

國立中山大學100學年度碩士班招生考試試題

科目：工程數學【海下海物所碩士班選考】

1. (a) Find the Fourier expansion of the function whose definition in one period is, (10 %)

$$f(t) = 4 - t^2 \quad -2 \leq t \leq 2$$

- (b) By the above results and simple sketch one period of $f(t)$, determine is this function odd or even? (10 %)

2. If the Laplace transform of $y(t)$ is (10 %)

$$\mathcal{L}\{y\} = \frac{s+1}{s^2+s-6}$$

What is $y(t)$?

3. Heat is generated at a constant rate r within a rod of finite length, the heat equation with boundary conditions are as follows, solve $u(x, t)$ (20 %)

$$k \frac{\partial^2 u}{\partial x^2} + r = \frac{\partial u}{\partial t}$$

$$u(0, t) = 0, \quad u(1, t) = u_0 \quad t > 0$$

$$u(x, 0) = f(x), \quad 0 < x < 1$$

4. S 為球面 $x^2 + y^2 + z^2 = 4$

$$\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k},$$

$$\text{求 } \iint_S \vec{F} \cdot d\vec{S} \text{ 。 (20\%)}$$

5. 求矩陣

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

的特徵值(eigen value)及特徵向量(eigen vector)。(15%)

6. 求解常微分方程式

$$\frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 5y = -\sin t \text{ 。 (15\%)}$$

微積分（海下海物研究所碩士班選考）

1. Find out the first derivative of the following function involving with a natural logarithm, (10 %)

$$f(x) = \ln(7x-14)$$

2. Find the integral of $\int e^x \cos x \, dx$ (10 %)

3. Find the integral of $\int \frac{dx}{x^2-4}$ (10 %)

4. Consider a surface described by the following function f in xyz -space. Calculate the volume of the solid defined by f with respect to the square region in the xy -plane whose edges are $x = -1$, $x = 1$, $y = -1$, and $y = 1$ (20 %)

$$f(x, y) = 3x^2 + 3y^2 + 1$$

5. 求 $\lim_{x \rightarrow 0} \frac{\sin^{-1} 2x}{\sin^{-1} x}$ 的極限值 (5%)。

6. 假設波速 c (單位 m/s) 為位置 x (單位) 的函數：

$$c = \frac{x^2}{1000}$$

求此波由 $x=40$ 傳到 $x=10$ 需要多少時間。(10%)

7. S 為球面 $x^2 + y^2 + z^2 = 4$

$$\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k},$$

求 $\iint_S \vec{F} \cdot d\vec{S}$ 。(20%)

8. 為計算積分 $I = \int_0^{\infty} e^{-x^2} dx$

先求其平方

$$I^2 = \left(\int_0^{\infty} e^{-x^2} dx \right) \left(\int_0^{\infty} e^{-y^2} dy \right) = \int_0^{\infty} \int_0^{\infty} e^{-x^2-y^2} dx dy$$

再轉換為極座標

$$I^2 = \int_0^a \int_0^b e^{-r^2} r dr d\theta$$

請問 (a) 上式之積分上限 a 、 b 是多少？(5%)

(b) 積分 I 是多少？(10%)

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科目：流體力學【海下海物所碩士班選考】

- (40%; 5% each) Define and explain the following terms (定義並解釋下列名詞):
 - Reynolds stress
 - Velocity potential
 - Steady state fluid flow
 - Doppler effect
 - Barotropic fluid
 - Boundary layer
 - Dynamic similarity
 - Streakline
- (10%) Consider the Bernoulli equation: $\frac{p}{\rho} + \frac{v^2}{2} + gz = \text{constant}$, where p , ρ , v , g are the pressure, density, speed, gravity constant, respectively; z is the vertical coordinate pointing upwards. List the assumptions that have to be made so that this Bernoulli equation is applicable.
- (15%) A spillway gate formed in the shape of a circular arc is w meter wide, as shown in Figure 1. Find the magnitude and line of action of the vertical component of the force due to all fluids acting on the gate.
- (15%) Consider a 30° reducing elbow as shown in Figure 2. The fluid is water. Evaluate the components of force that must be provided by the adjacent pipes to keep the elbow from moving.
- (20%) Consider a fluid flow that may be described by the following equation:

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla p + \mathbf{g}$$

where \mathbf{v} , p , \mathbf{g} , ρ are velocity, pressure, gravitational force, and density, respectively.

- What are the assumptions that have to be made so that the above equation is valid? (5%)
- Show that, if the flow starts irrotationally, then it remains irrotational all the time. (15%)

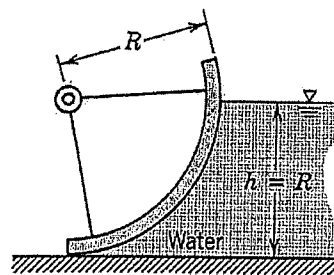


Figure 1

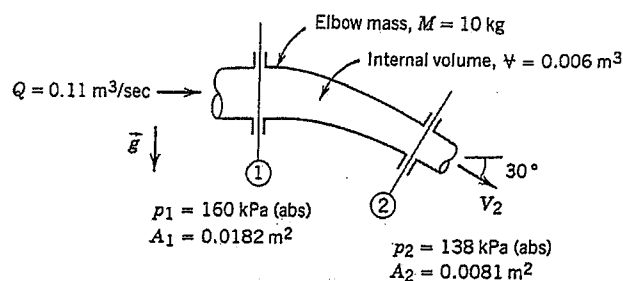


Figure 2

說明：本試卷共六題，總分 100 分。

1. (15%)

- (1) 請算出圖 1(a) F_1 與 F_2 之合力大小與方向，並算出合力對點 O 的合力矩大小。(5%)
 (2) 請算出圖 1(b) F_1 與 F_2 之合力大小與方向，並算出合力對 z 軸的合力矩大小。(10%)

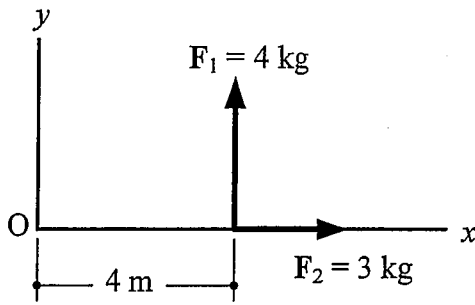


圖 1 (a)

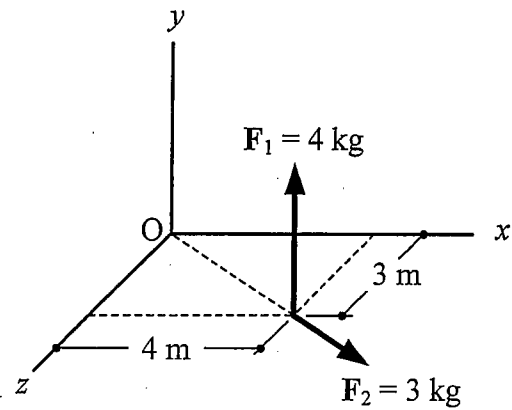


圖 1 (b)

2. (20%)

- (1) 請算出圖 2(a) 系統的質心位置。(10%)
 (2) 請算出圖 2(b) 系統的質心位置。(10%)

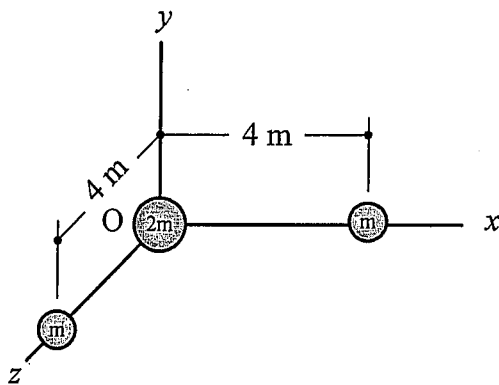


圖 2 (a)

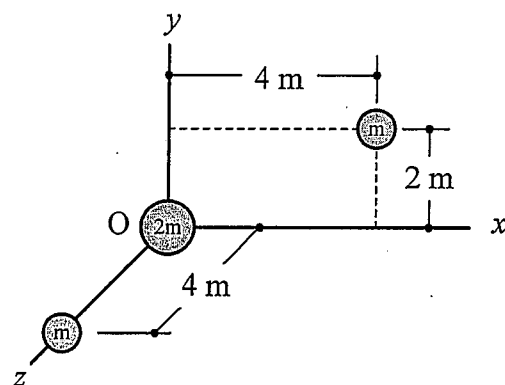


圖 2 (b)

3. (15%) 如圖 3 所示，爲了支撐 500 N 的負載，請問：

- (1) 圖 3(a) 之施力 P 爲何？ (5%)
- (2) 圖 3(b) 之施力 P 爲何？ (10%)

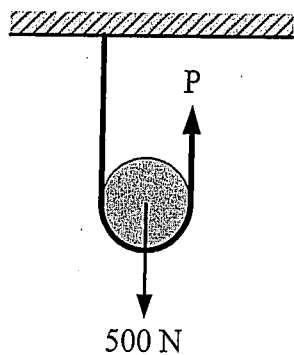


圖 3 (a)

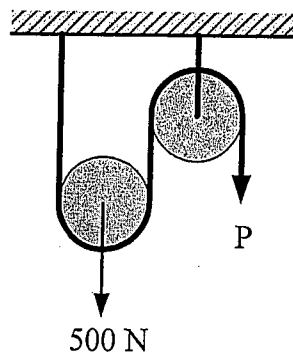


圖 3 (b)

4. (20%) 小孩站在獨木舟尾端，船首靠著大石邊，如圖 4 所示。大石上有一隻烏龜，小孩想走向船頭抓它。如果小孩跟獨木舟的質量各爲 50 公斤及 70 公斤，舟長 4 公尺(質心在船中央)，小孩在船首時手還可以向外伸長 1 公尺，請問他抓得到烏龜嗎？。(忽略所有的摩擦力)

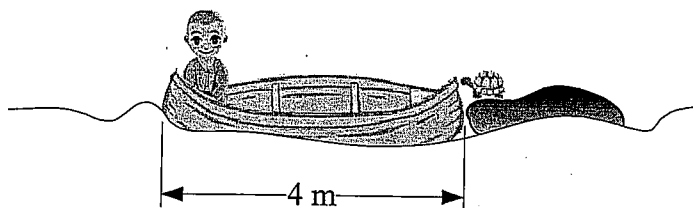


圖 4

5. (20%) 如圖 5 所示，一個半徑爲 1 公尺、1 公斤重的圓柱體，從圓心施力 F 要將其拉上高度爲 0.5 公尺的台階。請問：

- (1) 要拉動圓柱的話， F 與水平線夾角 θ 爲多少度時最省力？(10%)
- (2) 要將圓柱拉動最小的 F 爲多少？(10%)

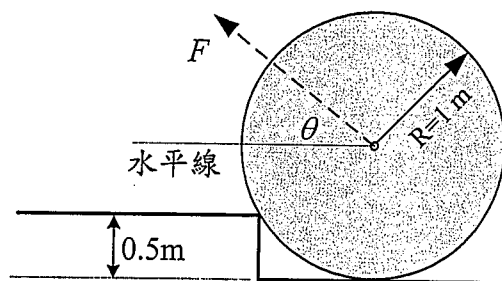


圖 5

6. (10%) 有一材質均勻 60 公斤的不等臂蹺翹板，如圖 6 所示。A 端點以抗張力 2000N 纜繩繫住，請問一公斤一個的水泥塊放幾個於 B 點時纜繩會繃斷。

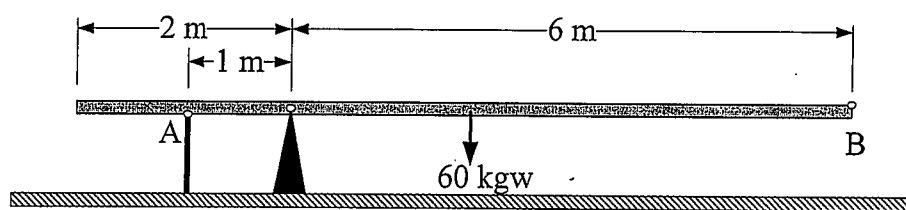


圖 6

國立中山大學100學年度碩士班招生考試試題

科目：電子學【海下海物所碩士班選考】

- (15%) For the circuit shown in Fig. 1, find the transfer function $T(s) = V_o(s) / V_i(s)$. Is this a high-pass or a low-pass network? What is its transmission at very high frequencies ($s \rightarrow \infty$)? What is the corner frequency ω_0 ? For $R_1 = 10 \text{ k}\Omega$, $R_2 = 40 \text{ k}\Omega$, and $C = 0.1 \text{ }\mu\text{F}$, find f_0 . What is the value of $|T(j\omega_0)|$?
- (15%) The circuit in Fig. 2 utilizes an ideal op amplifier. (a) Find I_1 , I_2 , I_3 and V_X . (5%) (b) If V_o is not to be lower than -13 V , find the maximum allowed value for R_L . (5%) (c) If R_L is varied in the range $100 \text{ }\Omega$ to $1 \text{ k}\Omega$, what is the corresponding change in I_L and in V_o ? (5%)
- (15%) The NMOS and PMOS transistors in the circuit of Fig. 3 are matched with $k_n' (W_n/L_n) = k_p' (W_p/L_p) = 1 \text{ mA/V}^2$ and $V_{tn} = -V_{tp} = 1 \text{ V}$. Assuming $\lambda = 0$ for both devices, find the drain currents i_{DN} and i_{DP} and the voltage v_o for $v_I = 0 \text{ V}$, $+2.5 \text{ V}$, and -2.5 V .
- (20%) For the common-emitter amplifier shown in Fig. 4, let $V_{CC} = 9 \text{ V}$, $R_1 = 27 \text{ k}\Omega$, $R_2 = 15 \text{ k}\Omega$, $R_E = 1.2 \text{ k}\Omega$, and $R_C = 2.2 \text{ k}\Omega$. The transistor has $\beta = 100$ and $V_A = 100 \text{ V}$. Calculate the dc bias current I_E . If the amplifier operates between a source for which $R_{sig} = 10 \text{ k}\Omega$ and a load of $R_L = 2 \text{ k}\Omega$, replace the transistor with its hybrid- π model, and find the values of R_{in} , the voltage gain v_o/v_{sig} , and the current gain i_o/i_i .
- (20%) The op amplifier in the circuit of Fig. 5 has an open-loop gain of 10^5 and a single-pole rolloff with $\omega_{3dB} = 10 \text{ rad/s}$. (a) Sketch a Bode plot for the loop gain. (6%) (b) Find the frequency at which $|A\beta| = 1$, and find the corresponding phase margin. (6%) (c) Find the closed-loop transfer function, including its zero and poles. Sketch a pole-zero plot. (8%)
- (15%) A BJT is specified to have $T_{Jmax} = 150 \text{ }^\circ\text{C}$ and to be capable of dissipating maximum power as follows:

$$40 \text{ W at } T_C = 25 \text{ }^\circ\text{C}$$

$$2 \text{ W at } T_A = 25 \text{ }^\circ\text{C}$$

Above $25 \text{ }^\circ\text{C}$, the maximum power dissipation is to be derated linearly with $\theta_{JC} = 3.12 \text{ }^\circ\text{C/W}$ and $\theta_{JA} = 62.5 \text{ }^\circ\text{C/W}$. Find the following: (a) The maximum power that can be dissipated safely by this transistor when operated in free air at $T_A = 50 \text{ }^\circ\text{C}$. (5%) (b) The maximum power that can be dissipated safely by this transistor when operated at an ambient temperature of $50 \text{ }^\circ\text{C}$, but with a heat sink for which $\theta_{CS} = 0.5 \text{ }^\circ\text{C/W}$ and $\theta_{SA} = 4 \text{ }^\circ\text{C/W}$. Find the temperature of the case and of the heat sink. (5%) (c) The maximum power that can be dissipated safely if an infinite heat sink is used and $T_A = 50 \text{ }^\circ\text{C}$. (5%) (note: θ_{JA} , θ_{JC} , θ_{CS} , and θ_{SA} are the thermal resistances between junction and ambience, junction and transistor case, transistor case and heat sink, and heat sink and ambience, respectively.)

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科目：電子學【海下海物所碩士班選考】

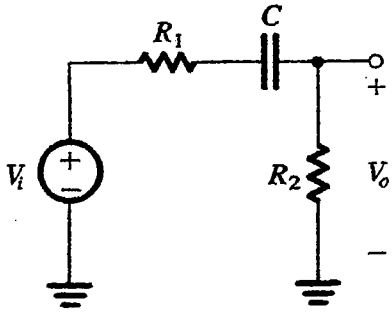


Figure 1

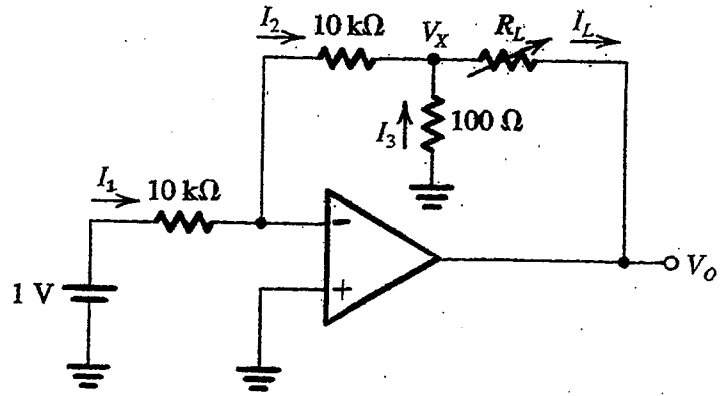


Figure 2

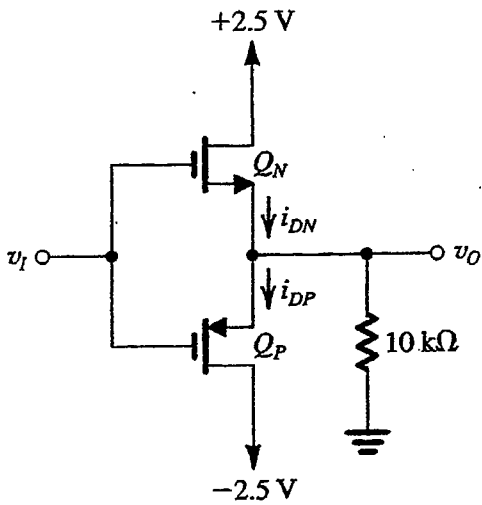


Figure 3

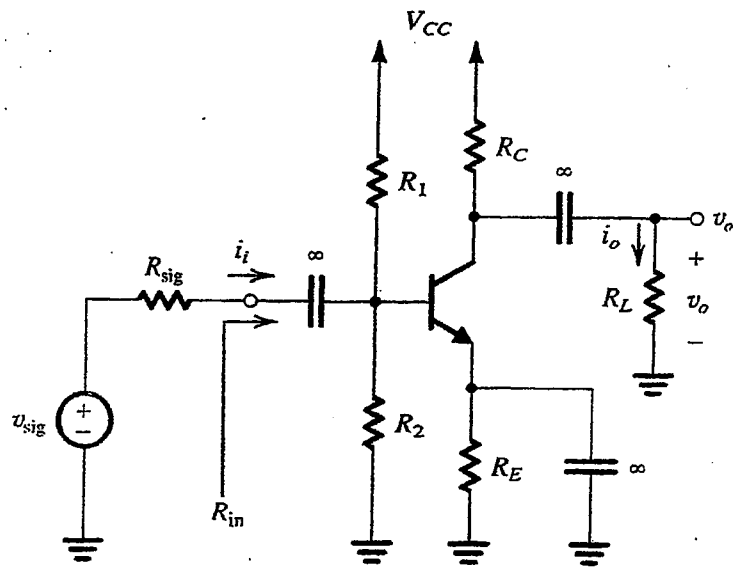


Figure 4

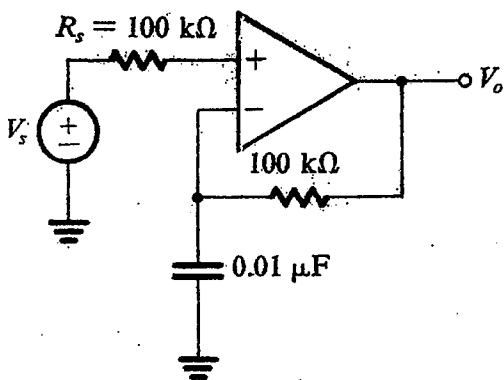


Figure 5

國立中山大學100學年度碩士班招生考試試題

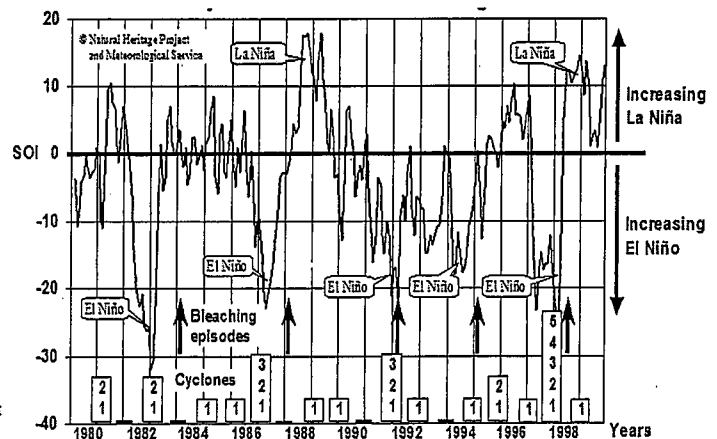
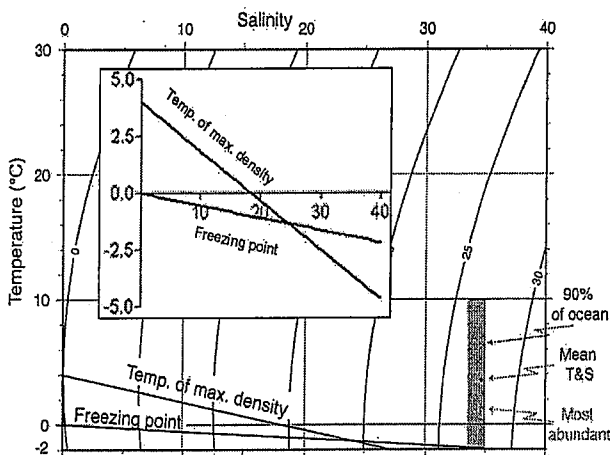
科目：海洋學【海下海物所碩士班選考】

一、解釋名詞 (10 題、每題 5 分共 50 分) (請在試卷上作答，否則不予計分)

1. Continental slope
2. Geostrophic current
3. Latent heat flux
4. Pycnocline
5. Storm surge
6. Eddy viscosity
7. Ekman transport
8. Estuarine circulation
9. Primary productivity
10. CODAR

二、申論題 (2 題共 50 分) (請勿在本試題紙上作答，否則不予計分)

1. 下圖左圖為海水密度之溫鹽特性圖。圖中之兩條直線特別說明最大密度(temp. of max. density)及結冰點(freezing point)隨溫度(y 軸)及鹽度(x 軸)之變化。(a)請敘述淡水湖(例如鹽度 10 psu 以下)表面受冷之結冰過程? (b)鹹水湖(例如鹽度 34 psu)表面受冷之結冰過程又有何不同?(20 分)



2. 上圖右圖(摘自南太平洋 Cook Islands Biodiversity 網站)為珊瑚白化程度(1 2 3 4 5)與聖嬰指數(SOI)從 1979 起約 20 年之時序圖。請問(a)甚麼是 SOI? (b)如何從 SOI 看出那幾年有聖嬰現象? (c)聖嬰現象跟珊瑚白化有何關聯?(30 分)