國立中山大學 109 學年度
碩士暨碩士專班招生考試試題

科目名稱：工程數學【海下所碩士班】

1. The determinant det($\mathbf{M}$) of a 2×2 matrix $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is defined by

$$\det(\mathbf{M}) = ad - bc .$$

(1) (5%) For which values of det($\mathbf{M}$) does $\mathbf{M}$ have an inverse?
(2) (5%) Write down all 2×2 bit matrices with determinant 1. (Bits are either 0 or 1.)
(3) (5%) Write down all 2×2 bit matrices with determinant 0.

2. (5%) What's the relation between a and b if the following vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal?

$$\mathbf{u} = \begin{bmatrix} 1 \\ a \\ 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 5 \\ -2 \\ b \end{bmatrix}$$

3. (5%) A robotic arm rotates around an axis with an angular velocity $\boldsymbol{\omega} = (1, 0, -1)^T$. What is the linear velocity at the position $\mathbf{r} = (-2, 3, 1)^T$?

4. (1) (5%) Show that $y(t) = 3e^{2t} - 1$ is solution of the equation $y' = 2y + 2$.
(2) (5%) Find the differential equation $y' = f(y)$ satisfied by $y(t) = 2e^{2t} - 3$.
(3) (5%) Find constants a, b, so that $y(t) = (t+3)e^{2t}$ is the solution of the initial value problem

$$y' = ay + e^{2t}, \quad y(0) = b .$$

5. (10%) Find all vectors $\mathbf{b}$ such that the system $\mathbf{Ax} = \mathbf{b}$ has solutions, where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

6. (10%) Solve the initial value problem:

$$y'' + 2.2y' + 0.4y = 0, \quad y(0) = 3.3, \quad y'(0) = -1.2$$

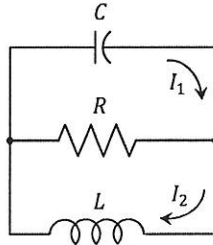7. (10%) Find the steady-state oscillation of the vibrating system governed by the following equation:

$$y'' + y = \cos(2t)$$

8. (1) (5%) Show that a model for the currents $I_1(t)$ and $I_2(t)$ in the network below is

$$\frac{1}{c}\int I_1 dt + R(I_1 - I_2) = 0, \quad LI_2' + R(I_2 - I_1) = 0$$

(2) (10%) Find a general solution, assuming that $R = 3$ ohms, $L = 4$ henrys, and $C = \frac{1}{12}$ farad.

9. (5%) Find the volume of the following region in space:

The region beneath $z = x^2 + y^2$ and above the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, $(0, 1)$ in the xy-plane.

10. (10%) Find the Fourier series of the periodic function $f(x)$, of period $p = 2L$:

$$f(x) = -1 \quad (-1 < x < 0), \quad f(x) = 1 \quad (0 < x < 1), \quad p = 2L = 2$$