(1) (20%) 求解下列微分方程式

  (a) $y' + \frac{1}{2}y = y^3$, $y(0) = 1$ (10%)

  (b) $y'' + 2y' - 3y = 8e^{-t} + \delta(t - \frac{1}{2}), y(0) = 3, y'(0) = -5$ ，$\delta(\ )$ 是單位脈衝函數（unit impulse function）. (10%)

(2) (10%) 已知矩陣 $A = \begin{bmatrix} 1 & 4 & 2 & 4 \\ 1 & 3 & 1 & 2 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$

  (a) 求矩陣 A 的 rank. (5%)

  (b) 假設有一個線性系統 $Ax = 0$, A 是本題所給的矩陣. 求矩陣 A 的 null space 以及它的 nullity. (5%)

(3) (10%) The velocity vector **v** of an incompressible fluid rotating in a cylindrical vessel is of the form $\mathbf{v} = \mathbf{w} \times \mathbf{r}$, where **w** is the rotation vector and **r** is the position vector. Show that there is no divergence in this flow field.

(4) (10%) 求面積分 $\iint\limits_{S} \mathbf{F} \bullet \mathbf{n}\, dA$, $\mathbf{F} = [x^3, y^3, z^3]$ ，S 為球面 $x^2 + y^2 + z^2 = 4$.

(5) (10%) 求 $\sin x, 0 < x < \pi$ 之 Fourier 級數.

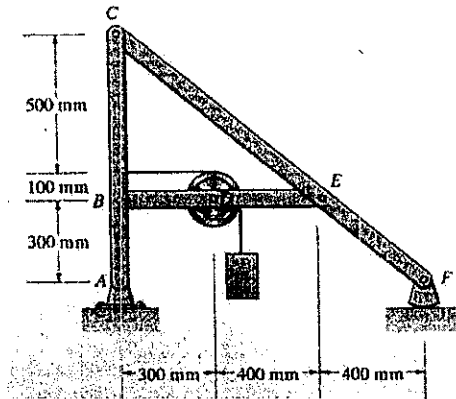(6) (15%) 求 $\sum\limits_{n=0}^{\infty} a^n \cos n\theta$ 之級數和.

(7) (15%) 導出 Fourier integral.

(8) (10%) 由分離變數法解 $\nabla^2 \phi = \dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} + \dfrac{\partial^2 \phi}{\partial z^2} = 0$ 之通解

1. 【靜力學】（25%）

   What is the largest mass m that the frame will safely supported, if the resultant force at pin support at B is 50-kN? (B&F, 1999)
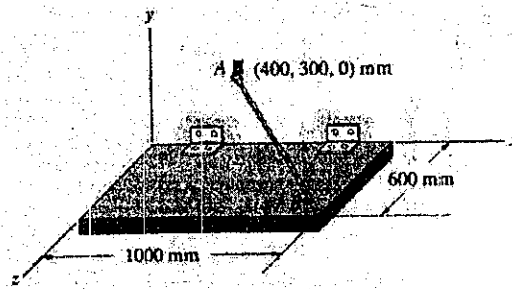
   【Hint：To solve this problem by taking different fee-body-diagrams】

2. 【靜力學】（25%）

   Determine the moment about the x-axis due to a tensile force of 5 kN along the cable AB on the hatch door at point B. (B&F, 1999)

   【Suggestion：It is more efficient to solve this problem using a vector approach.】

3. 【材料力學】（30%）

   A T-beam is subjected to the loading as shown below, (a) Draw the bending moment and shear force diagrams, and (b) Find the maximum normal stress in the beam.
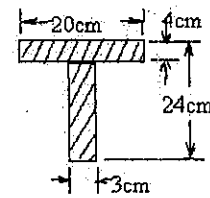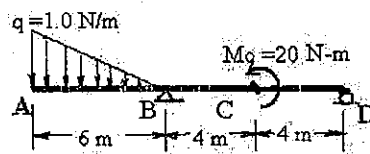
4. 【材料力學】（20%）

   A cantilever beam AB is subjected to a concentrated force P at end B as shown in Fig. 4(a).

   (a). Find the deflection δ_0 at B.

   (b). If a spring is added at B to reduce the potential deflection to 1/2 δ_0 calculated in part (a), what would be the stiffness of the spring?
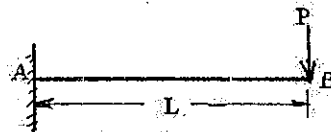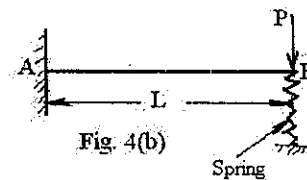
   Fig. 4(a)

   Fig. 4(b)

   Spring

(30%)1. Explain the following terms

    (1) Reynolds Number   (2) Pressure center

    (3) Froude Number   (4) Pascal's law

    (5) Gage pressure   (6) Continuum

    (7) Laminar boundary layer   (8) Discharge coefficient

    (9) Cavitation   (10) Piezometric head

(15%)2. A laminar flow occurs between two horizontal parallel plates under a pressure gradient dp/dx (p decreases in the positive x direction). The upper plate moves left (negative) at velocity u_t. The expression for local velocity u is given as

$$u = -\frac{1}{2\mu}\frac{dp}{dx}\left(Hy - y^2\right) + u_t\frac{y}{H}$$

    (1) Find the shearing stresses at the moving plate (y = H) and at the stationary plate (y = 0).

    (2) Which is the magnitude of shearing stress greater? Why?
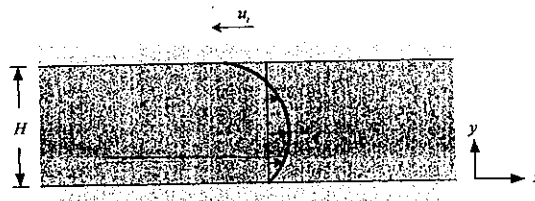
(10%) 3. Estimate the excess pressure inside a raindrop having a diameter of 3 mm. Assume the water temperature is 15.6 ℃, and its surface tension (σ) is 0.0734 N/m.

(15%) 4. Water flowing from a oscillating nozzle produces a projected water jet. Assume that its velocity field in a certain region is approximated by

$$u = u_0$$

$$v = v_0(1-\frac{y}{h}) \quad \text{for } 0<y<h ; \quad v = 0 \quad \text{for } y>h$$

    Determine the equation for streamline and plot the streamline that passes through the origin for values of $u_0/v_0 = 1$ and $h = 5$.

(15%) 5. Derive a general form of Bernoulli Equation and explain why it is so important in engineering applications.

(15%) 6. A certain spillway for a dam is 20 m wide and is designed to carry 125 m^3/s at flood stage. A 1:15 model is constructed to study the flow characteristics through the spillway. Determine the required model width and flowrate. What operating time for the model corresponds to a 24-hour period in the prototype? The effects of surface tension and viscosity are to be neglected.

一、解釋名詞：(20%)

1. biodiversity
2. ecological corridor
3. ecological resoureces
4. ecological footprint
5. sandy storm

二、問答題：(80%)

1. 試說明能夠維持地球生態系統中生命現象的三大因子，以及彼此之間的關聯性。(10%)

2. 何謂環境技術(Environmental Technology)？何謂生態技術(Ecotechnology)？何謂生物技術(Biotechnology)？試比較彼此之間之相關性及差異性。(10%)

3. 何謂「京都議定書」？其主要內容為何？此議定書對我國的的衝擊性為何？(10%)

4. (a)試描述「食物鍊(food chains)」，並討論其中的能量轉化效率（6%）
(b)試論核能發電的優勢與缺點。（8%）

5. (a)科學家經由基因改造工程(genetically engineered)發展出一種農作物在整個生長期間會產生天然殺蟲劑(natural pesticide)，如果大量推廣會有何疑慮？（6%)(b) 何謂環境荷爾蒙？主要影響為何？。（6%）

6. (a)試描述影響地球表面溫度的能量平衡。（6%）(b)試描述造成地球表面二氧化碳濃度升高的兩個主要成因。（6%） (c) 試論聖嬰現象（El Nino）及其影響。（6%）

7. 高雄港港內沉積物濬渫後打算海拋，試論在環境影響評估時需注意的重點，應進行何種測試？（6%）

國立中山大學海洋環境及工程系
九十四學年度碩士班乙組環境流體力學試題

一、試述柏努力公式（Bernoulli's Equation）原理，並舉例說明。(20%)

二、簡述一維水動力的控制方程式（或稱淺水方程式）。(20%)

三、以下管網的地面高程是 130m，埋管深度是 2m，經分析後其流量
如下圖，請計算其水力分佈，並分析其問題所在並提出可能之解
決方案。 (20%)

提示：A 點供水高程為 $H_A = 200m$，$\Delta H = KLQ^2$

BRANCH	K	L(m)	Q(final)
AB	1.42376	600	0.1553
BC	3.76475	800	0.0964
CD	12.37630	500	0.0495
DE	12.37630	200	0.0495
FE	12.37630	500	0.0505
GF	3.76475	800	0.1036
HG	1.42376	600	0.1447
AH	0.30697	200	0.2447
BG	1.42376	200	0.0589
CF	12.37630	200	0.0469

四、有一下水道圓管直徑 300mm，坡度是千分之三，n=0.013，請以
曼寧公式計算其滿管的流量及流速。(10%)

五、某城市內有一商業區，長 100m，寬 50m，降雨強度為 100mm/hr，
試計算其逕流量？其連接之下水道為直徑 200mm，長度為五公
里，若區內逕流至該管渠需時 5 分鐘，請計算其集流時間
(concentration time)? (20%)

六、新設過濾池一座，流量需求為 500CMD，濾率為 0.3cm/s，請問該
過濾池需要多大的面積？(10%)

1. 試列舉三個與微生物相關之環境污染議題，並敘述由何種類微生物所主導，以及說明為何該種微生物會引發此一環境污染問題。(10%)

2. 試依據微生物各種不同的代謝作用方式，對其加以分類。試說明該分類的名稱，並各舉出一例，包括參與微生物種類名稱、及其所進行之異化作用及同化作用的生化反應式。(10%)

3. 試說明飲用水水質所採用的微生物指標包含有那些？各個指標為何可以做為飲用水水質之指標，其所代表之意義又為何？(10%)

4. 試論 (a)疏水性有機污染物在土壤介質中之傳輸現象受哪些主要特性影響？(b)金屬如 Pb、Cd 等離子在土壤介質中之傳輸現象受哪些主要特性影響？（12%）

5. 某有機污染物在水環境中裂解(degradation)屬一級反應(first order reaction)，在一次意外洩漏到一水環境中 50 天後，發現該污染物濃度剩下原來 60%，請列式子說明如何估計需要多少天該污染物濃度會剩下 20%。（8%）

6. 請說明為何一樣的酸雨下在不同的湖泊會有不同的 pH 變化影響？(5%)

7. 氧化還原電位半反應式：

$Ag^+ + e^- = Ag \quad E^\circ=0.8V$
$Cu^{2+} + 2e^- = Cu \quad E^\circ=0.4V$

試求 $Cu + 2Ag^+ = 2Ag + Cu^{2+}$ 之 E° (5%)

8. 試舉任兩個環境中發生的氧化還原反應的例子並說明之。(8%)

9. 試以膠體(colloidal)化學理論解釋混凝現象。(5%)

10. 試計算 180mg/L 之 CH_3COOH 之總有機碳(TOC)與理論需氧量(theoretical oxygen demand)。(8%)

11. 請解釋熱力學第一定律 (First law of thermodynamics) (5%)

12. 請解釋 K_{ow} (octanol/water partition coefficient)，及其意義。(5%)

13. 請說明何謂初級處理(primary treatment)、二級處理(secondary treatment)與三級處理(tertiary treatment) (9%)

第一部份：數學公式/簡要說明題 【55分】

1. 【Normal distribution】【15分：6 @2.5】
   Repeated measurements made on large samples of x_i ($i = 1$ to N) may produce a bell-shaped frequency distribution curve, called *normal distribution* $N(\bar{x}, \sigma)$.
   (1) What are the physical meaning of $\bar{x}$ and σ?
   (2) Give a mathematical equation to calculate σ.
   (3) What should we do in order to convert the *normal distribution* $N(\bar{x}, \sigma)$ into the standard form of $N(1, 0)$.
   (4) Draw a normal distribution, and indicate the probability (in percentage) for the region covered between -3σ and $+3\sigma$.
   (5) Indicate the position of the *mean, median* and *mode* on a biased distribution curve.
   (6) Define the skewness (偏態) and kurtosis (峰態) mathematically using $x_i, \bar{x}$, σ and N.

2. 【Statistical tests】【15分：3 @5】
   Three major statistical tests are commonly in use, these being the *t*-test, the *F*-test and the χ^2-test. Give the mathematical equation and describe the main purposes of potential applications for each of the test method mentioned above.

3. 【Least squared method】【15分】
   A student wants to fit a straight line manually (without using EXCEL or MATLAB software) to a set of experimental data by the least squared method. Given the total number of data as "N" and the set of data points as $x_i = [x_1, x_2, ..., x_{N-1}, x_N]$ for $i = 1 \sim N$, and the straight line equation to be derived as $y_i = a + b\,x_i$, please derive the mathematical equation for variable a and b in this equation.

4. 【Covariance and correlation coefficient】【10分：2 @5】
   From two almost identical observation data sets x_i and y_i, $i = 1$ to N :
   (1) Define the *covariance C_{xy}* in words and express it in a mathematical form.
   (2) Define the *correlation coefficient ρ_{xy}* in words and express it in a mathematical form.

第二部份：計算題 【45分】

5. 【Normal distribution】【15分：1 @5 and 1 @ 10】
   A random survey result drawn from a sample of 8 university students found the number of times per month for each student visited a KTV in December 2004 are: 1, 4, 2, 1, 2, 3, 1 and 2, respectively. Calculate (1) the sample mean and variance, and (2) The skewness and kurtosis.

6. 【Test of hypothesis】【15分】
   A car battery manufacturer indicates that one of its products would last at least 3 years. A random sample of 10 customers who have used this particular battery found the number of years recorded are 3.5, 3.1, 3.4, 3.7, 3.2, 3.8, 2.9, 3.2, 3.1 and 2.1, respectively. If the variance of this product is 0.2 and for a level of significance at $\alpha = 0.05$, test statistically whether the statement of $\sigma^2 = 0.2$ is correct? 【Hint: use the χ^2 分配表 attached】

7. 【Confidence interval】【15 分： 3 @ 5】
UFO coffee is a new product on the market. The weight of the content indicted on its label is 200 grams. Random samples of 50 tins of this product found there are 2 tines with MEAN weight of 190 grams, 20 tines with 195 grams, 20 tines with 205 grams and 8 tines with 210 grams.
(1) Calculate the mean and standard deviation of the samples.
(2) Calculate the confidence interval for a 95% of confidence interval.
(3) Is it correct for the producer to indicate the content of the coffee as 200 grams on its label?
【Hint: use the 標準常態分配表 attached】

標準常態分配表

z	0	1	2	3	4	5	6	7	8	9
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9430	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9648	.9656	.9664	.9671	.9678	.9686	.9693	.9700	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9762	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9874	.9878	.9881	.9884	.9887	.9890

χ^2 分配

$1 - \alpha$
χ^2
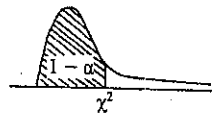
f	Probability $1 - \alpha$							
	.005	.010	.025	.050	.950	.975	.990	.995
1	---	---	---	.004	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	23.68	26.12	29.14	31.32
15	4.60	5.23	6.25	7.26	25.00	27.49	30.58	32.80

1. (20%) About Continuity.
   a) The temperature on a specific day at a given location is considered as a function of time. Is this special function a continuous function? Explain your answer. (7%)
   b) The charge for a taxi ride is considered as a function of mileage. Is this special function a continuous function? Explain your answer. (7%)
   c) Given a real function $f(x) = \begin{cases} \dfrac{\sin ax}{x} & if \ x < 0 \\ 5 & if \ x = 0 \\ x + b & if \ x > 0 \end{cases}$, find constants a and b so that $f(x)$ is continuous.

      (6%)

2. (15%) About Limit.
   a) Determine $\lim\limits_{x \to 0} \cos\left(\dfrac{1}{x}\right)$ (7%)
   b) Determine $\lim\limits_{x \to 0} \dfrac{e^{-4x} - 1}{e^{-2x} + e^{-x} - 2}$ (8%)

3. (20%) About Differentiation.
   a) Find the derivative $\dfrac{dy}{dx}$ in terms of x alone using $y = \dfrac{e^{2x}(2x-1)^6}{(x^3+5)^2(4-7x)}$. (10%)
   b) In a healthy person of height x inches, the average pulse rate in beats per minute is modeled by the formula $P(x) = 596/\sqrt{x}$, $30 \le x \le 100$. Estimate the change in pulse rate that corresponding to a height change from 59 to 60 inches. (10%)

4. (30%) About Integration.
   a) Evaluate $\int [\sin x \ln(2 + \cos x)]dx$ (15%)
   b) Evaluate $\int x^3 \sqrt{x^2 - 1} \, dx$ (15%)

5. (15%) About Infinite Series.
   a) Test the series $\sum\limits_{k=1}^{\infty} \dfrac{2k^3 + k + 1}{k^3 + k^2 + 1}$ for convergence or divergence. (7%)
   b) Find the interval of convergence for the power series $\sum\limits_{k=1}^{\infty} \dfrac{(x+1)^k}{3^k}$. (8%)

一、何謂環境工程技術(environmental technology)？何謂生物技術
(biotechnology)？何謂生態工程技術(ecotechnology)？三者
間之相關性為何？（25%）

二、何謂「京都議定書」？其主要內容為何？該議定書有些
大國諸如美國及中國尚未簽訂，其根節點為何？而該議
定書對我國之衝擊性為何？我國又有何因應的對策？
(25%)

三、最近釣魚台列嶼和東海春曉油田的開發引起很大的關
切，請問叢爾小島有何重要性？請以所知案例，說明島
嶼的保育利用在規劃上有何重點應該特別注意？
(25%)

四、台灣河川的主要問題何在？河川與海洋關係如何？對於
河川巡守有何改善建議？(25%)

1. 請說明下列段落所談內容，並說明 ODBC 之重要性及未來可能發展。(20 分)

「It fell to Microsoft to lay down a standard abstraction barrier in January 1993: Open Database Connectivity (ODBC). Then companies like Intersolv released ODBC drivers. These are programs that run on the same computer as the would-be database client, usually a PC. When the telephone operator's forms application wants to get some data, it doesn't connect directly to Oracle. Instead, it calls the ODBC driver which makes the Oracle connection. In theory, switching over to Sybase is as easy as installing the ODBC driver for Sybase. Client programs have two options for issuing SQL through ODBC. If the client program uses "ODBC SQL" then ODBC will abstract away the minor but annoying differences in SQL syntax that have crept into various products. If the client program wants to use a special feature of a particular RDBMS like Oracle Context, then it can ask ODBC to pass the SQL directly to the database management system. Finally, ODBC supposedly allows access even to primitive flat-file databases like FoxPro and dBASE. You'd expect programs like Microsoft Access to be able to talk via ODBC to various and sundry databases. However, this flexibility has become so important that even vendors like Oracle and Informix have started to incorporate ODBC interfaces in their fancy client programs. Thus you can use an Oracle-brand client program to connect to an Informix or Sybase RDBMS server. The point here is not that you need to rush out to set up all the database client development tools that they use at Citibank. Just keep in mind that your Web server doesn't have to be your only RDBMS client.」

2. 請說明下列段落所談之內容，並說明 Database 對 ecommerce 之重要性，資料庫在進行 ecommerce 之可能做法，並說明此問題未來可能之發展。(20 分)

「In the past four years, a particular kind of ecommerce site development project seems to cross my desk repeatedly. A manufacturing company has been selling to retailers for decades. The Web site will be the first time that they've sold directly to customers. Customers will get the ability to order semi-custom products and the factory will get real-time access to information about what customers want.

An obvious first cut at the problem involves three database installations:

1. an RDBMS on the Web server to hold catalog info, user personalization information, and orders,
2. an RDBMS at a bank or service bureau that holds credit card numbers to be billed,
3. an RDBMS in the factory that collects orders in batches from the Web server,

There are some good things about this solution. You aren't responsible for keeping customer credit card numbers secure. Data that are needed for real-time decisions are kept close to the programs that use the data.

Suppose, however, that the Web server needs a lot of data from the factory tables before it can offer delivery information. It is possible to keep copies of the necessary factory tables in the Web server's RDBMS. Every time an update or insert is made on the factory RDBMS, the transaction is duplicated on the Web server RDBMS. This is called *database replication* and companies like Oracle produce software to facilitate replication. However, you have to budget extra time and money to develop and maintain a replication strategy. So you might consider eliminating the RDBMS next to the Web server and configuring the Web server program to make an encrypted connection to the factory RDBMS.

Suppose that you are quickly turning out custom goods. You can't wait for batches of orders to be sent to the factory because products are supposed to be built and shipped on the same day as orders placed before 2:00 pm. This means that the factory RDBMS must have rapid access to the relevant tables from the Web server RDBMS. Again, you have the option of replication or eliminating the factory database installation and having the factory client programs talk directly to the Web server RDBMS.

Suppose that you want to offer customers the option to "bill the same credit card as on your last order, ending in 4561." In that case you either need to keep the old credit card numbers yourself or have some means of communication between your database and your transaction vendor (bank).

Suppose that your accounting system can run the credit card numbers. In that case, you will probably want to eliminate the RDBMS at the bank and tightly couple the Web site to whatever RDBMS installation is running the accounting system.

There are no correct answers to the questions of "how many RDBMS installations and where are they" but you need to address them nonetheless. 」

3. 請說明下列段落所談之內容，並說明分散式資料庫特性、作法及其未來可能之發展。(20 分)

『The Mariposa distributed database management system is an ongoing research project at the University of California at Berkeley. Mariposa addresses fundamental problems in the standard approach to distributed data management. We believe that the underlying assumptions traditionally made while designing distributed data managers do not apply to today's wide-area network (WAN) environments.

To date, distributed database management systems have been designed for local-area networks (LANs) with few servers operating within one administrative domain, such as one company or one department within a company. Furthermore, these systems assume uniformity of all processors and network connections within the system. Data movement in these systems is a very heavyweight operation and is performed manually by a database administrator. The explosive growth of distributed computing, illustrated by the World Wide Web, dictates an entirely different set of assumptions.

Mariposa allows DBMSs which are far apart and under different administrative domains to work together to process queries. Furthermore, we have introduced an economic paradigm in which processing sites buy and sell data and query processing services. Not only does this approach reflect the emerging reality of a commercialized Internet, it has also allowed us to address many of the problems inherent in designing a wide-area distributed DBMS. Mariposa has been designed with the following principles in mind:

- Scalability to a large number of cooperating sites. In a WAN environment, there may be a large number of sites. Our goal is to scale to 10,000 servers.
- Local autonomy. Each site must have control over its resources. This includes which objects to store and which queries to run. Query and data allocation cannot be done by a central, authoritarian query optimizer.
- Data mobility. It should be easy and efficient to change the home of an object. Preferably, the object should remain available during movement.
- No global synchronization. Updates and schema changes should not force a site to synchronize with all other sites. Otherwise, many common operations will have exceptionally poor response time.
- Easily configurable policies. It should be easy for a local database administrator to change the behavior of a Mariposa site. A Mariposa system should respond gracefully to changes in user activity and data access patterns to maintain low response time and high system throughput. 」

4. 試以任一種程式語言(例如：Fortran、Visual Basic、Visual C++等)計算一數列 1+ (1/3) + (1/5) + (1/7) + (1/9) +........之值(精度達到小數點第六位)，程式中須附註說明所使用各參數之意義。(20 分)

5. 在地理資訊系統(Geographic Information Systems)必備數值地圖(digital map)，但目前在網路搜尋引擎並不具備此功能，試說明可能解決之道。(20 分)

(1) (30%) The original linear system is written as following:
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$
If we apply

Gauss elimination to the system and the reduced system will have the form:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots\cdots + a_{1n}x_n = b_1 \\ \quad\quad c_{22}x_2 + \cdots\cdots + c_{2n}x_n = \tilde{b}_2 \\ \quad\quad\quad\quad \vdots \\ \quad\quad\quad\quad k_{rr}x_r + \cdots + k_{rn}x_n = \tilde{b}_r \\ \quad\quad\quad\quad\quad\quad 0 = \tilde{b}_{r+1} \\ \quad\quad\quad\quad\quad\quad\quad \vdots \\ \quad\quad\quad\quad\quad\quad 0 = \tilde{b}_m \end{cases}$$
, where $r \le m$ and $a_{11}, c_{22}, \cdots, k_{rr} \ne 0$.

Under the prescribed conditions, what will be the system solution(s)?

(a) If $r < m$ and one of the numbers $\tilde{b}_{r+1}, \cdots, \tilde{b}_m$ is not zero.

(b) If $r < n$ and $\tilde{b}_{r+1}, \cdots, \tilde{b}_m$, if present, are zero.

(c) If $r = n$ and $\tilde{b}_{r+1}, \cdots, \tilde{b}_m$, if present, are zero.

(2) (20%) An electrical network consists of nodes and branches. One node is the reference node (grounded node, whose voltage is zero). We number the other nodes, and number and direct the branches. Such network can be described by a so called nodal incidence matrix $\mathbf{A} = [a_{jk}]$, where

$$a_{jk} = \begin{cases} +1 & \textit{if branch k leaves node j} \\ -1 & \textit{if branch k enters node j} \\ 0 & \textit{if branch k does not touch node j} \end{cases}$$

(a) Find the nodal incidence matrix of the following network.
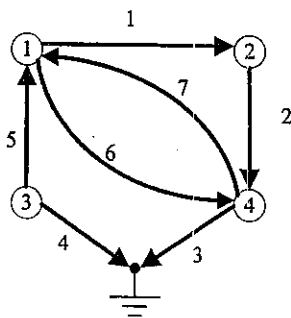
(b) Sketch the network corresponding to the nodal incidence matrix $A = \begin{bmatrix} 1 & -1 & 0 & 0 & 1 \\ -1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$.

(3) (30%) Given a matrix $A = \begin{bmatrix} 10 & -3 & 5 \\ 0 & 1 & 0 \\ -15 & 9 & -10 \end{bmatrix}$

(a) Find the eigenvalues and the corresponding eigenvectors.

(b) Given $T = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 5 \end{bmatrix}$ and $\hat{A} = T^{-1}AT$, please verify that A and $\hat{A}$ are similar in terms of their eigenvalues and eigenvectors.

(4) (20%) The Leslie model describes age-specified population growth. Let the oldest age, that the females in some animal population can live, be 6 years. Meanwhile, the 6 years life span is divided into three age classes, with 2 years each. Let the "*Leslie matrix*" be:

$L = [l_{jk}] = \begin{bmatrix} 0 & 2.3 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}$ where l_{1k} is the average number of daughters born by a single female

during the time she is in age class k. $l_{j,j-1}$ ($j=2,3$) is the fraction of females in age class $j-1$ that will survive and pass into class j. Answer the following questions:

(a) What is the number of females in each age class after 2, 4, 6 years if each age class initially has 500 females? (**Use matrix multiplication**)

(b) For what initial (number) distribution will the number of females in each class change by the same proportion? (**You only need to describe how to find the answer. No calculation is required**)