1. (10%) Find the angle between the straight lines $4x - y = 2$ and $x + 4y = 3$.

2. (15%) Let vectors $\mathbf{a} = [4, 3, 9]$, $\mathbf{b} = [2, \frac{1}{3}, -1]$. Find

   (a) $|2\mathbf{a} + 3\mathbf{b}|$
   (b) the inner product $\mathbf{a} \cdot \mathbf{b}$
   (c) the vector product $\mathbf{a} \times \mathbf{b}$
   (d) the component of $\mathbf{a}$ in the direction of $\mathbf{b}$
   (e) For what value of c_1 are $\mathbf{c} = [c_1, 2, 0]$ and $\mathbf{a}$ orthogonal?

3. (15%) Find the plane through points A(1, 2, 3), B(-1, 4, -5), and C(0, 1, -6).

4. (10%) Find the rank of the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 2 & 2 & -3 \end{bmatrix}$$

5. (a) (10%)

$$f(x) = x, -2 < x < 2; f(x+4) = f(x)$$

   求函數 f 的 Fourier Series。

   (b) (10%) 由以上結果利用 Parsevel 定理求級數和

$$\sum_{m=1}^{\infty} 1/m^2$$

6. (10%) 利用分離變數法求解 $u_t - x\, u_x = 0$ 的通解。

7. (10%) 求解

$$y'' + 2y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = 6$$

8. (10%) $F(\omega)$ 是 $f(t)$ 的 Fourier 轉換；若 $f(t)$ 是偶函數(即 $f(-t)=f(t)$)，請問 $F(\omega)$ 是偶函數還是奇函數？

1. Find the derivative of the following function, (10 %)
$$f(x) = \sqrt[3]{9 - x^2}$$

2. Evaluate the integral $\int_2^3 \frac{x}{(1-x)^3} dx$ (10 %)

3. Calculate the square root of 2 by expanding the Taylor series to its 7th derivative, and compute the percentage of error from the result of calculator. (10 %)

4. （10%）D 是由圓 $x^2 + y^2 = R^2$ 所圍成的區域，請計算以下的積分
$$\iint_D e^{-(x^2+y^2)} dxdy$$

5. (a)以 Taylor Series 將 $\sqrt[3]{8(1+x)}$ 對 x 展開至 $O(x^2)$ 階（5%）
   (b)利用以上結果求 $\sqrt[3]{9}$ 的近似值並探討其誤差（10%）

6. 求極限 $\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x$ （5%）。

7. Please evaluate $\int_0^{\pi/2} \dfrac{\sin x}{2\cos^2 x - 3\cos x - 2} dx$ (10%)

8. Please find the domain of x such that the following series converges. (10%)
$$S_n = 1 + \tan(x^2) + \tan^2(x^2) + \tan^3(x^2) + \ldots + \tan^n(x^2), \ n \to \infty$$

9. Please evaluate $\int_0^{4\pi} \left| \sin\theta \cos\theta - \dfrac{1}{2} \right| d\theta$ (10%)

10. Please evaluate $\int \dfrac{x}{(x+2)(x+3)(x+4)} dx$ (10%)

1. A velocity field is given by $\mathbf{V} = (V_0/l)(x\hat{\mathbf{i}} - y\hat{\mathbf{j}})$ where V_0 and l are constants.

   (a) At what location in the flow field is the speed equal to V_0? (5 %)

   (b) Make a sketch of the velocity field in the first quadrant ($x \geq 0, y \geq 0$) by drawing arrows representing the fluid velocity at representative locations. (10 %)

2. Air flows steadily at low speed through a horizontal nozzle, discharging to atmosphere. The area at the nozzle inlet is 0.1 m^2, and 0.02 m^2 at the nozzle exit. Determine the gauge pressure required at the nozzle inlet to produce an outlet speed of 50 m/s. (15 %)

3. Water flows steadily through the 90° reducing elbow pipe. At the inlet to the elbow, the absolute pressure is 220 kPa and the cross-sectional area is 0.01 m^2. At the outlet, the cross-sectional area is 0.0025 m^2 and the velocity is 16 m/s. The elbow discharges to the atmosphere. Determine the force required to hold the elbow in place. (20 %)

4. A viscous fluid is contained between wide, parallel plates spaced a distance h apart as shown in Fig.1. The upper plate is fixed, and the bottom plate oscillates harmonically with a velocity amplitude U and frequency ω. The differential equation for the velocity distribution between the plates is:

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2}$$

where u is the velocity, t is time, and ρ and μ are fluid density and viscosity, respectively. Rewrite this equation in a suitable non-dimensional form using h, U, and ω as reference parameters. (15%)
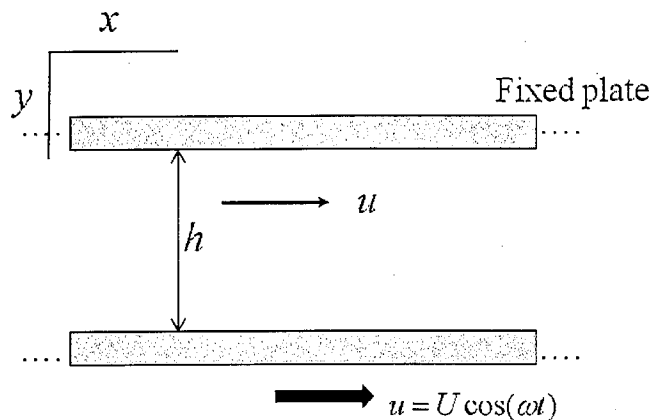

Fig. 1

5. A gate having the cross section shown in Fig.2 closes an opening 1.5 m and 1.2 m high in a water reservoir. The gate weighs 2.2kN and its center of gravity is 0.3 m to the left of AC and 0.6 m above BC. Determine the horizontal reaction that is developed on the gate at C. (20%)
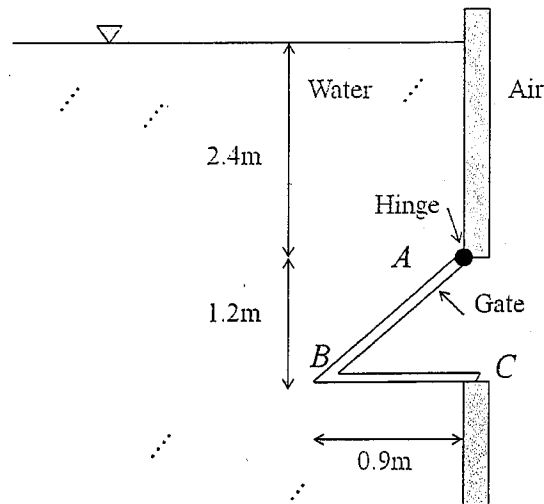

Fig. 2

6. A 1-m-diameter cylindrical mass, M, is connected to a 2-m-wide rectangular gate as shown in Fig. 3. The gate is to open when the eater level, h, drops below 2.5m. Determine the required value for M. Neglect friction at the gate hinge and pulley. (15%)
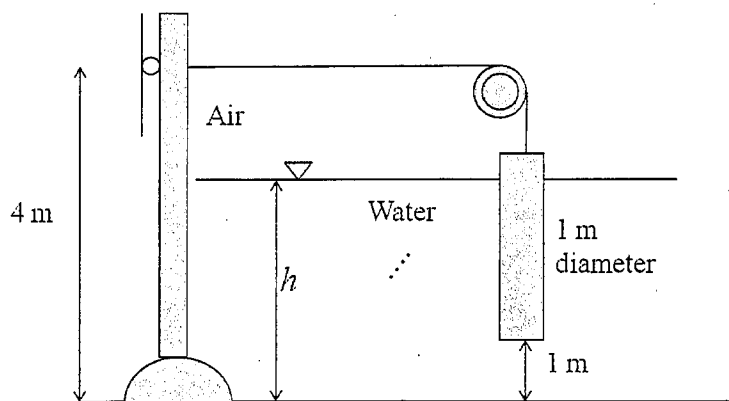
Fig. 3

一、解釋名詞 (10 題、每題 5 分共 50 分) (請在試卷上作答，否則不予計分)

1. Bathymetry

2. Thermocline

3. Cabbeling effect

4. Tidal current

5. Ekman transport

6. Estuarine circulation

7. Gravity wave

8. Ocean acidification

9. Benthic habitat

10. Acoustic tomography

二、申論題 (2 題共 50 分) (請勿在本試題紙上作答，否則不予計分)

1. 下圖左圖為在小琉球附近觀測到海水之溫度(x 上軸)及鹽度(x 下軸)隨深度(y 軸)之變化，圖中之兩條線為儀器下放及上收之記錄。請問(a)溫度及鹽度隨深度有何種變化？(b)溫鹽深資料說明海水有何種垂直分層結構？(20 分)
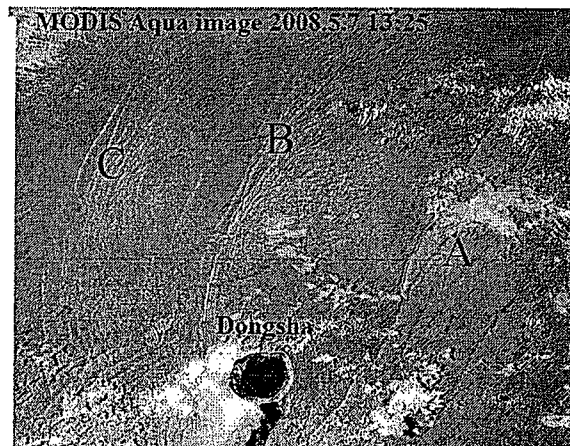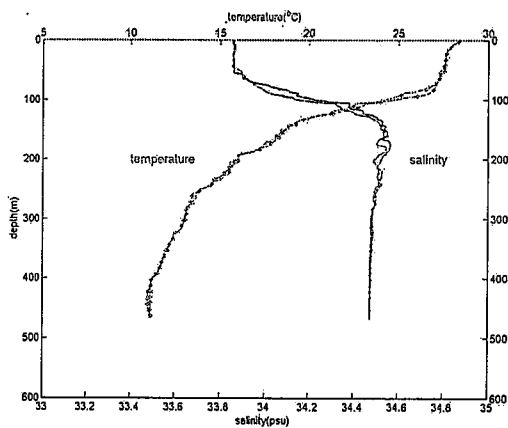
2. 上圖右圖為南海北部、東沙海域之衛星影像(MODIS Aqua 2008.5.7 13:25)；圖中東沙(Dongsha)環礁之直徑約 25 公里，ABC 分別標示內波反應在海面的粗糙度(白色塊狀部分是雲)，由東向西約每半天一組內波。請問(a)內波大小尺度大概是多少?(b)ABC 三群(組)內波有何不同?(c)根據水深資料顯示，東沙環礁東側較深(數百米至一千多米)東沙環礁西側較淺，內波運動會受到水深影響嗎? 有何種可能? (30 分)

1. (18%) For the circuit in Fig. 1, find the values of i_I, v_I, i_1, i_2, v_o, i_L, and i_o. Also find the voltage gain v_o/v_I, and the current gain i_L/i_I.

2. (20%) The 6.8-V zener diode in the circuit of Fig. 2 is specified to have $V_Z = 6.8$ V at $I_Z = 5$ mA, $r_z = 20\ \Omega$, and $I_{ZK} = 0.2$ mA. The supply voltage V^+ is nominally 10 V but can vary by ± 1 V. (a) Find V_O with no load and with V^+ at its nominal value. (b) Find the change in V_O resulting from the ± 1-V change in V^+. (c) Find the change in V_O resulting from connecting a load resistance R_L that draws a current $I_L = 1$ mA. (d) Find the change in V_O when $R_L = 2$ kΩ. (e) Find the value of V_O when $R_L = 0.5$ kΩ.

3. (16%) Figure 3 shows a discrete common-source MOSFET amplifier utilizing the drain-to-gate feedback biasing arrangement. Determine the small-signal voltage gain v_o/v_i, and the input resistance R_{in}. The transistor has $V_t = 1.5$ V, $k'_n(W/L) = 0.25$ mA/V^2, and $V_A = 50$ V. Assume the coupling capacitors to be sufficiently large so as to act as short circuits at the signal frequencies of interest.

4. (15%) A piezoelectric crystal, such as quartz, exhibits electromechanical-resonance characteristics that are very stable and high Q factors. The circuit symbol of a crystal is shown in Fig. 4(a) and its equivalent circuit model is given in Fig. 4(b). A 2-MHz quartz crystal is specified to have L = 0.52 H, $C_s = 0.012$ pF, $C_p = 4$ pF, and $r = 120\ \Omega$. Find the series resonance frequency f_s, parallel resonance frequency f_p, and the Q factor.

5. (15%) For the class B output stage of Fig. 5, let $V_{CC} = 6$ V and $R_L = 4\ \Omega$. If the output is a sinusoid with 4.5-V peak amplitude, find (a) the output power; (b) the average power drawn from each supply; (c) the power efficiency obtained at this output voltage; (d) the peak currents supplied by v_I, assuming that $\beta_N = \beta_P = 50$; (e) the maximum power that each transistor must be capable of dissipating safely.

6. (16%) For a pn junction with $N_A = 10^{17}$/cm^3 and $N_D = 10^{16}$/cm^3, find, at T = 300 K, the built-in voltage, the width of the depletion region, and the distance it extends in the p side and in the n side of the junction. Use $n_i = 1.5 \times 10^{10}$/cm^3.
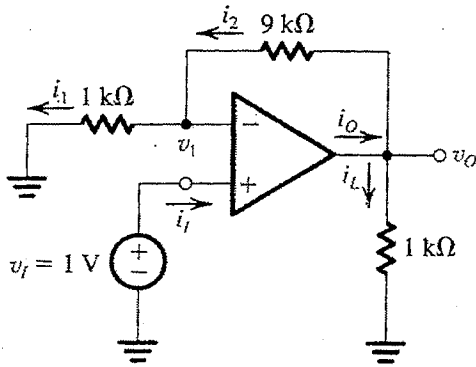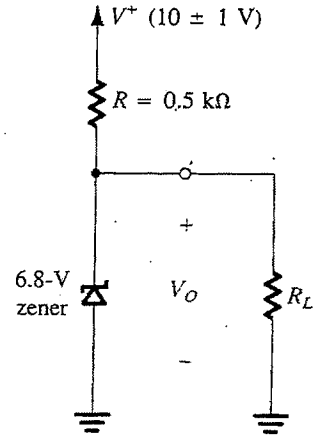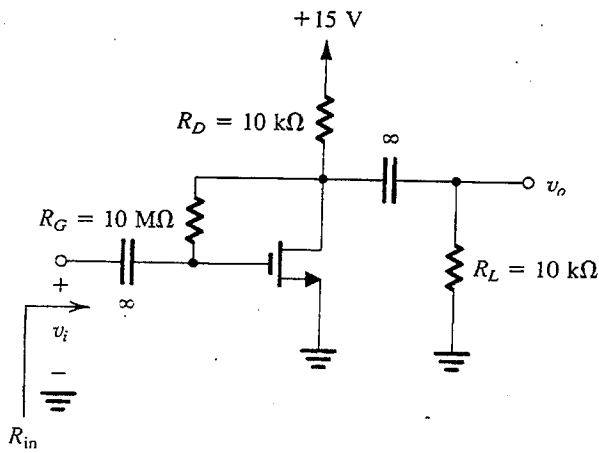
Fig. 1

Fig. 2

Fig. 3
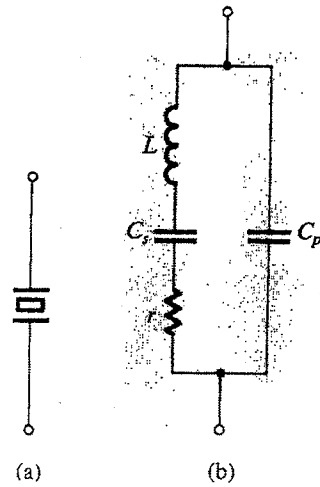
Fig. 4
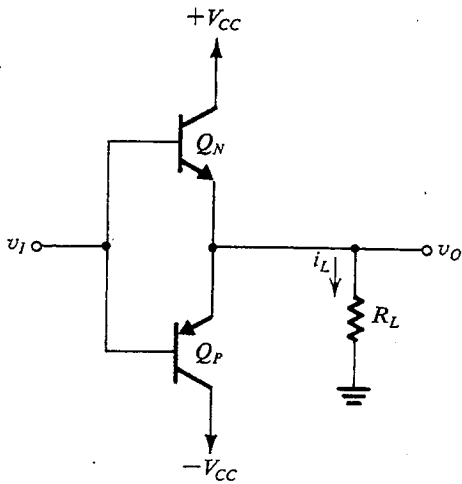
Fig. 5

說明：本試卷共六題，總分 **100** 分。

1.
   (1) 求出圖 1(a) 作用力 P 對點 O 產生的力矩大小與方向。(5%)
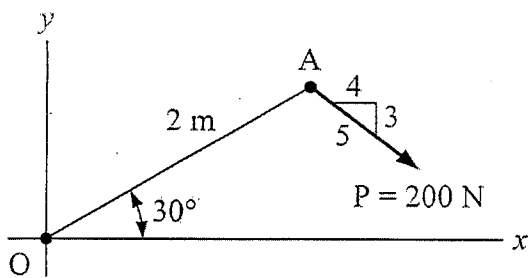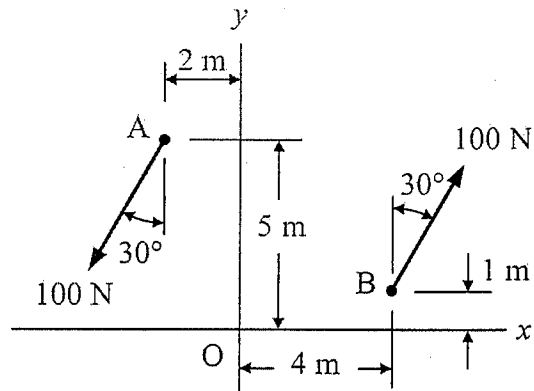   (2) 求出圖 1(b) 中之力偶矩大小與方向。(10%)

圖 1 (a)

圖 1 (b)

2. 有一半徑為 R 之光滑圓柱懸吊於彈簧末端，其重量為 W，如圖 2 所示。彈簧不計質量，彈簧之未拉伸長度為 L，彈簧常數為 k。當此一系統達成力平衡時，彈簧與垂直牆面之夾角為 θ。
   (1) 畫出圓柱之自由體圖。(5%)
   (2) 證明 W 與 θ 的關係式為 $W = k\cos\theta\left(\dfrac{R}{\sin\theta} - L\right)$。(15%)
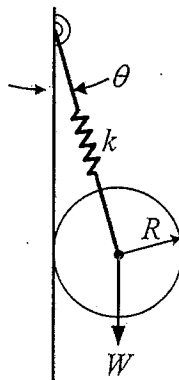
圖 2

3.
(1) 一均質梯形如圖 3(a)所示，求其形心距離 $\bar{y}$。(7%)
(2) 圓弧狀的均質鋼條如圖 3(b) 所示，求其形心位置 $\bar{x}$ (以半徑 r 和半弧角 θ 表示)。(8%)
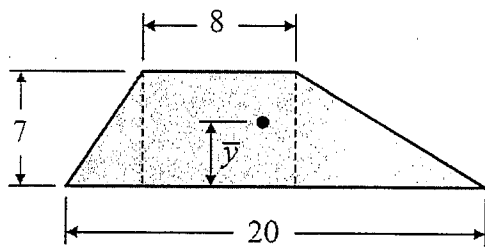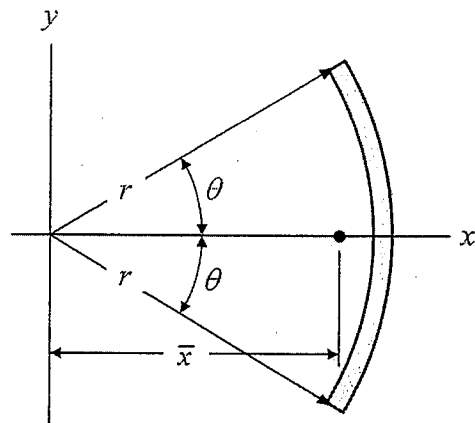
圖 3 (a)

圖 3 (b)

4. 一個兩邊等腰的梯子展成六十度置於光滑地面上，左右兩邊材質均勻但是左邊重量為 W，右邊重量為 3W。兩邊以一條不計重量的繩子支撐住，如圖 4 所示。請問：
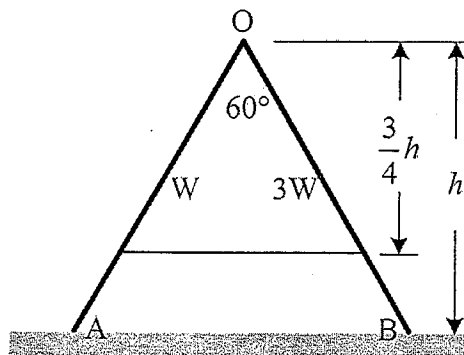(1) A 與 B 處地面的反作用力為多少？(7%)
(2) 繩子的張力為多少？(13%)

圖 4

5. 一條 1 m 長的細繩(重量不計)將重物 W 以一光滑套環吊掛於距離 0.8 m 的兩牆間，因此套環可以自由左右滑動，如圖 5 所示。若左邊吊掛點比右邊吊掛點高的話，請問當系統平衡時：
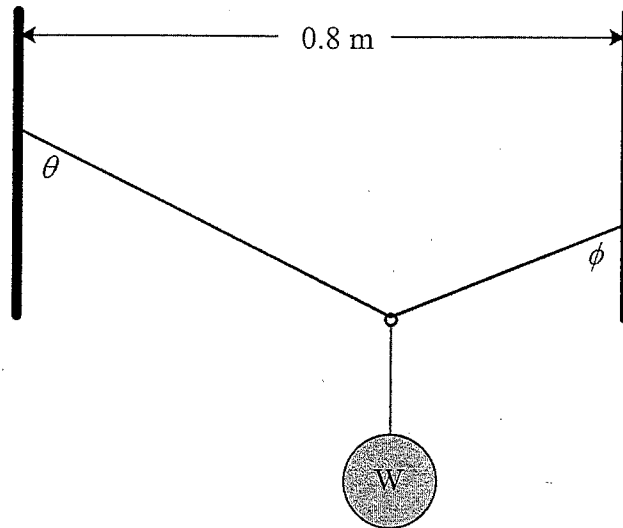
(1) $\sin\theta$ = ? (5%)

(2) 繩子中的張力為多少？(5%)

圖 5

6. 一個外力 $(10\,\text{N}, 5\,\text{N}, -8\,\text{N})$ 作用在剛體上，施力點為 $(1\,\text{m}, 2\,\text{m}, -1\,\text{m})$。請問：

(1) 以 $(1\,\text{m}, 1\,\text{m}, 1\,\text{m})$ 為參考點，力矩為多少。(10%)

(2) 如果剛體可以繞著通過 $(1\,\text{m}, 2\,\text{m}, -1\,\text{m})$ 與 $(2\,\text{m}, 1\,\text{m}, 0\,\text{m})$ 連線的一個軸旋轉的話，請問上述力矩在轉軸上的有效分量為多少。(10%)