1. (10%) For which values of k are there no solution, many solutions, or a unique solution to this system?
$$x - y = 1$$
$$3x - 3y = k$$

2. (5%) Does $(1, 0, 2, 1)$ lie on the line through $(-2, 1, 1, 0)$ and $(5, 10, -1, 4)$?

3. (1) (2%) Find the length of the vector $[3, 1]^T$.
   (2) (4%) Find the angle between the two vectors $[1, 2]^T$ and $[1, 4]^T$.
   (3) (4%) Find k so that these two vectors $[k, 1]^T$ and $[4, 3]^T$ are perpendicular.

4. (15%) Find the characteristic equation, and the eigenvalues and associated eigenvectors for this matrix.
$$\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$

5. Given that $y''' - 6y'' + 11y' - 6y = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 1$. Find:
   (5%) the characteristic equation's roots
   (5%) the particular solution

6. (10%) Use the Laplace transform to solve for $x(t)$ and $y(t)$ of the following system:
$$\begin{cases} x' + y = t \\ 4x + y' = 0 \end{cases}, \quad x(0) = 1, \; y(0) = 2$$

7. (10%) In a temperature field, heat flows in the direction of maximum decrease of temperature T. Find this direction at $P: (2, 1)$ when $T = x^3 - 3xy^2$.
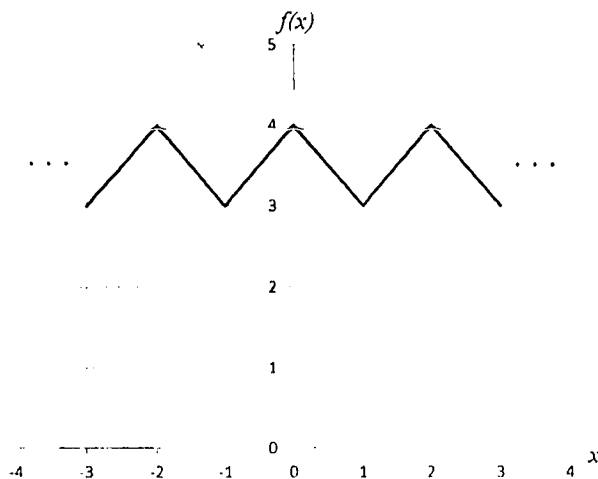
8. (15%) For a function $f(x)$ with period $p = 2L$, its Fourier series is

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x \right)$$

where $a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$, $a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx$, $b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx$, $n = 1,2,...$
Find the Fourier series of the periodic function $f(x)$ shown below.

9. (15%) The Fourier transform of $f(x)$ is $\hat{f}(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{-iwx}\,dx$. Find the Fourier transform of $f(x) = e^{-|x+3|}$.