。

2. (10%)【統計學的種類】

(a) 敘述統計 (descriptive statistics) 及其包括的項目。

(b) 推論統計 (inferential statistics) 及其包括的項目或推論步驟。

3. (10%)【假設與檢定】

(a) 列舉“假設與檢定”的五大步驟。(b) 列舉“假設與檢定”中設立“假設”的三大基本原則。

4. (10%)【常態分佈】

隨機抽樣數據 $x_i (i=1 \sim N)$ ，在 N 個大樣本數時，樣本個值出現之機率可能符合常態分佈 ($N(\bar{x}, \sigma)$ ； \bar{x} 為樣本平均數， σ 為樣本標準偏差)。簡扼表示或回答下列問題：

(a) 繪圖標示在右偏分佈情況下 mean、median 及 mode 的相對位置；

(b) 機率密度函數 (probability density function) $p(x)$ 的數學式；

(c) 在以 $z_i = (x_i - \bar{x})/\sigma$ 標準化後的對稱常態分佈圖 $N(0, 1)$ 上：請問在 $-\sigma \leq z_i \leq +\sigma$ 、 $-2\sigma \leq z_i \leq +2\sigma$ 及 $-3\sigma \leq z_i \leq +3\sigma$ 之三個區間，累積機率各為多少%？

(d) 以 x_i 、 \bar{x} 、 σ 及 N 表示 skewness (偏態) 及 kurtosis (峰態)。

5. (10%)【最小二乘法及迴歸分析】

假設某物理現象觀測值 $y_i (i=1, \dots, N)$ 與其相對的自變數 x_i 有線性關係： $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ 。若選用一階迴歸線 $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$ 表示 y_i ，請推導計算迴歸係數 $\hat{\beta}_0$ 及 $\hat{\beta}_1$ 的數學式。

Part 2：計算題【50%】

6. (20%)【次數分配表及圖形】

某高科技公司各分廠在 2008 年 12 月份的產品超高速攝影機的生產數量 (單位：千台) 如下：

$x = [62 \ 66 \ 57 \ 64 \ 61 \ 58 \ 67 \ 65 \ 70 \ 58 \ 67 \ 63 \ 64 \ 64 \ 65$
 $59 \ 68 \ 71 \ 60 \ 65 \ 71 \ 59 \ 66 \ 66 \ 61 \ 68 \ 60 \ 55 \ 69 \ 56]$

請選擇適當的組數，編製該月生產數量的次數分配表及直方圖。【建議：組數 = $1 + 3.32 \log(\text{分廠數})$ 】

7. (20%)【假設與檢定、差異值檢定；使用第 2 頁所附 t 表】

某大學教授以水槽模型研究波浪發電，試驗兩種機電能傳輸的錨定配置。每種配置皆在多種不同的波浪條件下，先後進行 10 次試驗；所得出力的機械扭距 (bending moment) 各為：

配置 1 = [2.23, 4.09, 9.62, 8.98, 0.82, 10.83, 1.54, 10.75, 5.79, 7.38];

配置 2 = [1.82, 3.46, 9.77, 8.88, 0.87, 11.20, 1.33, 10.32, 5.87, 7.41];

請以兩種配置結果的差異值， $\alpha = 0.05$ 及適當的假設，matched pairs t-test，檢定 (a) 該兩種錨定配置方法的機械扭距是否有所不同，及 (b) 該兩種機械扭距差值在 95% 的信賴區間。

8. (10%)【變異數檢定、ANOVA 表；使用第 2 頁所附 F 表】

某實驗室測定四組不同成份的混凝土成品強度，每組各採 6 個試體，所有數據經計算後填入 ANOVA 表如下 (請填入缺項部份)。請以 $\alpha = 0.05$ 及適當的假設，檢定該四種成份的混凝土成品強度是否不同？

變異來源	平方和	自由度	均方差	拒絕區 (CR)	判斷/結論說明
組間 SSC	382.79			統計量 $F_0 =$ 臨界值 $F =$	
組內 SSE		20			
總變異 SST	512.96				

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Tables 2.11 and 2.14a below are taken from Davis, J.C. (1986)

TABLE 2.11 Critical Values of t for ν Degrees of Freedom and Selected Levels of Significance

Number of Degrees of Freedom, ν	Significance Level, α (%)					
	10	5	2.5	1	0.5	0.1
1	3.078	6.314	12.706	31.821	63.657	318.310
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144
11	1.363	1.796	2.201	2.718	3.106	4.025
12	1.356	1.782	2.179	2.681	3.055	3.930
13	1.350	1.771	2.160	2.650	3.012	3.852
14	1.345	1.761	2.145	2.624	2.977	3.787
15	1.341	1.753	2.131	2.602	2.947	3.733
16	1.337	1.746	2.120	2.583	2.921	3.686
17	1.333	1.740	2.110	2.567	2.898	3.646
18	1.330	1.734	2.101	2.552	2.878	3.610
19	1.328	1.729	2.093	2.539	2.861	3.579
20	1.325	1.725	2.086	2.528	2.845	3.552

TABLE 2.14a Critical Values of F for ν_1 and ν_2 Degrees of Freedom and 5% ($\alpha = 0.05$) Level of Significance

Degrees of Freedom for Denominator, ν_2	Degrees of Freedom for Numerator, ν_1												
	1	2	3	4	5	6	7	8	9	10	12	15	20
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.84	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12

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Answer each of the following problems. Each problem carries 10%. The total is 100 %. Show details of your work.

(1) Evaluate the following limits

(a) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$; (5%)

(b) $\lim_{n \rightarrow \infty} \frac{3^n + (-2)^n}{3^{n+1} + (-2)^{n+1}}$. (5%)

(2) Evaluate the limit

$$\lim_{x \rightarrow 1^-} \frac{\int_x^1 \sqrt{t-1} \cos t \, dt}{(x-1)^{3/2}}.$$

(3) Evaluate the derivative $\frac{d}{dx} \tan^{-1}(5 \tan x)$.

(4) Evaluate the derivative $\frac{d}{dx} x^2|x|$ at $x = 0$.

(5) Let $z = x \sin y - y^2$. Evaluate $\frac{\partial y}{\partial z}$ when $(x, y) = (1, \pi)$.

(6) Evaluate the integral $\int x^5 e^{-x^3} dx$.

(7) Evaluate the integral $\int \frac{1}{\sqrt{-x^2+4x-3}} dx$.

(8) Evaluate the integral for $\Omega = \{(x, y) : x^2 + y^2 \leq 1\}$

$$\iint_{\Omega} \frac{1}{(1+x^2+y^2)^2} dx dy.$$

(9) Show that $\pi^e < e^\pi$. (Hint: Consider the function $\frac{\ln x}{x}$.)

(10) Find all values of $q \in \mathbf{R}$ such that the improper integral $\int_0^\infty \frac{x^2}{1+x^q} dx$ is convergent. Explain your answer.

End of Paper