

國立中山大學 95 學年度碩士班招生考試試題

科目：工程數學【海工系碩士班甲組】

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1. (20%) 求解下列常微分方程式。

(a) (10%) $x^3 y' + x^2 y = 2y^{-4/3}$

(b) (10%) $y'' + y' - 2y = -6\sin 2x - 18\cos 2x$, $y(0) = 2$, $y'(0) = 2$

2. (10%) 利用兩種不同的方法找出拉普拉斯(Laplace)的逆轉換 $\mathcal{L}^{-1}\left[\frac{1}{s^2}\left(\frac{s-1}{s+1}\right)\right]$

3. (10%) 回答下列有關矩陣的問題。

(a) (2%) 找出矩陣 $A = \begin{bmatrix} 5 & 0 & -15 \\ -3 & -4 & 9 \\ 5 & 0 & -15 \end{bmatrix}$ 的秩(rank).

(b) (6%) 找出矩陣 A 的特徵值(eigenvalues) 與特徵向量(eigenvectors).

(c) (2%) 你能將矩陣 A 對角化嗎 (diagonalize the matrix A)? 說明你的理由?

4. (10%) 已知一向量函數 $F = z\mathbf{i} - xz\mathbf{j} + y\mathbf{k}$, 在給定的曲面(surface) S 上 $S: x^2 + 9y^2 + 4z^2 = 36$

, $x \geq 0, y \geq 0, z \geq 0$, 請計算其通量積分(flux integral) $\iint_S \mathbf{F} \cdot \mathbf{n} \, dA$

5. (5%) 已知 $\frac{1}{a^2 + x^2}$ 的 Fourier Transform 為 $-\sqrt{\frac{\pi}{2}} \frac{e^{a|w|}}{a}$, 求 $\frac{x}{(a^2 + x^2)^2}$, $\text{Re}(a) < 0$ 的 Fourier Transform.

6. (15%) 有關 PDE

(a) (10%) 推導 PDE

一弦之縱向張力為常數 T , 密度為 $\rho(x)$, 單位長度橫向受力為 $f(x)$, 請推導其控制方程式;
弦之橫向位移請以 y 表示。

(b) (5%) 以上方程是橢圓/拋物線/雙曲線中的哪一類方程?

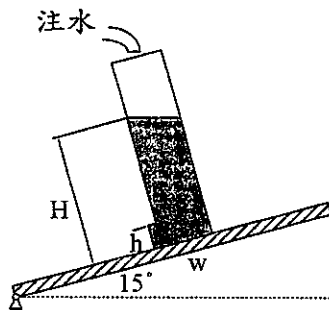
7. (20%) Fourier Series 與 Parseval Relation

(a) (10%) 函數 $f = x, -\pi < x < \pi; f(x + 2\pi) = f(x)$, 求 f 的 Fourier Series。

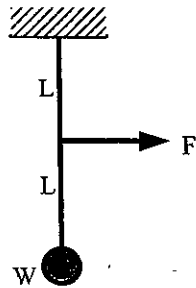
(b) (10%) 求級數和 $\sum_{m=1}^{\infty} \frac{1}{m^2}$

8. (10%) 請用直接積分的方式求複變積分 $\oint_C \frac{1}{z^2} dz$, 其中 C 是以原點為圓心的單位圓。

1. 一個開口為正方形的量杯站立於 15° 光滑斜面上，將水徐徐注入。求 H 為多少時，量杯會傾倒。（不計量杯的質量；量杯開口 $5\text{ cm} \times 5\text{ cm}$ ，擋板 h 高度為 0.5 cm 。） (25%)



2. $L=100\text{ cm}$, $w=10\text{ N}$ ，請問要施力 F 多少時，可將 w 上抬 20 cm 。 (25%)



3. Please draw the shear force and bending moment diagram and also the corresponding deflected shape of the beam shown in Fig. P3. (25%)

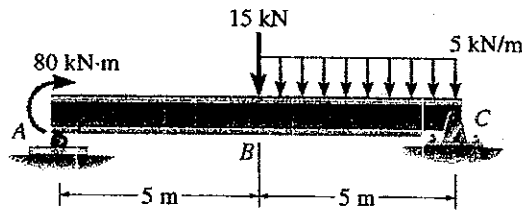
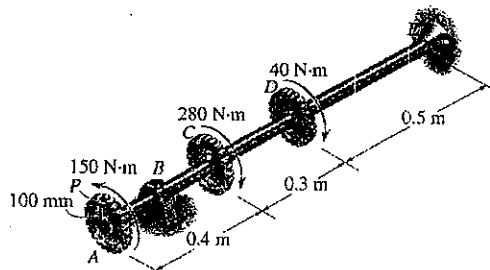


Fig. P3

4. The gears attached to the fixed-end steel shaft are subjected to the torques shown in Fig. P4. If the shear modulus of elasticity is 80 GPa and the shaft has a diameter of 14 mm , please determine the displacement of the tooth P on gear A . The shaft turns freely within the bearing at B . (25%)



- 20% 1. Explain the technical terms.
 (1) No-slip condition, (2) vapor pressure (3) pressure center
 (4) gage pressure (5) mass flowrate (6) ideal gas law
 (7) inviscid flow (8) Darcy-Weisbach equation (9) Drag force
 (10) Reynolds number

- 15% 2. A tank has the shape as Fig. 1. Determine the resultant force on the curved gate AB.

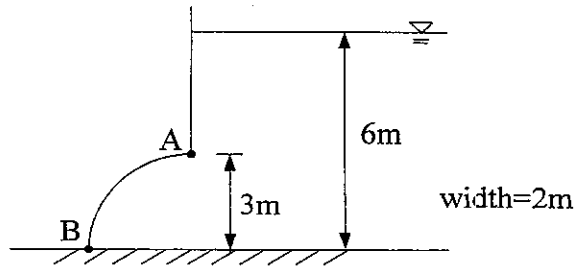


Fig. 1

- 15% 3. A cylindrical buoy has a diameter of 0.8 m and 1.2 m long, weighs 3 kN, and is anchored to the sea floor with a cable shown in Fig. 2. Although most of time it floats on the water surface, at certain time the water depth may increase such that it is completely submerged into water. For this condition what is the tension of the cable, and determine the minimum weight of the concrete block is needed to secure (繫牢) the buoy not to be washed away.

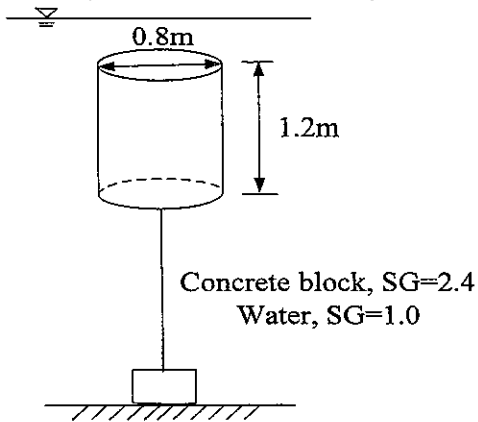


Fig. 2

15% 4. What is the Bernoulli equation and write its assumptions and equation. Use a simple example to illustrate how it works.

15% 5. A flow passing around a cylinder, its stream function can be expressed as

$$\psi = Ur \left(1 - \frac{a^2}{r^2}\right) \sin \theta$$

where U is the velocity of the uniform flow, a is the radius of the cylinder, r and θ are polar coordinates. Determine the corresponding potential function ϕ and the velocity around the cylinder.

20% 6. Water flows from an open channel and is diverted by an inclined plate as illustrated in Fig.3. When the velocity at section (1) is 3 m/s, what horizontal and vertical forces are required to hold the plate in position? At section (1) the pressure distribution is hydrostatic, and the fluid acts as a free jet at section (2). Neglect friction.

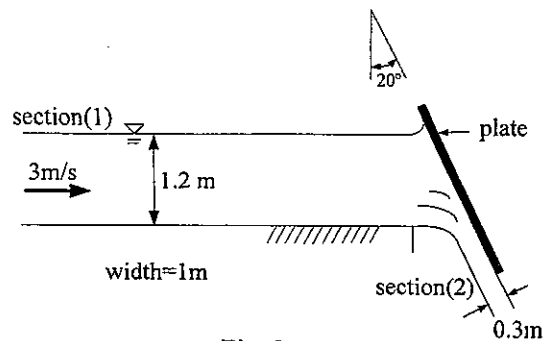


Fig. 3

1. 試解釋說明為何太陽系中只有地球這個星球有各類的生物物種生存的原因。(20%)
2. 何謂海洋污染？海洋污染的種類及其來源有哪些？如何防止海洋污染發生？(20%)
3. 近來地球幾乎每個地區及國家都遭逢非常嚴重的天然災害及極端的季節性及氣候變化，包括風災、水災、嚴寒的冬季及酷熱的夏季、森林及草原火災及土石流等災害，以及民眾飲用水的正常供應，不但困擾人類的日常生活及生命財產上的損失，也嚴重影響了其他生物所棲息的生態環境。試問這些災害發生的原因究竟是天災？人禍？還是兩者都有？試舉出一些實際案例並加以分析解釋之。(20%)
4. 近年來石油不斷的漲價，像這種化石能源(fossil fuels)屬非再生性能源(nonrenewal energy)終有用盡的一天，而目前全世界都在找替代能源，我國也不例外，也正在積極尋找石油以外的方式獲取能源。試問我國正在發展，或未來計畫將進行的替代能源方案有哪些？(20%)
5. 目前我國有關地下水及土壤污染較為嚴重的場址，在北部有 RCA 桃園廠，在南部則有台南市中石化安順廠。兩個舊廠區皆被我國環保署公告為污染整治場址。試說明這兩個場址污染的情況，其影響性，以及我國目前所採取的對策及整治方式。(20%)

國立中山大學 95 學年度碩士班招生考試試題

科目：環境微生物學與環境化學【海工系碩士班乙組】

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1. 試舉三個例子說明環境中具有與他種生物具有互利共生特性(mutualistic)微生物種類的名稱。這些共生體對環境的影響性如何？(10%)
2. 何謂活性污泥(activated sludge)？活性污泥中包含哪些微生物物種？這些物種在廢水處理中所扮演的角色各為何？(10%)
3. 試說明環境中的碳(C)、氫(H)、氧(O)、氮(N)、磷(P)及硫(S)等元素的物質循環(matter cycling)，有哪些微生物物種參與？其參與所扮演的角色為何？試寫出其參與的反應式。(10%)
4. 請寫出 TBT 化合物的化學式或畫出結構式(3%)，並說明環境中這類有機錫主要來源為何？(3%) 對環境生態影響為何？(4%)
5. 請由下列半反應式推導海水中氧的活性(activity)與 pE 值之相關性 (設海水 pH 值為 8)。
$$0.25 \text{ O}_2 + \text{H}^+ + \text{e}^- = 0.5 \text{ H}_2\text{O} \quad \text{pE}^0 = 20 \quad (10\%)$$
6. 含有乙醇($\text{C}_2\text{H}_5\text{OH}$) 100mg/l 之水樣，其理論需氧量(Theoretical Oxygen Demand or Theoretical COD) 值為多少？若此化合物可以很容易被其中之微生物分解，請估計 ($k=0.2/\text{day}$) 水樣之 BOD_5 值 (10%)
7. 請說明一個化合物的 octanol/water partition coefficient (K_{ow}) 的意義，並討論此值的高低對該化合物在環境中的傳輸與宿命有何影響。(10%)
8. 何謂緩衝強度？(3%) 假設有一工廠生產程序連續產生廢酸(約 $\text{pH}=2$ ，但不甚穩定)，排放前需中和至 $\text{pH}=6.5-7.5$ ，緩衝強度在此操作上的重要性為何？(3%) 應如何進行？請詳細說明步驟與需要收集之資訊(4%)
9. 何謂 pzc (point of zero charge or zero of point charge)？對於 pzc 為 9 的顆粒在自然水體中帶正電或負電？如何決定？(10%)
10. 何謂色層分析法(chromatography)？(4%) 利用色層分析法之分析儀器有哪些？分別在環境領域之應用各為何？(6%)

第一部份：數學公式/簡要說明題 【70 分】

1. 【General】【5 分】

What are the FIVE main topics in Statistics. (請列舉統計學的五項內容)

2. 【Regression analysis】【10 分】

Mean value \bar{x} and regressed values \hat{x}_i can be calculated from a set of statistical data x_i ($i = 1$ to N).

- What is the basic principle behind a regression analysis?
- What is the *goodness-of-fit* R^2 and express it in terms of x_i , \bar{x} and \hat{x}_i .

3. 【Covariance and correlation coefficient】【10 分】

From two sets of observation data x_i and y_i , $i = 1$ to N :

- Define the *covariance* C_{xy} in words and express it in a mathematical form.
- Define the *correlation coefficient* ρ_{xy} in words and express it in a mathematical form.

4. 【Terminology】【10 分】

Stevens (1951) proposed four scales of measurements (測量尺度) for statistics analysis. (a) What are the names of these scales? (b) Give a short definition and one example for each of the four scales stated in part (a).

5. 【Normal distribution】【10 分】

Repeated measurements made on large samples of x_i ($i = 1$ to N) may produce a bell-shaped frequency distribution curve, called *normal distribution* $N(\bar{x}, s)$, where \bar{x} is the mean and s is the standard distribution of the samples.

- Indicate the position of the *mean*, *median* and *mode* on a biased distribution curve.
- Define the *skewness* (偏態) and *kurtosis* (峰態) mathematically using x_i , \bar{x} , s and N .

6. 【Normal distribution】【10 分】

- Give a general equation of the probability density function $p(x)$ for the normal distribution $N(\bar{x}, s)$.

- Upon using the z -score to standardize each sample value, i.e., $z_i = (x_i - \bar{x}) / \sigma$, the standardized normal distribution $N(0, 1)$ has zero mean and the curve covers 99.73% within the range of $-3\sigma \leq z_i \leq +3\sigma$, where σ is the standard deviation of the population. What is the value of probability within the range $-2\sigma \leq z_i \leq +2\sigma$ and $-\sigma \leq z_i \leq +\sigma$?

7. 【Statistical tests】【15 分】

Give the mathematical equation for each of the three major statistical tests, namely the (a) t -test, (b) F -test, and (c) χ^2 -test, and the main purpose of their applications.

第二部份：計算題 【30 分】

8. 【Linear Regression】【20 分: 10, 5, 5】

- a) Given a data set of 5 pairs of (x, y) values, in which $x = [-2, 0, 1, 3, 4]$ and $y = [0, 1, 3, 6, 5]$; find the value for constants b_1 and b_0 in the equation $y = b_1x + b_0$ that would best-fit the data. [Graph (x, y) values and the line that best-fit the data to help make sure your answer is correct.]
- b) Compute the standard deviation for x and y .
- c) Compute the covariance between x and y , C_{xy} .

9. 【Test of hypothesis and t -distribution】【10 分】

An environmental scientist wishes to test the hypothesis that a set of 10 beach sediment samples, $x_i = [0.19, 0.23, 0.19, 0.23, 0.27, 0.24, 0.18, 0.22, 0.25, 0.21]$ come from a parent population having a mean median diameter of 0.205 mm and standard deviation of 0.03. Assuming the samples were randomly collected from a normal population, and upon calculating the t -statistic, he wishes to test the null hypothesis

$$\begin{array}{l} H_0: \bar{x} \leq 0.205 \text{ mm} \\ \text{against} \quad H_1: \bar{x} > 0.205 \text{ mm} \end{array}$$

for a significance level α of 0.05 (i.e., it is true only one time in twenty).

- a) STATE clearly what is the possible outcome of the test, given the critical t value equals to 1.833 for degree of freedom 9 and level of significance $\alpha = 0.05$.
- b) What is the interval of estimation (信賴區間) for the sample mean?

【Hint: 應用公式 (a) t -statistic is given by $t = \frac{\bar{x} - \mu_0}{s\sqrt{1/N}}$; (b) $\bar{x} \pm t_\alpha(N-1)\frac{s}{\sqrt{N}}$ 】

國立中山大學 95 學年度碩士班招生考試試題

科目：微積分【海工系碩士班丙組選考】

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1. (20%) Find the derivative for the following given functions with respect to x or θ or λ :

(a) $f(x) = \frac{x+1}{\sqrt{x}}$; (b) $f(\theta) = \frac{\theta}{1-\sin\theta}$; (c) $f(x) = \sqrt[3]{(x^2-1)^2}$; (d) $f(\lambda) = \ln \frac{e^{-\lambda} \lambda^x}{x!}$.

2. (10%) Given (a) $x^2 + y^2 = 25$, find $\frac{d^2y}{dx^2}$; (b) $z = ye^{2x} + x \ln y^2$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial^2 z}{\partial x \partial y}$.

3. (5%) Find the limits for:

(a) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$; (b) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$.

4. (5%) If the bacteria growth in a culture can be expressed as a function in time t , such as

$$P(t) = 1000 \left(1 + \frac{4t}{50 + t^2} \right),$$

where the initial number of bacteria was 1000 and t is for the time in hours.

Find the rate of population growth for the bacteria when $t = 2$ hours.

5. (10%) Find the relative extrema for function $f(x) = -3x^5 + 5x^3$, using first and second order derivatives to assist the classification of either a maximum or minimum.

6. (40%) Find the integral for each of the following functions:

(a) (15%) $\int \frac{dx}{x^{1/3} + x^{1/2}}$.

(b) (10%) $\int [\sin x \cdot \ln(2 + \cos x)] dx$.

(c) (15%) $\int \sqrt{16x - 2x^2 - 23} dx$.

7. (10%) About Series

(a) (5%) Test the series $\sum_{k=1}^{\infty} \left| \frac{\sin k}{2^k} \right|$ for convergence or divergence.

(b) (5%) Find the interval of convergence for the power series $\sum_{k=1}^{\infty} \frac{2^k x^k}{k}$.

1. 請就資訊工業在海洋環境及工程的應用與發展提出你的看法，並舉幾個例子說明。(20%)
2. 目前電腦作業系統的三大主流 MS Windows 與 Linux，請評述其各自的特色及優缺點，並就未來的應用發展提出你自己的觀點。(20%)
3. 請比較 FORTRAN77 與 FORTRAN90 的差異，舉例說明，並評估其應用上的發展。(20%)
4. 解釋名詞 (40%)：(是解釋，不是翻譯，請注意！)
 - (1) Supercomputing
 - (2) PC Cluster
 - (3) Cache Memory
 - (4) Binary Tree
 - (5) ASCII
 - (6) HTTP
 - (7) GIS
 - (8) GPS
 - (9) Data mining
 - (10) High level language

1. 何謂京都議定書？試述京都議定書與溫室效應的關係，以及台灣應該如何努力才能達成目標？(15%)
2. 請描述台灣目前的海洋環境現狀及你的看法。(20%)
3. 請就環境保護與永續發展的基本概念及相互的關係論述。(15%)
4. 人類影響生物物種的演化方式包括有人工育種(artificial selection)及基因工程(genetic engineering)等技術，試解釋兩種技術的差異性。此種技術未來可能對生態環境及人類文明社會造成的衝擊性有哪些？(20%)
5. 目前我國有關地下水及土壤污染較為嚴重的場址，在北部有 RCA 桃園廠，在南部則有台南市中石化安順廠。兩個舊廠區皆被我國環保署公告為污染整治場址。試說明這兩個場址污染的情況，其影響性，以及我國目前所採取的對策及整治方式。(15%)
6. 近兩年來，桃園地區皆面臨石門水庫滿水位，但卻缺水的夢魘，其原因為何？政府相關水資源單位又何因應的對策？你認為該方式是否可以解決此一問題？不論是否請申述你的理由。如不適當，你又有何建議？(15%)

1. (10%)

(a) Let A be a square matrix. Will $(A^n)^T = (A^T)^n$? Explain your answer. (5%)(b) Let A be a square matrix. Will the determinant $|\mathbf{A}^{-1}\mathbf{A}^T\mathbf{A}| = |\mathbf{A}|$? Explain your answer. (5%)

2. (15%)

(a) Determine the inverse of the matrix $\mathbf{A} \begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$, if it exists, using the method of

Gauss-Jordan elimination. (10%)

(b) If the matrix A in (a) is the coefficient matrix of a homogeneous linear system, how many solution(s) can you expect? (5%)

3. (15%)

(a) Show the set of vectors $\{(1,3,-4), (3,-1,4), (1,0,-2)\}$ is linearly independent or dependent. (10%)

(b) Can the set of vectors in (a) form a basis of a vector space? Explain your answer. (5%)

4. (30%)

(a) Please list two real world applications of eigenvalues and eigenvectors. (5%)

(b) Find the eigenvalues and the corresponding eigenvectors of the matrix $\mathbf{A} \begin{bmatrix} 15 & 7 & -7 \\ -1 & 1 & 1 \\ 13 & 7 & -5 \end{bmatrix}$. (10%)(c) Compute \mathbf{A}^3 by diagonalizing the matrix. (15%)

5. (15%)

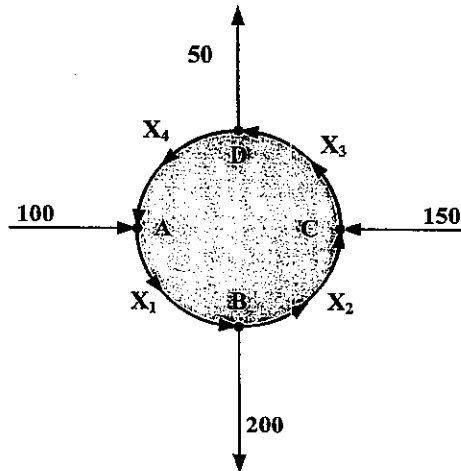
(a) The following matrix defines a communication network. Plot the network (digraph) using the vertices and oriented arcs. (5%)

$$\begin{array}{c}
 P1 \ P2 \ P3 \ P4 \ P5 \\
 P1 \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\
 P2 \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\
 P3 \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\
 P4 \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\
 P5 \begin{bmatrix} 1 & 0 & 0 & 0 & 0
 \end{array}$$

(b) Find the shortest paths for sending a message from $P2$ to $P5$ and $P3$ to $P2$. (10%)

6. (15%)

- (a) The following figure represents the traffic entering and leaving a "roundabout" road junction. Construct a mathematical model that describes the flow of traffic along various branches. (7%)



- (b) What is the minimum flow theoretically possible along the branch BC? Is this flow ever likely to be realized in practice? (8%)