1. 【Ordinary Differential Equations】 (20%)

(a) Use **two** different methods to solve the ODE: $-2x\sin(x^2)\,dx+\dfrac{\cos(x^2)}{y}\,dy=0$.

(b) Use Laplace Transformation to the IVP (Initial Value Problem)
$y''+5y'+6y=\delta(t-\frac{1}{2}\pi)+u(t-\pi)\cos t$, $y(0)=0$, $y'(0)=0$, where δ () is an unit impulse function and u() is an unit step function.

2. 【Linear Algebra】 (10%)

(a) Given a homogeneous linear system, whose number of variables is 54 and the number of equations is 30, will this linear system have non-trivial (i.e. non-zero) solutions and what's the dimension of the solution space (you may make some assumption here)? Explain your answer?

(b) Given $\hat{\mathbf{A}}=\mathbf{T}^{-1}\mathbf{A}\,\mathbf{T}$, where $\mathbf{A}=\begin{bmatrix}7&0&3\\2&1&1\\2&0&2\end{bmatrix}$, $\mathbf{T}=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$, Find the eigenvalues for both $\mathbf{A}$ and $\hat{\mathbf{A}}$.

3. 【Vector Calculus】 (20%)

(a) Given a curve $C: x^2+y^2=4$, $z=6arc\tan(y/x)$, represent the curve by the parametric form. Find the length along the curve from (2, 0, 0) to (2, 0, 24π).

(b) Evaluate the surface integral $\iint_S curl\,\mathbf{F}\bullet\mathbf{n}\,dA$, where $\mathbf{F}=\langle y^3,-x^3,0\rangle$, $S: x^2+y^2\le 1$, $z=0$. Then verify your answer by Stokes's theorem.

4. 【Partial differential equation】 20%

Linear partial differential equations, $Au_{xx}+2Bu_{xy}+Cu_{yy}=F(x,y,u,u_x,u_y)$ can be classified into one of the three types: elliptic, parabolic or hyperbolic, depending on the condition of B^2-4AC.

(a) Indicate how the condition of B^2-4AC is linked to each of these three types and also provide a typical mathematical equation for each of them. (15%)

(b) Prove $u=2xy/(x^2+y^2)^2$ is a solution to the Laplace equation. (5%)

5. 【Fourier analysis】 10%

Find the Fourier series for a periodic square wave given by the function:

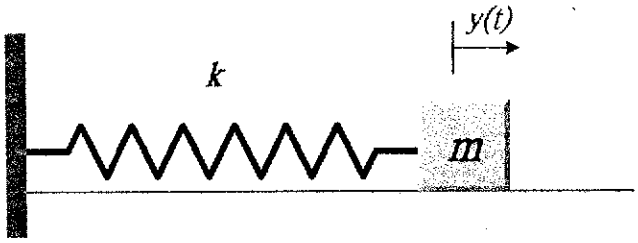
$$f(x)=\begin{cases}0, if -2<x<-1\\ k, if -1<x<1; p=2L=4, L=2\\ 0, if 1<x<2\end{cases}$$

6. 【Residue integration】10%

Evaluate the following integral $\oint \frac{e^{-z^2}}{\sin 4z} dz$

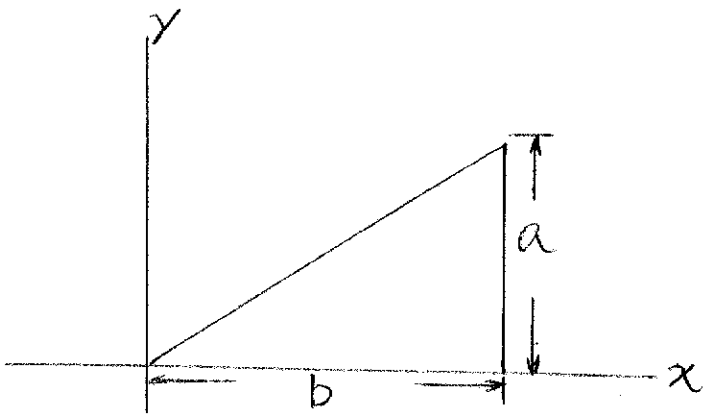
7. 【Modeling spring motion】10%

Suppose that a body of mass m slides without friction on a horizontal surface as shown in the following figure. The body is attached to a spring with spring constant k (the damping force of the spring is excluded), and is also subject to viscous air resistance with coefficient γ (the higher speed, the more air resistance). Formulate a differential equation to simulate this spring motion system, and briefly describe how to solve it.



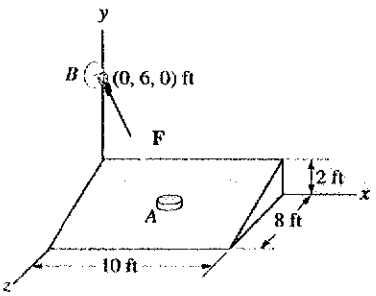
Problem 1

According to the following figure, please calculate the moment of inertia about y-axis I_y. (10%)



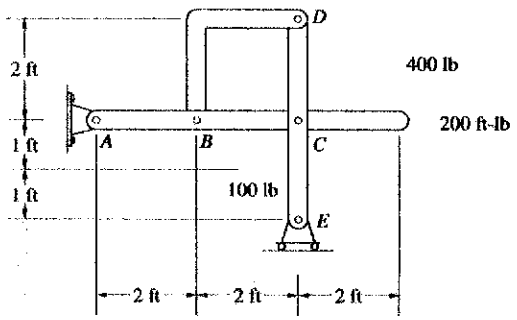
Problem 2.

The disk A is at the midpoint of the sloped surface. The string from **A** to **B** exerts a 2-lb force **F** on the disk. If you resolve F into vector components parallel and normal to the sloped surface, what is the component normal to the surface? (20%)



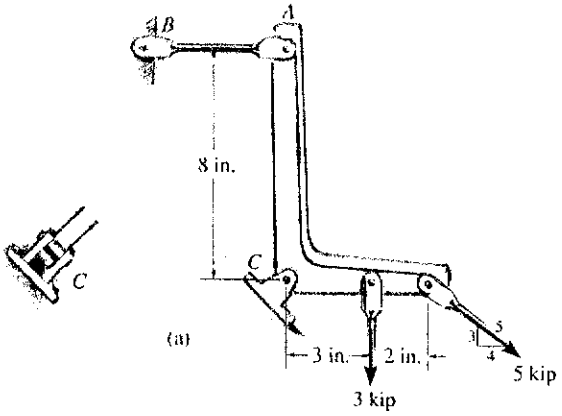
Problem 3

According to the following figure, please determine the forces on joint **A**, joint **B** and joint **C** in member ABC. (20%)



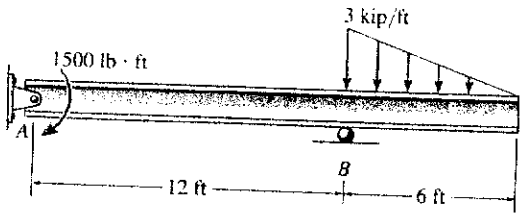
Problem 4

The control arm is subjected to the loading shown. Please determine to the nearest 1/4 in. the required diameter of the steel pin at C if the allowable shear stress for the steel is τ_{allow}= 8 ksi. (25%)



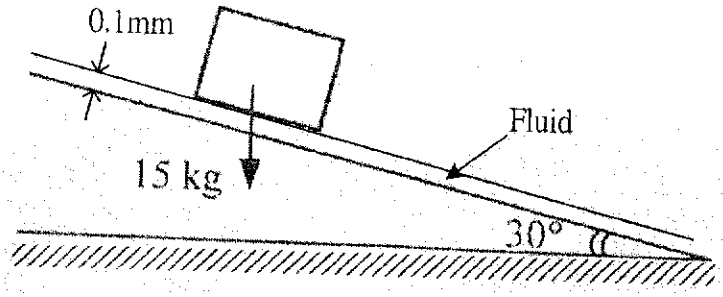
Problem 5

Please draw the shear and bending moment diagrams for the beam and determine the location of maximum moment. (25%)

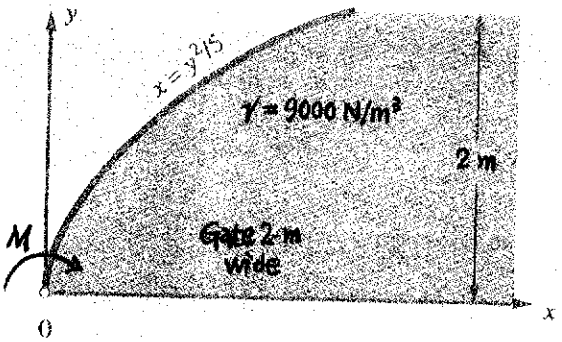


10% 1. Explain the following terms:
(1) cavitation, (2) Froude number, (3) irrotational flow,
(4) dimensional analysis, (5) wake

15% 2. A 15 kg cube slides down an inclined plane making an angle of 30° with the horizontal, shown in the figure. A fluid film 0.1 mm thick separates solid and surface. The fluid viscosity is 0.04 $\text{N}\sec/\text{m}^2$. Assuming that the velocity distribution in the film is linear, find the terminal velocity of the block. The area of the cube is constant with the film is 0.25 m^2.



15% 3. Determine the moment M to hold the gate, neglecting its weight.



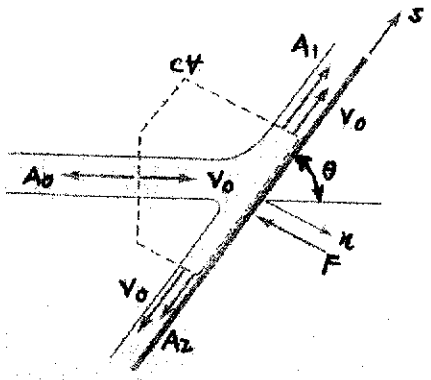
15% 4. The two-dimensional velocity field for an incompressible, Newtonian fluid is described by the relationship

$$\mathbf{V} = (12xy^2 - 6x^3)\mathbf{i} + (18x^2y - 4y^3)\mathbf{j}$$

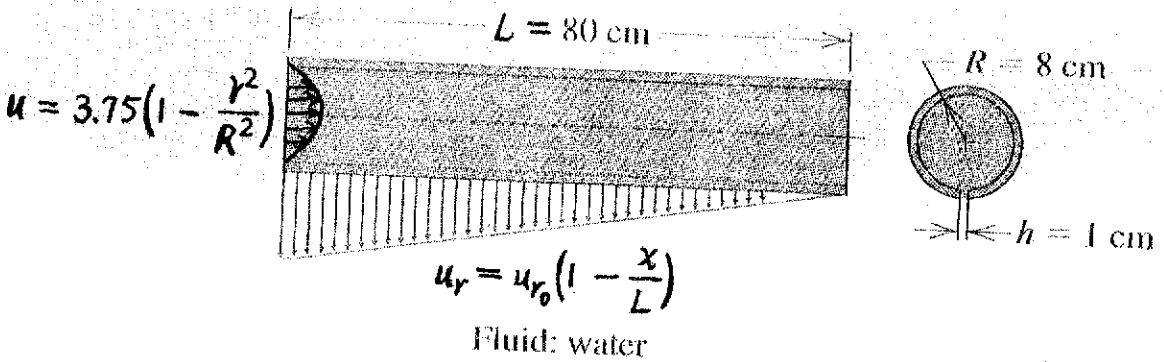
where the velocity has units of meters per second when x and y are in meters. Determine the stresses σ_{xx}, σ_{yy}, and τ_{xy} at the point $x = 0.5$ m, $y = 1.0$ m, if pressure at this point is 6 kPa and the fluid is water at 20 ℃. ($\mu_{\text{water},20^\circ\text{C}} = 1.002 \times 10^{-3}$ N·s/m²)

15% 5. Fluid jet flows from a long slot and strikes against a smooth inclined flat plate determine the division of flow and the force exerted on the plate, neglecting losses due to impact.

(Hint: $Q_1 = \frac{Q_0}{2}(1+\cos\theta)$ $Q_2 = \frac{Q_0}{2}(1-\cos\theta)$)



15% 6. Flow enters a circular pipe as shown in Figure and leaves through a narrow slit along the length of the pipe. If the velocity of the fluid varies linearly, calculate u_{r_0} the maximum velocity at the entrance.



15% 7. Prove the Bernoulli equation and describe its assumptions.

國立中山大學９７學年度碩士班招生考試試題

科目：基礎環境科學【海工系碩士班乙組】

一、解釋名詞：(40%)

1. biomes
2. natural capital
3. ecological footprint
4. industry ecology
5. low-waste economy
6. ecosystem
7. natural greenhouse effect
8. net primary productivity
9. sandy storm
10. ecological engineering

二、問答題：(60%)

1. What are the environmental problems occurring on the Earth? Please, describe the causes of environmental problems。(12%)

2. What is sustainability? What are the scientific principles of sustainability? (12%)

3. Through which processes that ecosystems and biosphere can be sustained? (12%)

4. What is biodiversity ? Can biodiversity be divided into which categories? Why is biodiversity important? (12%)

5. What is hypoxic zone in ocean? How does it occur? What would happen in hypoxic zone in oceans? Give some examples. (12%)

1. 請寫出下述溶液之電荷平衡式(charge balance): 0.005M NaH_2PO_4 (6%)

2. 在一個酸性水溶液中鐵物種濃度分佈為 10^{-5} M in Fe^{+3} and 10^{-3}M in Fe^{+2} 請計算此一水溶液之 pE 值
已知: $Fe^{+3} + e^- = Fe^{+2}$ $pE^0 = 16$ (6%)

3. 一般而言，酸雨(acid deposition) 的主要來源有哪兩個? (6 %)

4. 試述活性碳與離子交換樹酯在淨水方面的功用各為何。(8%)

5. 對於一個瀉湖如大鵬灣而言，請討論在一天 24 小時裡，灣域表層水中 pH 值可能的變化與原因？ (10%)

6. 何謂 Henry's law constant (亨利定律常數)? (3%) 單位為何? (2%) 何謂離子強度? (4%) 亨利定律常數受離子強度影響？(離子強度愈高，亨利定律常數愈高、愈低或不變?) (2%) 亨利定律常數受溫度影響? (溫度愈高，亨利定律常數愈高、愈低或不變?) (2%)

7. 何謂 K_{ow} (辛醇水分配係數)？K_{ow} 值的高低代表的意義為何?(6%)

8. 請寫出下列化合物的化學式或畫出結構式 (9%)
(1)五氯酚(pentachlorophenol) (2) PCBs (polychlorinated biphenyls) (3) BTEX (please give at least two of these aromatic compounds)

9. Assume that gasoline can be represented by C_8H_{18}. How much oxygen is needed to completely burn this fuel (into CO_2 and H_2O) and how much CO_2 is produced. Give your answer in grams of oxygen needed per gram of fuel and grams of CO_2 produced per gram of fuel. (atomic weight: H=1; C=12; O=16) (6%)

10. What are the differences among aerobic respiration, anaerobic respiration, and fermentation? Please, give an example for each of them by using microbial species. (10%)

11. What is nitrification? What is heterotrophic denitrification? What is autotrophic denitrification? Please write down these biochemical reactions and name of the bacterial species involving in the reactions. (10%)

12. In biological wastewater treatments, what is AO System? What is A_2O (or AAO) System? What are their functions for wastewater treatments? Why are their designs like such way? Please, write down their mechanisms. (10%)

國立中山大學９７學年度碩士班招生考試試題

科目：環境保護概論【海工系碩士班丙組選考】

1. 人類社會的貧窮(poverty)與富裕(affluence)皆可能會對環境造成影響，試敘分別述其影響性，並舉例說明之。（25%）

2. 今年的冬天，台灣又再度面臨了來自中國大陸的沙塵暴問題，請問沙塵暴是如何形成的？它給我們帶來的危害性是什麼？我們該如何解決這個環境問題？（25%）

3. 今年的冬天在中國大陸中部地區也發生了嚴重的冰災及雪災，這些地方原本不會發生這麼嚴重的災害，請問是何原因？究竟是天災，還是人禍？（25%）

4. 在海洋某些區域，有時形成所謂的「死亡區(hypoxic zone)」，請問何謂「死亡區」？如何形成？對海洋生態環境造成的危害性為何？那些海域容易形成「死亡區」？試舉一些實際的案例，進行分析。(25%)

1. 請說明下列段落所談之內容，並說明 RFID 之限制與資料庫的關係。(25 分)

Radio-frequency identification (RFID) is an automatic identification method, relying on storing and remotely retrieving data using devices called RFID tags or transponders.

An RFID tag is an object that can be applied to or incorporated into a product, animal, or person for the purpose of identification using radiowaves. Some tags can be read from several meters away and beyond the line of sight of the reader.

Most RFID tags contain at least two parts. One is an integrated circuit for storing and processing information, modulating and demodulating a (RF) signal, and other specialized functions. The second is an antenna for receiving and transmitting the signal. A technology called chipless RFID allows for discrete identification of tags without an integrated circuit, thereby allowing tags to be printed directly onto assets at a lower cost than traditional tags.

Today, a significant thrust in RFID use is in enterprise supply chain management, improving the efficiency of inventory tracking and management. However, a threat is looming that the current growth and adoption in enterprise supply chain market will not be sustainable. A fair cost-sharing mechanism, rational motives and justified returns from RFID technology investments are the key ingredients to achieve long-term and sustainable RFID technology adoption

An advanced automatic identification technology such as the Auto-ID system based on the Radio Frequency Identification technology has significant value for inventory systems. Notably, the technology provides an accurate knowledge of the current inventory. In an academic study performed at Wal-Mart, RFID reduced Out-of-Stocks by 30 percent for products selling between 0.1 and 15 units a day. Other benefits of using RFID include the reduction of labor costs, the simplification of business processes, and the reduction of inventory inaccuracies.

Wal-Mart and the United States Department of Defense have published requirements that their vendors place RFID tags on all shipments to improve supply chain management. Due to the size of these two organizations, their RFID mandates impact thousands of companies worldwide. The deadlines have been extended several times because many vendors face significant difficulties implementing RFID systems. In practice, the successful read rates currently run only 80%, due to radio wave attenuation caused by the products and packaging. In time it is expected that even small companies will be able to place RFID tags on their outbound shipments.

2. 請說明下列段落所談之內容，並說明 VR 之應用潛力及可能之困難。(25 分)

Virtual reality (VR) is a technology, which allows a user to interact with a computer-simulated environment, be it a real or imagined one. Most current virtual reality environments are primarily visual experiences, displayed either on a computer screen or through special or stereoscopic displays, but some simulations include additional sensory information, such as sound through speakers or headphones. Some advanced, haptic systems now include tactile information, generally known as force feedback, in medical and gaming applications. Users can interact with a virtual environment or a virtual artifact (VA) either through the use of standard input devices such as a keyboard and mouse, or through multimodal devices such as a wired glove, the Polhemus boom arm, and omnidirectional treadmill. The simulated environment can be similar to the real world, for example, simulations for pilot or combat training, or it can differ significantly from reality, as in VR games. In practice, it is currently very difficult to create a high-fidelity virtual reality experience, due largely to technical limitations on processing power, image resolution and communication bandwidth. However, those limitations are expected to eventually be overcome as processor, imaging and data communication technologies become more powerful and cost-effective over time.

Virtual reality has been heavily criticized for being an inefficient method for navigating non-geographical information. At present, the idea of ubiquitous computing is very popular in user interface design, and this may be seen as a reaction against VR and its problems. In reality, these two kinds of interfaces have totally different goals and are complementary. The goal of ubiquitous computing is to bring the computer into the user's world, rather than force the user to go inside the computer. The current trend in VR is actually to merge the two user interfaces to create a fully immersive and integrated experience. See simulated reality for a discussion of what might have to be considered if a flawless virtual reality technology was possible. Another obstacle is the headaches due to eye strain, caused by VR headsets.

3. 請說明何謂 web 2.0？其特色為何？可能之應用？ (20 分)

4. 請說明資料庫管理系統(Database Management System)與試算表(spreadsheet)在功能上之差異？ (20 分)

5. Google Map 之網頁開發採用 AJAX (Asynchronous JavaScript and XML) 技術，請說明其原因為何？ (10 分)

1. (15%) Given a linear system $\begin{cases} w-2x+5y-3z=0 \\ -3w+6x+y+z=0 \\ 2w-4x+3y-z=3 \end{cases}$, answer the following questions.

   (a) (5%) Before really solving the linear system, please judge the solution status based on some theorems.

   (b) (10%) Solve the linear system by Gauss Elimination.

2. (20%) Given a matrix $\mathbf{A}=\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

   (a) (5%) Predict if the matrix **A** has an inverse matrix without really solving it.

   (b) (10%) If the matrix **A** has an inverse matrix, find it using Gauss-Jordan Elimination.

   (c) (5%) Also find the inverse matrix of $\mathbf{A}^T$.

3. (15%) Given a matrix $\mathbf{A}=\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (a) (5%) Verify the matrix **A** is an orthogonal matrix.

   (b) (10%) Verify the determinant of the matrix **A** has the value +1 or -1. Also verify the eigenvalues of the matrix **A** are real number or complex conjugates in pairs and have absolute value 1.

4. (15%) Given a matrix $\mathbf{A}=\begin{bmatrix} -5 & 0 & 15 \\ 3 & 4 & -9 \\ -5 & 0 & 15 \end{bmatrix}$, $\mathbf{P}=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. If $\hat{\mathbf{A}}=\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$, find the eigenvalues and eigenvectors of both **A** and $\hat{\mathbf{A}}$.

5. (15%) Given a matrix $\mathbf{A}=\begin{bmatrix} -6 & -6 & 10 \\ -5 & -5 & 5 \\ -9 & -9 & 13 \end{bmatrix}$, find $\mathbf{A}^m$, where m is a large positive integer.

6. (20%) Given a linear system $\begin{cases} 2x_1+x_2+3x_3=-1 \\ 4x_1+x_2+7x_3=5 \\ -6x_1-2x_2-12x_3=-2 \end{cases}$

   (a) (10%) Solve the linear system using *LU* decomposition.

   (b) (10%) Verify the solution in (a) using Cramer's Rule.

第一部份：數學公式/簡要說明題 【60 分】

1. 【Normal distribution】【20 分】

Repeated measurements made on large quantity of sample for x_i ($i = 1$ to N) may produce a frequency distribution curve, called *normal distribution* $N(\bar{x}, \sigma)$, where $\bar{x}$ is the mean and σ is the standard deviation of the samples.

a). Indicate the position of the *mean*, *median* and *mode* on a biased distribution curve.

b). Define the skewness (偏態) and kurtosis (峰態) mathematically using $x_i, \bar{x}$, σ and N.

c). Give a general equation for the probability density function $p(x)$ in $N(\bar{x}, \sigma)$.

d). Upon using the z-score to standardize each sample value, i.e., $z_i = (x_i - \bar{x})/\sigma$, the standardized normal distribution $N(0, 1)$ has zero mean and the curve covers 99.73% within the range of $-3\sigma \le z_i \le +3\sigma$, where σ is the standard deviation of the population. What is the value of probability within the range $-2\sigma \le z_i \le +2\sigma$ and $-\sigma \le z_i \le +\sigma$?

2. 【Regression analysis】【15 分】

Mean value $\bar{x}$ and regressed values $\hat{x}_i$ can be calculated from a set of statistical data x_i ($i = 1$ to N).

a). Define the variance $Var(x)$ in words and express it in a mathematical form.

b). What is the basic principle behind a regression analysis?

c). What is the *goodness-of-fit* R^2 and express it in terms of $x_i, \bar{x}$ and $\hat{x}_i$.

3. 【Covariance and correlation coefficient】【10 分】

From two sets of data x_i and y_i ($i = 1$ to N) observated：

a). Define the *covariance* C_{xy} in words and express it in a mathematical form.

b). Define the *correlation coefficient* ρ_{xy} in words and express it in a mathematical form.

4. 【Statistical tests】【15 分】

Give the mathematical equation for each of the three major statistical tests, namely the (a). t-test, (b). F-test, and (c). χ^2–test, and the main purpose of their applications.

第二部份：計算題 【40 分】

5. 【Test of hypothesis and χ^2-distribution】【20 分】

A well known car company claims that one of its models is guaranteed for 3 years. Random samples taken from 10 drivers indicate this model had first repair in [3.5, 3.1, 3.4, 3.7, 3.2, 3.8, 2.9, 3.2, 3.1, 3.2] years, respectively. Given the standard deviation of the years of usage is 0.2, and for a 95% confidence interval, test whether the claim given by this company is true?

【 Hint: Set up hypothesis H_0 and H_1 and use χ^2–table provided】

6. 【t-distribution and Interval of estimation】【20 分】

A private hospital took six random samples of the weight for the new-born baby, these being 3.2, 2.9, 3.4, 4.0, 3.4 and 3.8 kilograms, respectively.

a). Calculate a 95% confidence interval for the estimation of the mean weight of new-born baby.

b). Given the standard deviation of new-born baby weight as 0.4, find the mean weight for a 90% confidence interval.

【 Hint: Use t-table provided and (a) t-statistic given by $t = \dfrac{\bar{x} - \mu_o}{s\sqrt{1/N}}$; (b) $\bar{x} \pm t_{\alpha(N-1)} \dfrac{s}{\sqrt{N}}$ 】

Part of χ^2-table and t-table for questions 5 and 6, respectively in page 1.

TABLE Critical Values of t for ν Degrees of Freedom and Selected Levels of Significance

Number of Degrees of Freedom, ν	Significance Level, α (%)					
	10	5	2.5	1	0.5	0.1
1	3.078	6.314	12.706	31.821	63.657	318.310
2	1.886	2.920	4.303	6.965	9.925	22.327
3	1.638	2.353	3.182	4.541	5.841	10.215
4	1.533	2.132	2.776	3.747	4.604	7.173
5	1.476	2.015	2.571	3.365	4.032	5.893
6	1.440	1.943	2.447	3.143	3.707	5.208
7	1.415	1.895	2.365	2.998	3.499	4.785
8	1.397	1.860	2.306	2.896	3.355	4.501
9	1.383	1.833	2.262	2.821	3.250	4.297
10	1.372	1.812	2.228	2.764	3.169	4.144

TABLE Critical Values of χ^2 for ν Degrees of Freedom and Selected Levels of Significance

Number of Degrees of Freedom, ν	Significance Level, α (%)				
	20	10	5	2.5	1
1	1.64	2.71	3.84	5.02	6.63
2	3.22	4.61	5.99	7.38	9.21
3	4.64	6.25	7.81	9.35	11.34
4	5.99	7.78	9.49	11.14	13.28
5	7.29	9.24	11.07	12.83	15.09
6	8.56	10.64	12.59	14.45	16.81
7	9.80	12.02	14.07	16.01	18.48
8	11.03	13.36	15.51	17.53	20.09
9	12.24	14.68	16.92	19.02	21.67
10	13.44	15.99	18.31	20.48	23.21
11	14.63	17.28	19.68	21.92	24.72
12	15.81	18.55	21.03	23.34	26.22
13	16.98	19.81	22.36	24.74	27.69
14	18.15	21.06	23.68	26.12	29.14
15	19.31	22.31	25.00	27.49	30.58

Part 1: Differentiation and limits (50%)

1. (20%) Find the derivative for each of the following given functions with respect to x or θ or λ:

(a) $f(x)=\dfrac{x+1}{\sqrt{x}}$; (b) $f(x)=\sqrt[3]{(x^2-1)^2}$;

(c) $f(\theta)=\dfrac{\theta}{1-\sin\theta}$; (d) $f(\lambda)=\ln\dfrac{e^{-\lambda}\lambda^y}{y!}$.

2. (10%) (a) Given $x^2+y^2=25$, find $\dfrac{d^2y}{dx^2}$;

(b) Given $z=ye^{2x}+x\ln y^2$, find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial^2 z}{\partial x\partial y}$.

3. (10%) Water discharges into a large conical tank with its top open. The radius of the top is 5 m and vertical height of the tank is 10 m. If water is running at the constant rate of 2 m^3 per minute, how fast is the water level rising when the water is 6 m deep from its bottom tip? 【Hint: Volume of a whole conical shape = $\pi r^2 h/3$】

4. (10%) Find the limits for:

(a) $\lim\limits_{x\to\infty}\dfrac{2x-1}{\sqrt{3x^2+x+1}}$; (b) $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}-1}{x}$

Part 2: Integration (50%)

5. (10%) $\displaystyle\int_0^4 \frac{x+2}{\sqrt{2x+1}}dx$

6. (10%) $\displaystyle\int \frac{\cos 2x}{\sin^3 2x}dx$

7. (10%) $\displaystyle\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$

8. (10%) $\displaystyle\int x^2e^x dx$

9. (10%) Find the area of the region bounded by the graph of $y=\dfrac{2x}{\sqrt{x^2+9}}$, with $y=0$, $x=0$, and $x=4$.