

國立中山大學 109 學年度 碩士暨碩士專班招生考試試題

科目名稱：工程數學【海下所碩士班】

— 作答注意事項 —

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- 答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，不得另攜帶紙張，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，其後果由考生自行負擔。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

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科目名稱：工程數學【海下所碩士班】

題號：454001

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 2 頁第 1 頁

1. The determinant $\det(\mathbf{M})$ of a 2×2 matrix $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is defined by

$$\det(\mathbf{M}) = ad - bc.$$

- (1) (5%) For which values of $\det(\mathbf{M})$ does \mathbf{M} have an inverse?
 (2) (5%) Write down all 2×2 bit matrices with determinant 1. (Bits are either 0 or 1.)
 (3) (5%) Write down all 2×2 bit matrices with determinant 0.

2. (5%) What's the relation between a and b if the following vectors \mathbf{u} and \mathbf{v} are orthogonal?

$$\mathbf{u} = \begin{bmatrix} 1 \\ a \\ 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 5 \\ -2 \\ b \end{bmatrix}$$

3. (5%) A robotic arm rotates around an axis with an angular velocity $\boldsymbol{\omega} = (1, 0, -1)^T$. What is the linear velocity at the position $\mathbf{r} = (-2, 3, 1)^T$?

4. (1) (5%) Show that $y(t) = 3e^{2t} - 1$ is solution of the equation $y' = 2y + 2$.
 (2) (5%) Find the differential equation $y' = f(y)$ satisfied by $y(t) = 2e^{2t} - 3$.
 (3) (5%) Find constants a, b , so that $y(t) = (t+3)e^{2t}$ is the solution of the initial value problem

$$y' = ay + e^{2t}, \quad y(0) = b.$$

5. (10%) Find all vectors \mathbf{b} such that the system $\mathbf{Ax} = \mathbf{b}$ has solutions, where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

6. (10%) Solve the initial value problem:

$$y'' + 2.2y' + 0.4y = 0, \quad y(0) = 3.3, \quad y'(0) = -1.2$$

7. (10%) Find the steady-state oscillation of the vibrating system governed by the following equation:

$$y'' + y = \cos(2t)$$

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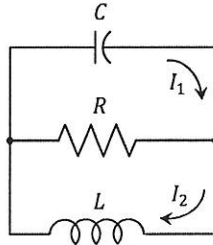
※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 2 頁第 2 頁

8. (1) (5%) Show that a model for the currents $I_1(t)$ and $I_2(t)$ in the network below is

$$\frac{1}{C} \int I_1 dt + R(I_1 - I_2) = 0, \quad LI_2' + R(I_2 - I_1) = 0$$

- (2) (10%) Find a general solution, assuming that $R = 3$ ohms, $L = 4$ henrys, and $C = \frac{1}{12}$ farad.



9. (5%) Find the volume of the following region in space:

The region beneath $z = x^2 + y^2$ and above the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$ in the xy -plane.

10. (10%) Find the Fourier series of the periodic function $f(x)$, of period $p = 2L$:

$$f(x) = -1 \quad (-1 < x < 0), \quad f(x) = 1 \quad (0 < x < 1), \quad p = 2L = 2$$