

I.

1. (15%) If the equipotential lines in a region of the  $xy$ -plane are  $4x^2 + y^2 = c$ , what are the curves of the electrical force? (10%) Sketch both families of curves. (5%)
2. (10%) Find all solutions or indicate that no solution exists in the following linear system.

$$3x + 5y - 8z = 18$$

$$x + 2y - 3z = 6$$

II.

1. (15%) (a) Find the residue at  $z = 0$  of the function (a)  $\frac{\sinh z}{z^4(1-z^2)}$ ; (b)  $z \cos(1/z)$ ; (c)  $\frac{1}{z+z^2}$ .
2. (10%) Solve the following Euler-Cauchy equation, showing the details of your work.

$$x^2 y'' + 0.6xy' + 16.04y = 0, \quad y(1) = 1, \quad y'(1) = 0.$$

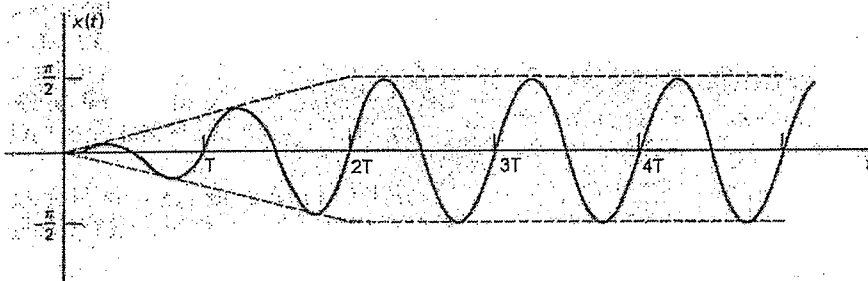
III.

1. Please answer the following questions:
- (a) Find the projection of the vector  $\mathbf{v} = -2\mathbf{j} + 2\mathbf{k}$  onto  $\mathbf{u} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$ . (2%)
- (b) Find the area of triangle with vertices  $(1,0,2)$ ,  $(3,2,1)$ , and  $(2,1,3)$ . (2%)
- (c) Find the vector of length 6 in the direction of  $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ . (2%)
2. Let  $f(x, y) = e^{xy} \sin(x + y)$ .
- (a) In what direction, starting at  $(0, \pi/2)$ , is  $f$  changing the fastest? (3%)
- (b) In what directions, starting at  $(0, \pi/2)$ , is  $f$  changing at 50% of its maximum rate? (3%)

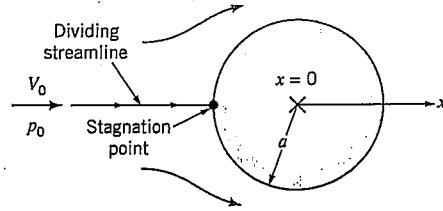
3. Let  $\mathbf{G}(x, y) = (xe^{x^2+y^2} + 2xy)\mathbf{i} + (ye^{x^2+y^2} + x^2)\mathbf{j}$ .
- Show that  $\mathbf{G} = \nabla f$  for some  $f$ ; find such an  $f$ . (5%)
  - Use (a) to show that the line integral of  $\mathbf{G}$  around the edge of the triangle with vertices  $(0,0)$ ,  $(0,1)$ ,  $(1,0)$  is zero. (4%)
  - State Green's theorem for the triangle in (b) and vector field  $\mathbf{F}$  and verify it for the vector field  $\mathbf{G}$  above. (4%)

## IV.

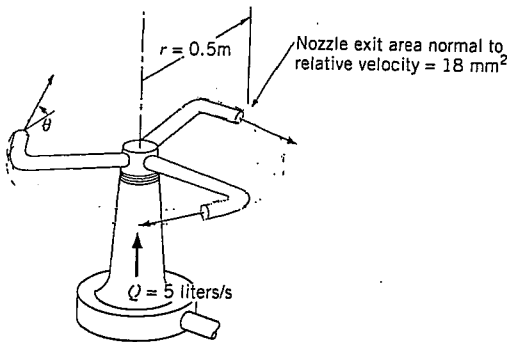
- (6%) Assuming that  $x(t)$  is the Fourier series representation of  $t^2 - 2t + 2$  for  $0 \leq t \leq 1$ . Determine the value for  $x(0)$  and  $x(1)$  and explain your answers.
- (6%) It is known that  $g(t) = g(t+L)$ ,  $f(t) = f(t+2L)$  and  $g(t) = f(2t)$ . Assuming  $G(t)$  is the Fourier series representation of  $g(t)$  for  $0 \leq t \leq L$  and  $F(t)$  is the Fourier series representation of  $f(t)$  for  $0 \leq t \leq 2L$ , will  $G(t)$  converge faster than  $F(t)$ ? Why or why not?
- (13%) With  $f(t) = \cos 2t$  for  $0 \leq t < 2\pi$  and  $f(t) = 0$  for  $t \geq 2\pi$ , The following figure gives the solution for  $x''(t) + cx(t) = f(t)$ . Note that  $c$  is an unknown constant coefficient and  $x(0) = x'(0) = 0$ .
  - By treating  $c$  as a parameter left to be determined, find  $X(s)$ . That is, find the Laplace transform of  $x(t)$ .
  - Determine the value of  $T$  which is shown in the given figure.
  - What is the value of  $c$ ?



1. 15%. An incompressible fluid flows steadily past a circular cylinder of radius  $a$  with an upstream speed of  $V_0$  as shown in the following figure. The fluid velocity along the dividing streamline ( $-\infty \leq x \leq -a$ ) is found to be  $V = V_0(1 - a^2/x^2)$ . (a) Determine the pressure gradient along this streamline. (b) If the upstream pressure is  $p_0$ , integrate the pressure gradient to obtain the pressure  $p(x)$  for  $-\infty \leq x \leq -a$ . (c) Show from the result of part (b) that the pressure at the stagnation point ( $x = -a$ ) is  $p_0 + \rho V_0^2/2$ , as expected from the Bernoulli equation..

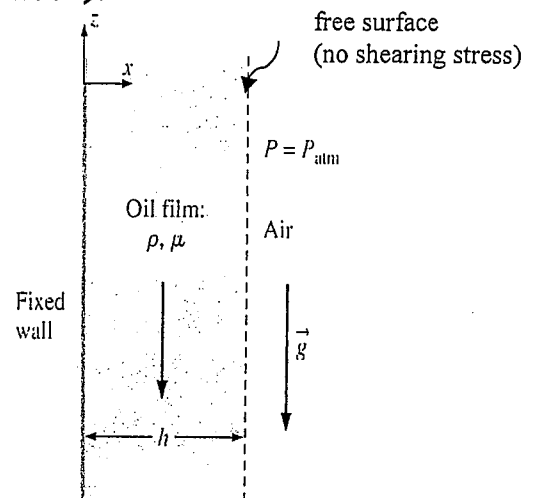


2. 20%. Five liters/s of water enter the rotor along the axis of rotation shown in the following figure. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is  $18 \text{ mm}^2$ . How large is the resisting torque required to hold the rotor stationary? How fast will the rotor spin steadily if the resisting is reduced to zero and (a)  $\theta = 0^\circ$ , (b)  $\theta = 30^\circ$ .



3. (15%) Consider steady, incompressible, parallel, laminar flow of a film of oil falling slowly down an infinite vertical wall. The oil film thickness is  $h$ , and gravity acts in the negative  $z$ -direction. There is no applied (forced) pressure driving the flow—the oil falls by gravity along. Assume the wall is infinite in the  $yz$ -plane, pressure  $p = p_{atm} = \text{constant}$  at the free surface.

- (a) What is the  $z$ -momentum equation? i.e. the equation of  $w$ ? (5%)  
 (b) What are the boundary conditions for  $w$ ? (5%)  
 (c) Solve for the velocity distribution  $w(x) = ?$  (5%)



4. (15%) For a long structural component of a bridge has the cross section shown in Figure 4.  $D = 0.2\text{m}$ ,  $H = 0.6\text{m}$ , and a representative wind velocity is  $V = 10\text{ m/sec}$ . Standard air can be assumed. The shedding frequency is to be determined through the use of a small-scale model that is to be tested in a water tunnel. We expect shedding frequency  $\omega = f(D, H, V, \rho, \mu)$ , where  $\rho$  is the density of the fluid and  $\mu$  is the viscosity.

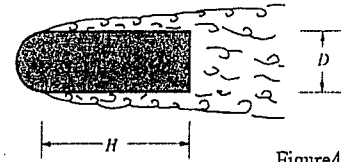


Figure 4.

(a) What dimensionless parameters would you use to organize these data? i.e. write the above equation in dimensionless form (5%)

(b) For the model,  $H_m = 120\text{mm}$  and the water temperature is  $20^\circ\text{C}$ ,

$$\mu = 1 \times 10^{-3} \text{ kg / m} \cdot \text{s}, \quad \rho = 998 \text{ kg / m}^3.$$

Determine the model dimension  $D_m$ , and the velocity at which the test should be performed. (5%)

(c) If the shedding frequency for the model is found to be  $49.9\text{Hz}$ , what is the corresponding frequency for the prototype? (5%)

(For standard air  $\mu = 1.79 \times 10^{-5} \text{ kg / m} \cdot \text{s}$ ,  $\rho = 1.23 \text{ kg / m}^3$ )

5. (5%)

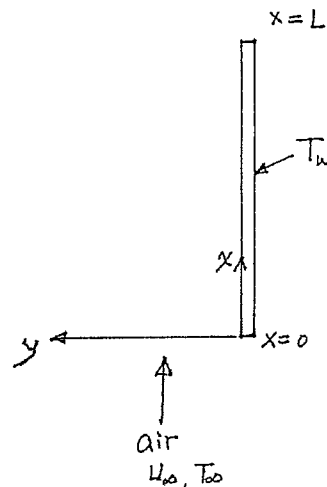
Give the definition of Prandtl number. What can you say about the nature of a fluid with large Prandtl number.

6. (10%)

A hot plate is vertically placed in an air stream with velocity  $U_\infty$  vertically up as shown in the figure. The free stream temperature of the air is  $T_\infty$ , and the temperature of the plate is kept at  $T_w$ . Assume 2D, steady state

(a) Draw the velocity profile  $u(y)$  and temperature profile  $T(y)$  in the boundary layer of the plate at  $x = L/2$  (in this case, the air velocity is large enough to neglect the gravitational effect) (5%)

(b) if  $U_\infty = 0$ , repeat (a) (5%)



## 國立中山大學99學年度碩士班招生考試試題

科目：流體力學及熱對流【機電系碩士班甲組】

7. (10%)

In the case (a) of the above problem, the temperature in the boundary is a function of  $x$  and  $y$ . Derive the governing equation of  $T(x, y)$  by considering the energy balance of a fluid element  $dx \cdot dy$ . Assume all physical properties are temperature-independent and viscous dissipation can be neglected.

8. (10%)

For a cool air (with free stream velocity  $U_\infty$  and temperature  $T_\infty$ ) flowing over a hot plate at constant temperature  $T_w$ , the temperature distribution in the thermal boundary layer is assumed to be  $T(y) = e + fy + gy^2 + hy^3$  for a streamwise location.

The constants  $e$ ,  $f$ ,  $g$  and  $h$  are evaluated by applying the boundary conditions.

- (a) What are the boundary conditions require to evaluate the four unknown coefficients,  $e$ ,  $f$ ,  $g$  and  $h$  (6%)
- (b) Under what condition do you expect the non-dimensional temperature profile is equal to the non-dimensional velocity profile in the boundary? (4%)

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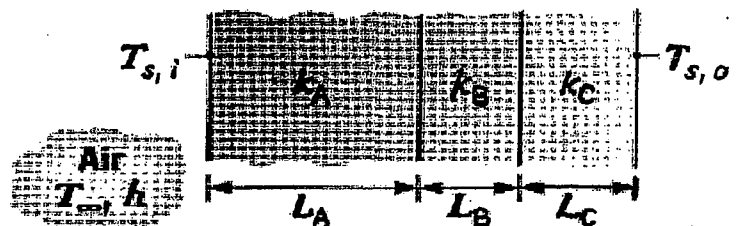
科目：熱力及熱傳導、熱輻射學【機電系碩士班甲組】

**Part I: Thermodynamics (65%)**

- (10%) Please briefly describe the Zeroth, First and Second laws of thermodynamics. What are their contributions, respectively? What properties are defined due to these basic laws introduced?
- (10%) An open system is interactive with its surroundings through the mechanisms of mass, heat or work transfer. Please describe what mechanisms can cause the changes of the energy, the entropy, and the exergy of a control volume, respectively?
- (15%) An ideal gas with constant specific heat from a given initial state expands to a fixed final volume undergoes three polytropic processes in a closed system which are adiabatic, isothermal, and isobaric processes, respectively? For which case is the work done greatest and which case is the work done least? If the closed system changes into a control volume, please discuss the work done from the greatest to the least for this similar case.
- (10%) Air enters a window air conditioner at 102 kPa, 35°C, and 80 percent relative humidity at a rate of 10 m<sup>3</sup>/min, and it leaves as saturated air at 100 kPa, 10°C. Part of the moisture in the air that condenses during the process is also removed at 10°C. Determine the rates of heat and moisture removal from the air. ( $C_{p0}=1.004$  kJ/kg K,  $R=0.287$  kJ/kg K for air)
- (10%) A Rankine cycle using steam as the working fluid in which the boiler pressure is 1.5 MPa and the condenser pressure is 15 kPa. The steam leaves the boiler as saturated vapor. Determine the efficiency of this Rankine cycle.
- (10%) 蒸氣壓縮式冷凍循環(vapor compression refrigeration cycle) 包含那四個過程，試畫出冷凍循環 T-s 圖，請詳述各個過程並列出其能量平衡式(6%)。冷暖氣機吹出冷氣與暖氣分別是四個過程中那一個過程的作用(2%)? 冷凍機(refrigerator)的 COP=?(1%) 熱泵(heat pump)的 COP=?(1%)

**Part II: Conduction and Radiation (35%)**

- (15%) The composite wall of an oven consists of three materials, two of which are of known thermal conductivity,  $k_A = 20$  W/m K and  $k_C = 50$  W/m K, and known thickness,  $L_A = 0.3$  m and  $L_C = 0.15$  m. The third material, B, which is sandwiched between materials A and C, is of known thickness,  $L_B = 0.15$  m, but unknown thermal conductivity  $k_B$ .



Under steady-state operating conditions, measurements reveal an outer surface temperature of  $T_{s,o} = 20^\circ\text{C}$ , an inner surface temperature of  $T_{s,i} = 600^\circ\text{C}$ , and an oven air temperature of  $T_{\infty} = 800^\circ\text{C}$ . The inside convection coefficient  $h$  is known to be  $25$  W/m<sup>2</sup> K. What is the value of  $k_B$ ? Draw the thermal circuit diagram.

- (15%) Carbon steel ( $\rho = 7832$  kg/m<sup>3</sup>,  $k = 51.2$  W/m K,  $c = 541$  J/kg K) shafts of 0.1 m diameter are heat treated in a gas-fired furnace whose gases are at 1200 K and provide a convection coefficient of  $100$  W/m<sup>2</sup> K. If the shafts enter the furnace at 300 K, how long must they remain in the furnace to achieve a centerline temperature of 800 K?
- (5%) Explain the green house effect from the radiation heat transfer point of view.

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科目：熱力及熱傳導、熱輻射學【機電系碩士班甲組】

Saturated water—Pressure table

| Press.,<br>P kPa | Sat.<br>temp.,<br>T <sub>sat</sub> °C | Specific volume,<br>m <sup>3</sup> /kg |                                  | Internal energy,<br>kJ/kg         |                           |                                  | Enthalpy,<br>kJ/kg                |                           |                                  | Entropy,<br>kJ/kg · K             |                           |                                  |
|------------------|---------------------------------------|--|----------------------------------|-----------------------------------|---------------------------|----------------------------------|-----------------------------------|---------------------------|----------------------------------|-----------------------------------|---------------------------|----------------------------------|
|                  |                                       | Sat.<br>liquid,<br>v <sub>f</sub>      | Sat.<br>vapor,<br>v <sub>g</sub> | Sat.<br>liquid,<br>u <sub>f</sub> | Evap.,<br>u <sub>fg</sub> | Sat.<br>vapor,<br>u <sub>g</sub> | Sat.<br>liquid,<br>h <sub>f</sub> | Evap.,<br>h <sub>fg</sub> | Sat.<br>vapor,<br>h <sub>g</sub> | Sat.<br>liquid,<br>s <sub>f</sub> | Evap.,<br>s <sub>fg</sub> | Sat.<br>vapor,<br>s <sub>g</sub> |
| 1.0              | 6.97                                  | 0.001000                               | 129.19                           | 29.302                            | 2355.2                    | 2384.5                           | 29.303                            | 2484.4                    | 2513.7                           | 0.1059                            | 8.8690                    | 8.9749                           |
| 1.5              | 13.02                                 | 0.001001                               | 87.964                           | 54.686                            | 2338.1                    | 2392.8                           | 54.688                            | 2470.1                    | 2524.7                           | 0.1956                            | 8.6314                    | 8.8270                           |
| 2.0              | 17.50                                 | 0.001001                               | 66.990                           | 73.431                            | 2325.5                    | 2398.9                           | 73.433                            | 2459.5                    | 2532.9                           | 0.2606                            | 8.4621                    | 8.7227                           |
| 2.5              | 21.08                                 | 0.001002                               | 54.242                           | 88.422                            | 2315.4                    | 2403.8                           | 88.424                            | 2451.0                    | 2539.4                           | 0.3118                            | 8.3302                    | 8.6421                           |
| 3.0              | 24.08                                 | 0.001003                               | 45.654                           | 100.98                            | 2306.9                    | 2407.9                           | 100.98                            | 2443.9                    | 2544.8                           | 0.3543                            | 8.2222                    | 8.5765                           |
| 4.0              | 28.96                                 | 0.001004                               | 34.791                           | 121.39                            | 2293.1                    | 2414.5                           | 121.39                            | 2432.3                    | 2553.7                           | 0.4224                            | 8.0510                    | 8.4734                           |
| 5.0              | 32.87                                 | 0.001005                               | 28.185                           | 137.75                            | 2282.1                    | 2419.8                           | 137.75                            | 2423.0                    | 2560.7                           | 0.4762                            | 7.9176                    | 8.3938                           |
| 7.5              | 40.29                                 | 0.001008                               | 19.233                           | 168.74                            | 2261.1                    | 2429.8                           | 168.75                            | 2405.3                    | 2574.0                           | 0.5763                            | 7.6738                    | 8.2501                           |
| 10               | 45.81                                 | 0.001010                               | 14.670                           | 191.79                            | 2245.4                    | 2437.2                           | 191.81                            | 2392.1                    | 2583.9                           | 0.6492                            | 7.4996                    | 8.1488                           |
| 15               | 53.97                                 | 0.001014                               | 10.020                           | 225.93                            | 2222.1                    | 2448.0                           | 225.94                            | 2372.3                    | 2598.3                           | 0.7549                            | 7.2522                    | 8.0071                           |
| 20               | 60.06                                 | 0.001017                               | 7.6481                           | 251.40                            | 2204.6                    | 2456.0                           | 251.42                            | 2357.5                    | 2608.9                           | 0.8320                            | 7.0752                    | 7.9073                           |
| 25               | 64.96                                 | 0.001020                               | 6.2034                           | 271.93                            | 2190.4                    | 2462.4                           | 271.96                            | 2345.5                    | 2617.5                           | 0.8932                            | 6.9370                    | 7.8302                           |
| 1100             | 184.06                                | 0.001133                               | 0.17745                          | 779.78                            | 1805.7                    | 2585.5                           | 781.03                            | 1999.6                    | 2780.7                           | 2.1785                            | 4.3735                    | 6.5520                           |
| 1200             | 187.96                                | 0.001138                               | 0.16326                          | 796.96                            | 1790.9                    | 2587.8                           | 798.33                            | 1985.4                    | 2783.8                           | 2.2159                            | 4.3058                    | 6.5217                           |
| 1300             | 191.60                                | 0.001144                               | 0.15119                          | 813.10                            | 1776.8                    | 2589.9                           | 814.59                            | 1971.9                    | 2786.5                           | 2.2508                            | 4.2428                    | 6.4936                           |
| 1400             | 195.04                                | 0.001149                               | 0.14078                          | 828.35                            | 1763.4                    | 2591.8                           | 829.96                            | 1958.9                    | 2788.9                           | 2.2835                            | 4.1840                    | 6.4675                           |
| 1500             | 198.29                                | 0.001154                               | 0.13171                          | 842.82                            | 1750.6                    | 2593.4                           | 844.55                            | 1946.4                    | 2791.0                           | 2.3143                            | 4.1287                    | 6.4430                           |
| 1750             | 205.72                                | 0.001166                               | 0.11344                          | 876.12                            | 1720.6                    | 2596.7                           | 878.16                            | 1917.1                    | 2795.2                           | 2.3844                            | 4.0033                    | 6.3877                           |
| 2000             | 212.38                                | 0.001177                               | 0.099587                         | 906.12                            | 1693.0                    | 2599.1                           | 908.47                            | 1889.8                    | 2798.3                           | 2.4467                            | 3.8923                    | 6.3390                           |

Superheated water (Continued)

| T<br>°C                 | v<br>m <sup>3</sup> /kg | u<br>kJ/kg | h<br>kJ/kg | s<br>kJ/kg · K          | v<br>m <sup>3</sup> /kg | u<br>kJ/kg | h<br>kJ/kg | s<br>kJ/kg · K          | v<br>m <sup>3</sup> /kg | u<br>kJ/kg | h<br>kJ/kg | s<br>kJ/kg · K |
|-------------------------|-------------------------|------------|------------|-------------------------|-------------------------|------------|------------|-------------------------|-------------------------|------------|------------|----------------|
| P = 1.00 MPa (179.88°C) |                         |            |            | P = 1.20 MPa (187.96°C) |                         |            |            | P = 1.40 MPa (195.04°C) |                         |            |            |                |
| Sat.                    | 0.19437                 | 2582.8     | 2777.1     | 6.5850                  | 0.16326                 | 2587.8     | 2783.8     | 6.5217                  | 0.14078                 | 2591.8     | 2788.9     | 6.4675         |
| 200                     | 0.20602                 | 2622.3     | 2828.3     | 6.6956                  | 0.16934                 | 2612.9     | 2816.1     | 6.5909                  | 0.14303                 | 2602.7     | 2803.0     | 6.4975         |
| 250                     | 0.23275                 | 2710.4     | 2943.1     | 6.9265                  | 0.19241                 | 2704.7     | 2935.6     | 6.8313                  | 0.16356                 | 2698.9     | 2927.9     | 6.7488         |
| 300                     | 0.25799                 | 2793.7     | 3051.6     | 7.1246                  | 0.21386                 | 2789.7     | 3046.3     | 7.0335                  | 0.18233                 | 2785.7     | 3040.9     | 6.9553         |
| 350                     | 0.28250                 | 2875.7     | 3158.2     | 7.3029                  | 0.23455                 | 2872.7     | 3154.2     | 7.2139                  | 0.20029                 | 2869.7     | 3150.1     | 7.1379         |
| 400                     | 0.30661                 | 2957.9     | 3264.5     | 7.4670                  | 0.25482                 | 2955.5     | 3261.3     | 7.3793                  | 0.21782                 | 2953.1     | 3258.1     | 7.3046         |
| 500                     | 0.35411                 | 3125.0     | 3479.1     | 7.7642                  | 0.29464                 | 3123.4     | 3477.0     | 7.6779                  | 0.25216                 | 3121.8     | 3474.8     | 7.6047         |
| 600                     | 0.40111                 | 3297.5     | 3698.6     | 8.0311                  | 0.33395                 | 3296.3     | 3697.0     | 7.9456                  | 0.28597                 | 3295.1     | 3695.5     | 7.8730         |
| 700                     | 0.44783                 | 3476.3     | 3924.1     | 8.2755                  | 0.37297                 | 3475.3     | 3922.9     | 8.1904                  | 0.31951                 | 3474.4     | 3921.7     | 8.1183         |
| 800                     | 0.49438                 | 3661.7     | 4156.1     | 8.5024                  | 0.41184                 | 3661.0     | 4155.2     | 8.4176                  | 0.35288                 | 3660.3     | 4154.3     | 8.3458         |
| 900                     | 0.54083                 | 3853.9     | 4394.8     | 8.7150                  | 0.45059                 | 3853.3     | 4394.0     | 8.6303                  | 0.38614                 | 3852.7     | 4393.3     | 8.5587         |
| 1000                    | 0.58721                 | 4052.7     | 4640.0     | 8.9155                  | 0.48928                 | 4052.2     | 4639.4     | 8.8310                  | 0.41933                 | 4051.7     | 4638.8     | 8.7595         |
| 1100                    | 0.63354                 | 4257.9     | 4891.4     | 9.1057                  | 0.52792                 | 4257.5     | 4891.0     | 9.0212                  | 0.45247                 | 4257.0     | 4890.5     | 8.9497         |
| 1200                    | 0.67983                 | 4469.0     | 5148.9     | 9.2866                  | 0.56652                 | 4468.7     | 5148.5     | 9.2022                  | 0.48558                 | 4468.3     | 5148.1     | 9.1308         |
| 1300                    | 0.72610                 | 4685.8     | 5411.9     | 9.4593                  | 0.60509                 | 4685.5     | 5411.6     | 9.3750                  | 0.51866                 | 4685.1     | 5411.3     | 9.3036         |
| P = 1.60 MPa (201.37°C) |                         |            |            | P = 1.80 MPa (207.11°C) |                         |            |            | P = 2.00 MPa (212.38°C) |                         |            |            |                |
| Sat.                    | 0.12374                 | 2594.8     | 2792.8     | 6.4200                  | 0.11037                 | 2597.3     | 2795.9     | 6.3775                  | 0.09959                 | 2599.1     | 2798.3     | 6.3390         |
| 225                     | 0.13293                 | 2645.1     | 2857.8     | 6.5537                  | 0.11678                 | 2637.0     | 2847.2     | 6.4825                  | 0.10381                 | 2628.5     | 2836.1     | 6.4160         |
| 250                     | 0.14190                 | 2692.9     | 2919.9     | 6.6753                  | 0.12502                 | 2686.7     | 2911.7     | 6.6088                  | 0.11150                 | 2680.3     | 2903.3     | 6.5475         |
| 300                     | 0.15866                 | 2781.6     | 3035.4     | 6.8864                  | 0.14025                 | 2777.4     | 3029.9     | 6.8246                  | 0.12551                 | 2773.2     | 3024.2     | 6.7684         |
| 350                     | 0.17459                 | 2866.6     | 3146.0     | 7.0713                  | 0.15460                 | 2863.6     | 3141.9     | 7.0120                  | 0.13860                 | 2860.5     | 3137.7     | 6.9583         |
| 400                     | 0.19007                 | 2950.8     | 3254.9     | 7.2394                  | 0.16849                 | 2948.3     | 3251.6     | 7.1814                  | 0.15122                 | 2945.9     | 3248.4     | 7.1292         |
| 500                     | 0.22029                 | 3120.1     | 3472.6     | 7.5410                  | 0.19551                 | 3118.5     | 3470.4     | 7.4845                  | 0.17568                 | 3116.9     | 3468.3     | 7.4337         |
| 600                     | 0.24999                 | 3293.9     | 3693.9     | 7.8101                  | 0.22200                 | 3292.7     | 3692.3     | 7.7543                  | 0.19962                 | 3291.5     | 3690.7     | 7.7043         |
| 700                     | 0.27941                 | 3473.5     | 3920.5     | 8.0558                  | 0.24822                 | 3472.6     | 3919.4     | 8.0005                  | 0.22326                 | 3471.7     | 3918.2     | 7.9509         |
| 800                     | 0.30865                 | 3659.5     | 4153.4     | 8.2834                  | 0.27426                 | 3658.8     | 4152.4     | 8.2284                  | 0.24674                 | 3658.0     | 4151.5     | 8.1791         |
| 900                     | 0.33780                 | 3852.1     | 4392.6     | 8.4965                  | 0.30020                 | 3851.5     | 4391.9     | 8.4417                  | 0.27012                 | 3850.9     | 4391.1     | 8.3925         |
| 1000                    | 0.36687                 | 4051.2     | 4638.2     | 8.6974                  | 0.32606                 | 4050.7     | 4637.6     | 8.6427                  | 0.29342                 | 4050.2     | 4637.1     | 8.5936         |
| 1100                    | 0.39589                 | 4256.6     | 4890.0     | 8.8878                  | 0.35188                 | 4256.2     | 4889.6     | 8.8331                  | 0.31667                 | 4255.7     | 4889.1     | 8.7842         |
| 1200                    | 0.42488                 | 4467.9     | 5147.7     | 9.0689                  | 0.37766                 | 4467.6     | 5147.3     | 9.0143                  | 0.33989                 | 4467.2     | 5147.0     | 8.9654         |
| 1300                    | 0.45383                 | 4684.8     | 5410.9     | 9.2418                  | 0.40341                 | 4684.5     | 5410.6     | 9.1872                  | 0.36308                 | 4684.2     | 5410.3     | 9.1384         |

1. A specimen of medium-carbon steel having an initial diameter of  $d_0$  inch was tested in tension using a gauge length of  $L_0$  inch. The following data were obtained for the states 1 and 2:

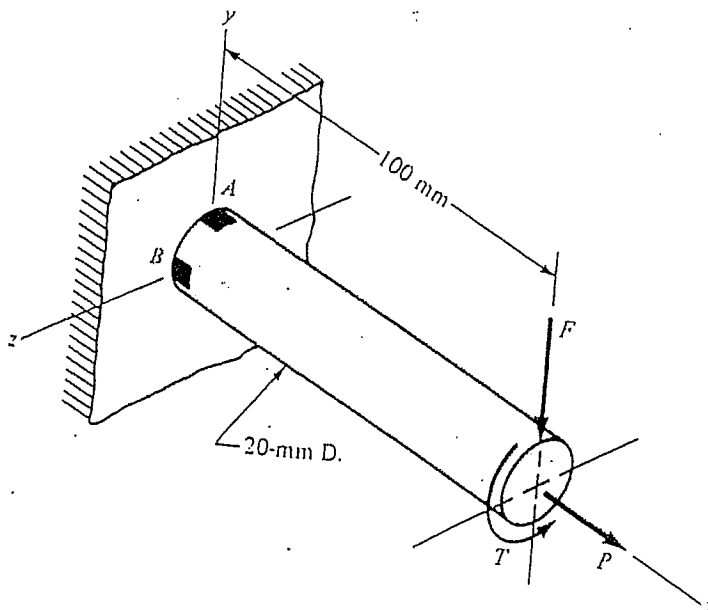
| <u>Cases</u> | <u>Load <math>P</math> (lb)</u> | <u>Measured Diameters</u> |
|--------------|---------------------------------|---------------------------|
| 1            | $P_1$                           | $d_1$                     |
| 2            | $P_2$                           | $d_2$                     |

- (a) To express the engineering stress  $\sigma$  and the engineering strain  $\varepsilon$  at case 1. (5%)
- (b) To express the true stress  $\sigma'$  and the true strain  $\varepsilon'$  at case 2. (5%)

2. Assume the stress tensor at a point can be expressed as

$$[\sigma_{ij}] = \begin{bmatrix} 8 & 12 & 4 \\ 12 & 18 & 6 \\ 4 & 6 & 36 \end{bmatrix} \quad (\text{MPa})$$

- (i) To calculate the principal stresses  $\sigma_1, \sigma_2, \sigma_3$  at this point. (10%)
- (ii) To calculate the maximum shear stress  $\tau_{\max}$  at this point. (5%)
- (iii) To calculate the von Mises stresses  $\sigma'$  at this point. (5%)
3. This solid bar is made of steel with a diameter 20 mm. The loads are  $F = 0.55 \text{ kN}$ ,  $P = 8.0 \text{ kN}$  and  $T = 30 \text{ Nm}$ . To find the stress tensors at points A and B of the member as shown in the figure. (20%)

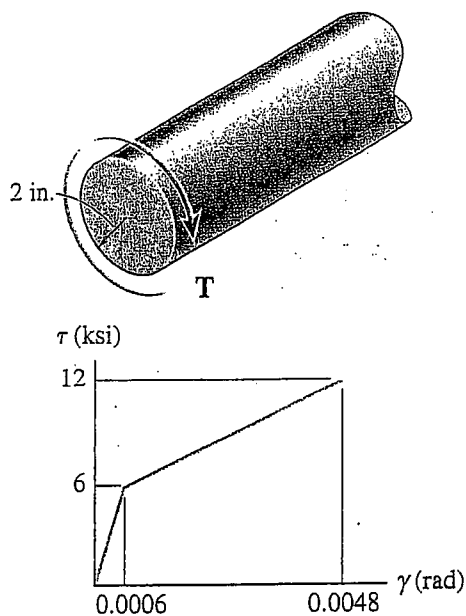




4.

The shaft is subjected to a maximum shear strain of 0.0048 rad. Determine the torque applied to the shaft if the material has strain-hardening as shown by the shear stress-strain diagram.

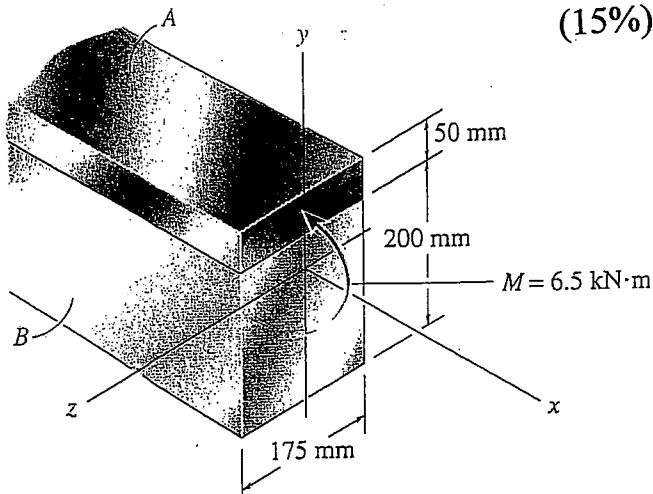
(15%)



5.

The composite beam is made of steel (A) bonded to brass (B) and has the cross section shown. If the allowable bending stress for the steel is  $(\sigma_{\text{allow}})_{\text{st}} = 180 \text{ MPa}$ , and for the brass  $(\sigma_{\text{allow}})_{\text{br}} = 60 \text{ MPa}$ , determine the maximum moment  $M$  that can be applied to the beam.  $E_{\text{br}} = 100 \text{ GPa}$ ,  $E_{\text{st}} = 200 \text{ GPa}$ .

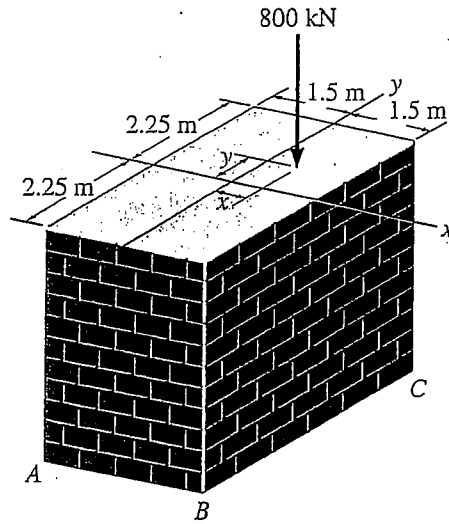
(15%)



6.

The masonry pier is subjected to the 800-kN load. If  $x = 0.25$  m and  $y = 0.5$  m, determine the normal stress at each corner  $A, B, C, D$  (not shown) and plot the stress distribution over the cross section. Neglect the weight of the pier.

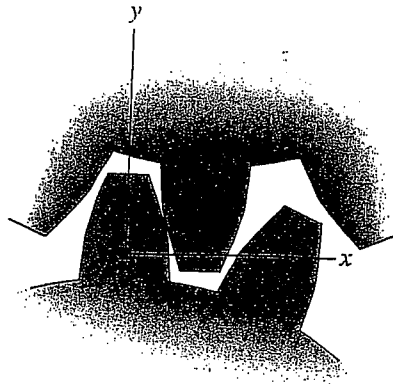
(10%)



7.

The state of strain at the point on the gear tooth has the components  $\epsilon_x = 520(10^{-6})$ ,  $\epsilon_y = -760(10^{-6})$ ,  $\gamma_{xy} = -750(10^{-6})$ . Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the  $x$ - $y$  plane.

(10%)



## 國立中山大學99學年度碩士班招生考試試題

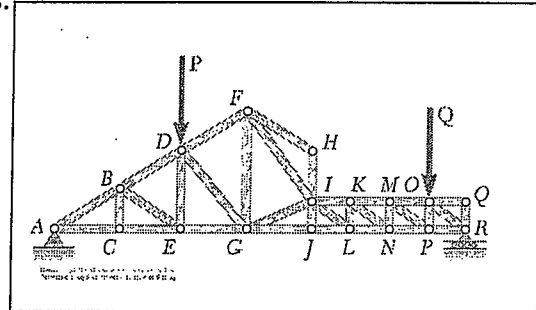
科目：應用力學【機電系碩士班乙組、丙組】

Please choose the correct answers for problem 1 to problem 3

Please be noted that the correct answers for each problem may be more than one

1. Consider the loaded truss shown, which following statements are correct? (5%)

- (A) There are 8 zero-force members in this loaded truss.  
 (B) Both members BE and DE are zero-force members.  
 (C) Both members AB and DG are in compression.  
 (D) Both members AC and EG are in tension  
 (E) Both members OR and PR are in tension  
 (F) None of the previous statements is correct.



2. Consider a balanced sphere with radius  $r$  rolls on the ground. Let  $F$  and  $N$  be defined as the friction force and normal force, respectively, between the sphere and the ground;  $\mu_s$  and  $\mu_k$  are the coefficients of static friction and kinetic friction, respectively, between the sphere and the ground;  $\omega$  and  $\alpha$  are the angular velocity and angular acceleration, respectively, of the sphere;  $v$  and  $a$  are the velocity and acceleration, respectively, of the sphere center. Then which following statements are correct? (5%)

- (A) If the sphere rolls and slides on the ground at the same time, then there is no relative motion between the point of the sphere in contact with the ground and the ground itself.  
 (B) If the sphere rolls and slides on the ground at the same time, then  $F = \mu_k N$ .  
 (C) If the sphere rolls and slides on the ground at the same time, then  $a = r\alpha$ .  
 (D) If the sphere rolls without sliding on the ground, then  $v = r\omega$ .  
 (E) If the sphere rolls without sliding on the ground, then  $F$  must equal to  $\mu_s N$ .  
 (F) None of the previous statements is correct

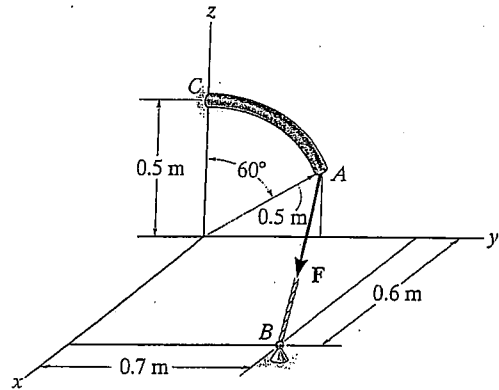
3. A 200-kg flywheel is at rest when a constant  $300 \text{ N} \cdot \text{m}$  couple is applied. After executing 640 revolutions, the flywheel reaches its rated speed of 2400 rpm. Knowing that the radius of gyration of the flywheel is 400 mm, and the kinetic friction in the bearing results in a constant friction couple of magnitude  $M_f$ . Which following statements are correct? (15%)

- (A) The centroidal moment of inertia,  $I$ , of the flywheel is  $32 \times 10^6 \text{ kg} \cdot \text{m}^2$ .  
 (B) The kinetic energy of the flywheel can be calculated as  $(I\omega^2 + mv^2)/2$ , where  $I$  = centroidal moment of inertial,  $\omega$  = angular velocity,  $m$  = mass,  $v = r\omega$ , and  $r$  = radius of the flywheel.  
 (C) When the flywheel reaches its rated speed of 2400 rpm, the kinetic energy of the flywheel is  $102400\pi^2$  Joules.  
 (D) The value of the friction couple,  $M_f$ , can be obtained by using the principle of impulse and momentum.  
 (E) The value of the friction couple,  $M_f$  is less than  $50 \text{ N} \cdot \text{m}$ .  
 (F) None of the previous statements is correct.

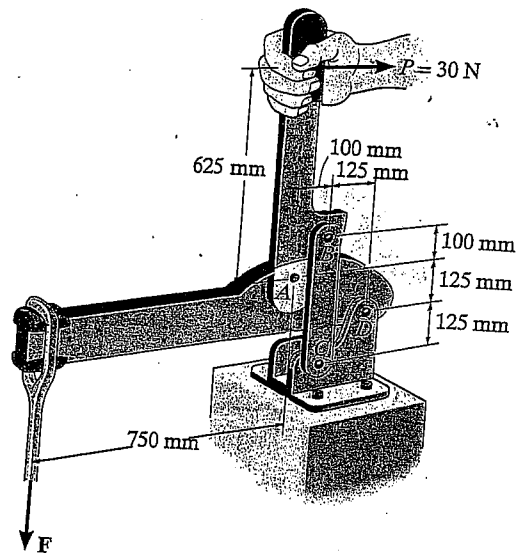
國立中山大學99學年度碩士班招生考試試題

科目：應用力學【機電系碩士班乙組、丙組】

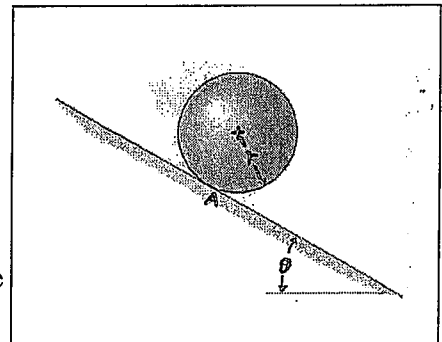
4. Determine the smallest force  $F$  that must be applied along the rope in order to cause the curved rod, which has a radius of 0.5 m, to fail at the support C. This requires a moment of  $M = 200 \text{ N} \cdot \text{m}$  to be developed at C. (10%)



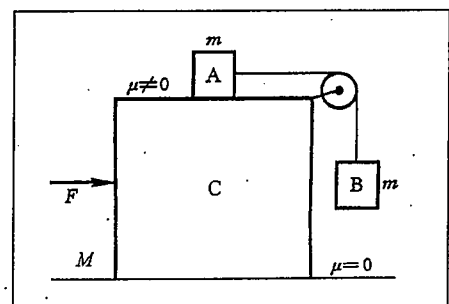
5. If a force of  $P = 30 \text{ N}$  is applied perpendicular to the handle of the mechanism, determine the magnitude of force  $F$  for equilibrium. The members are pin-connected at A, B, C, and D. (15%)



6. The disk has a radius of  $r = 0.3\text{-m}$  and a mass of  $8\text{-kg}$ . If the disk is released from rest on a  $\theta = 30^\circ$  inclined plane, determine the angular acceleration of the disk. The coefficients of static and kinetic friction between the disk and inclined plane are  $\mu_s = 0.15$  and  $\mu_k = 0.1$ , respectively. Please solve this problem by the following procedures: (1) setup the coordinate system, (2) draw the free body diagram, (3) write down the three equations of motion including their positive direction, basic equation, and their content, (4) how many unknowns are in this three equations? (5) if the disk rolls without slipping, what is the fourth equation including the positive direction, basic equation, and their content? (6) solve for the answer and check this assumption, (7) do you need to rework the problem? (8) in the case of slipping, what is the content of this slipping condition? (9) solve for the right answer. (25%)



7. 如圖所示，質量為  $M$  的滑塊 C 放置在光滑桌面上。質量均為  $m$  的兩個重物 A 與 B 用細繩相連。A 平放在滑塊上，與滑塊間的靜摩擦係數為  $\mu$ ，細繩跨過滑輪後將 B 束直懸掛。細繩和滑輪的質量均忽略不計，滑輪轉軸不受摩擦力。以水平推力  $F$  作用於滑塊，為使重物 A 和 B 與滑塊保持相對靜止，試問  $F$  至少應多大？(25%)



國立中山大學99學年度碩士班招生考試試題

科目：自動控制【機電系碩士班丙組】

- (10%) Obtain the transfer functions  $X_1(s)/U(s)$  and  $X_2(s)/U(s)$  of the mechanical system shown in Fig. 1.

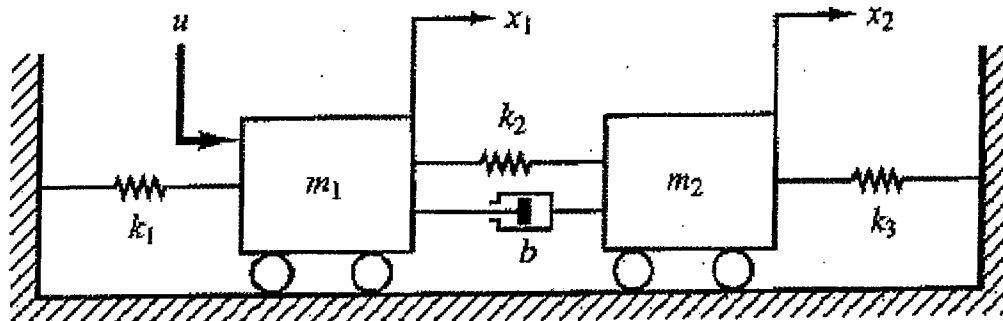


Fig. 1

- (20%) The block diagram of a linear system is shown in Fig. 2.
  - (5%) When  $r(t) = tu_s(t)$  and  $T_d(t) = 0$ , determine how the values of  $K_1$  and  $K_2$  affect the steady-state value of  $e(t)$ .
  - (10%) Construct a parameter plane ( $K_1$  versus  $K_2$ ) to show the stable region.
  - (5%) Let  $r(t) = 0$  and  $T_d(t) = u_s(t)$ . Determine how the values of  $K_1$  and  $K_2$  affect the steady-state value of  $y(t)$ .

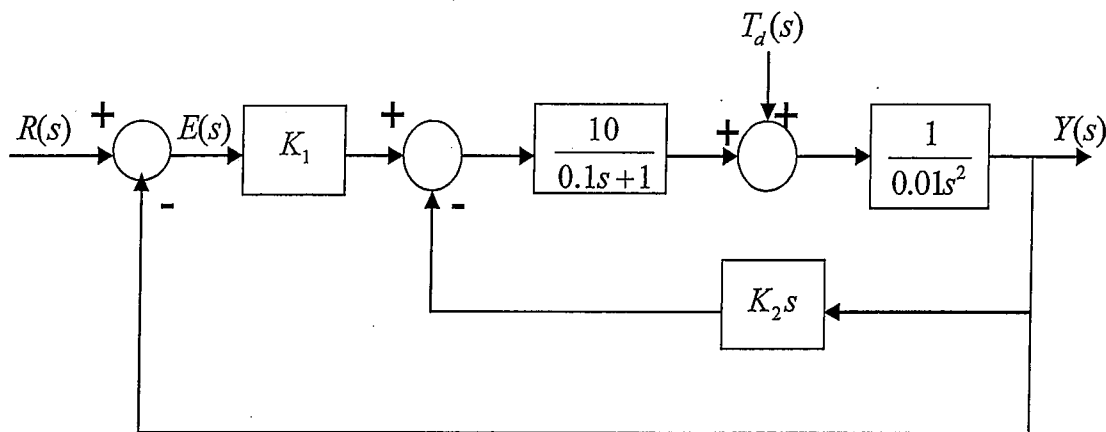


Fig. 2

- (20%) Consider a system as shown in Fig. 3. Please use the root-locus approach to design a proportional-plus-derivative (PD) controller such that the damping ratio  $\zeta$  of the closed-loop system is 0.7 and the undamped natural frequency  $\omega_n$  is 0.5 rad/sec. Also plot the root loci for the system.

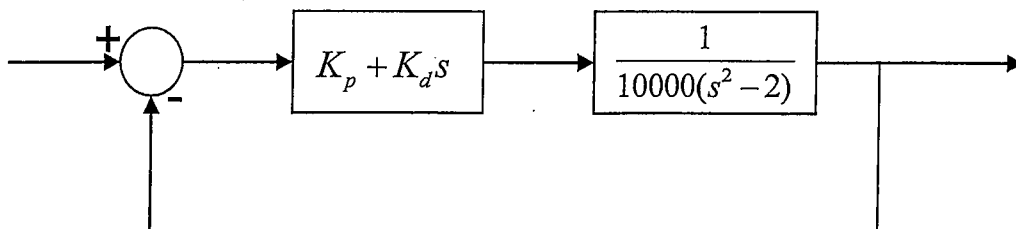


Fig. 3

4. (10%) Consider two dynamic systems modeled by the transfer functions  $\frac{120 \cdot (s+12)}{(s^2 + 4s+100) \cdot (s+350)}$  and  $\frac{5 \cdot (s+3) \cdot e^{-0.004s}}{s \cdot (s^2 + 6s + 400)}$ , respectively. Now, these two systems are subjected to an input (or excitation) described by the function  $1.5 \cdot \cos(12t - 0.24) - 0.48 \cdot \sin(19t + 0.2)$ . Please write down the steady-state responses of the two systems, respectively.

5. (10%) Consider a dynamic system modeled by the transfer function  $\frac{12 \cdot e^{-0.03s}}{s \cdot (s+10)}$ . Please draw the polar plot and Bode diagram of the system.

6. (10%) It is given the polar plot of a dynamic system as shown in Fig. 4. Please try to guess the possible transfer function of the system from the plot.

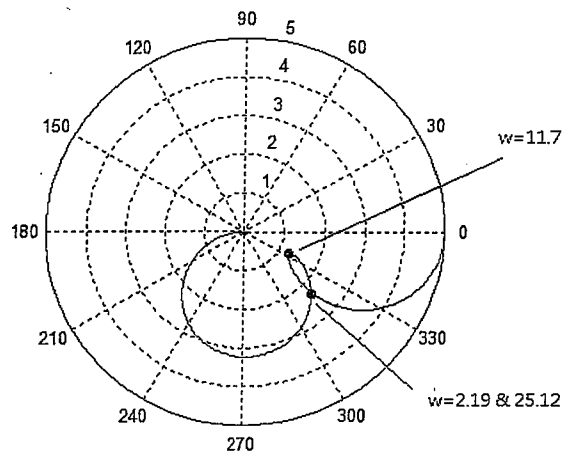


Fig. 4

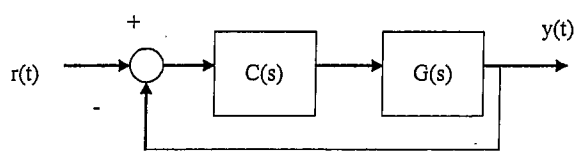


Fig. 5

7. (10%) Consider a unit feedback control system as indicated in Fig. 5. It is known that the system  $G(s)$  has a polar plot as shown in Fig. 6. Now, a P controller is applied to control the system (that is  $C(s) = K_p$ ). If the gain of the controller is set to 1, can we obtain a control performance with good stability? Why? What will be happened if the gain is set to 5?

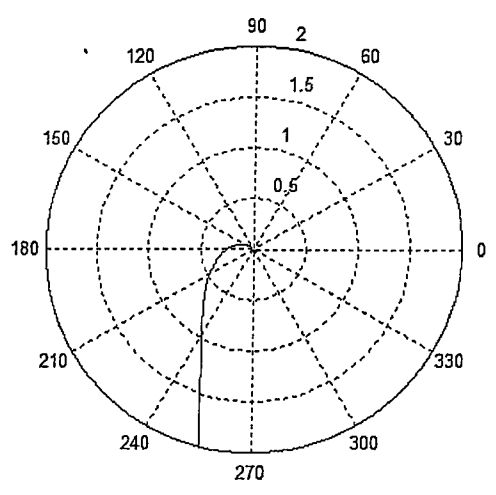


Fig. 6

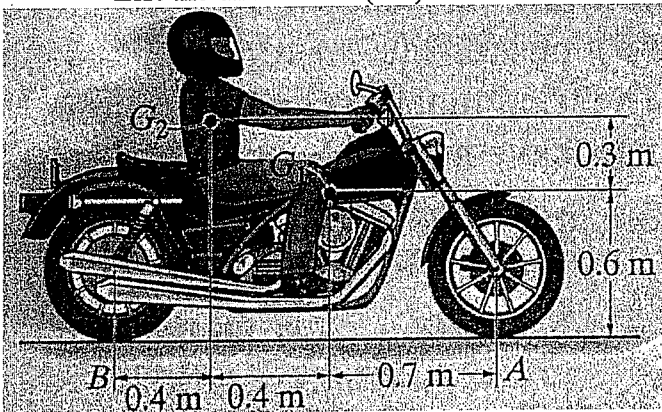
8. (10%) Consider a unit feedback control system as shown in Fig.5. It is desired that the closed-loop control system has a good control performance with a bandwidth greater than 20 Hz. Please draw a polar plot of  $G(s) \cdot C(s)$  that can achieve the desired control performance.

I SINGLE OR MULTIPLE CHOICES for Problems 1 – 12. (Single: 2% each problem, multiple: 1% each choice. You have to determine by yourself which ones are single or multiple-choice problems. 35% in total).

1. Dynamics has two distinct parts. Among which *kinematics* is the study of motion without reference to the [(a) gravitational field, (b) elasticity of the body, (c) forces, (d) inertia].
2. The acceleration would be [(a) indefinite, (b) positive, (c) negative, (d) zero] if the particle had a negative velocity which was becoming less negative. That is, the particle is decelerating.
3. Using *polar coordinates* to describe planar curvilinear motion, we have two unit vectors,  $\mathbf{e}_r$  and  $\mathbf{e}_\theta$ , which are established in the positive  $r$ - and  $\theta$ - directions, respectively. The following relations then hold: [(a)  $\dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e}_r$ , (b)  $\dot{\mathbf{e}}_r = \dot{\theta}\mathbf{e}_\theta$ , (c)  $\dot{\mathbf{e}}_r = \theta\dot{\mathbf{e}}_\theta$ , (d)  $\dot{\mathbf{e}}_\theta = -r\dot{\mathbf{e}}_r$ ].
4. When a particle falls from rest (relative to the earth) at a height  $h$  above the ground, [(a) the rotation of the earth gives rise to an acceleration relative to the earth, (b) commonly called *Euler acceleration*. (c) The particle will fall to the ground a distance  $(2/3)\omega(2h^3/g)^{1/2}\cos\gamma$  (d) South] away from the point on the ground directly under the point from which it was dropped. (Note that  $\omega$  is the angular velocity of the earth, and  $\gamma$  is the latitude.)
5. The ratio of the work done *by* a machine to the work done *on* it during a time interval is called the mechanical [(a) advantage, (b) gain, (c) effectiveness, (d) efficiency] of the machine.
6. If the resultant force on a particle is zero during an interval of time, we see from equation  $\Sigma\mathbf{F} = m\dot{\mathbf{v}} = (d/dt)(m\mathbf{v})$  that its linear momentum is [(a) constant, (b) also zero, (c) invariably increasing, (d) monotonously decreasing].
7. If the coefficient of restitution  $e$  is defined as |relative velocity of separation|/|relative velocity of approach|, then  $e = 1$  means that the capacity of the two particles to recover [(a) exceeds, (b) enhances, (c) equals, (d) reduces] their tendency to deform.
8. *D'Alembert's principle* amounts to rewriting the equation of motion as  $\Sigma\mathbf{F} - m\mathbf{a} = 0$ , [(a) taking the form of a zero force summation, (b) in which  $-m\mathbf{a}$  is treated as an inertia, (c) known as the *force inertia*, (d) and the principle transforms a problem in dynamics to one in statics].
9. In plane kinematics of rigid bodies, translation is defined as any motion in which [(a) every line in the body remains parallel to its original position at all times. (b) However, there could be rotation of certain lines in the body. (c) In *rectilinear translation*, all points in the body move in parallel straight lines. (d) In *curvilinear translation*, all points move on circular arcs].
10. As the body changes its position, the instantaneous center also changes its position both in space and on the body. [(a) The locus of the instantaneous centers in space is called the *space centrode*, and that on the body is called the *body centrode*. (b) At each instant the two centrods are tangent to each other at the instantaneous center. (c) The two centrods also slide on each other during the motion of the body].
11. The equation for transferring the moment of inertia of a body between 2 axes is [(a)  $I = I^* + md^2$ , where  $m$  is the mass, (b)  $d$  is the distance between the two axes. (c)  $I^*$  is a known moment of inertia about any axis, (d) and  $I$  is that of the goal axis that does not intersect the other axis].
12. A symmetrical rotor is spinning about the  $z$ -axis with a large angular velocity. We then apply to the rotor axle a couple whose vector is directed along the  $x$ -axis. As a result, the rotor shaft will start to rotate about [(a) the  $x$ -axis, (b) the  $y$ -axis, (c) the  $z$ -axis, (d) a non-axial axis] with a relatively slow angular velocity known as the *precession velocity*.

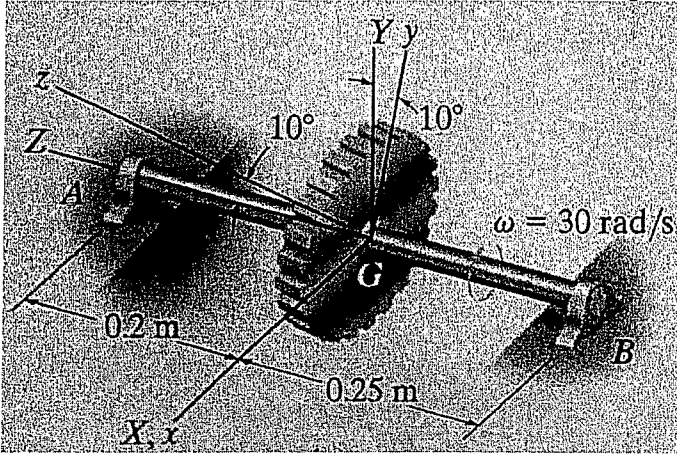
II Find solutions for Problems 13 – 19. (65% in total)

13. An accelerometer mounted on a building frame indicates the frame is vibrating harmonically at 15 cps, with a maximum acceleration of 0.5 g.  
 (a) Determine the amplitude (m) of the building frame. (4%)  
 (b) Determine the maximum velocity (m/s) of the building frame. (4%)
14. Show that any linear combination of  $\sin\omega t$  and  $\cos\omega t$  such that  $x(t) = A_1\cos\omega t + A_2\sin\omega t$  ( $A_1, A_2 =$  constant) represents a simple harmonic motion. (8%)
15. A spring-mass system has a natural period of 0.21 s. What will be the new period if the spring constant is  
 (a) increased by 50 percent. (4%)  
 (b) decreased by 50 percent. (4%)
16. An automobile having a mass of 2,000 kg deflects its suspension springs 0.02 m under static conditions. Determine the natural frequency of the automobile in the vertical direction by assuming damping to be negligible. (8%)
17. The motorcycle shown in the figure below has a mass of 125 kg and a center of mass at  $G_1$ , while the rider has a mass of 75kg and a center of mass at  $G_2$ . We would like to determine the acceleration necessary for the rider to lift the front wheel off the ground. Neglect the mass of the wheels and assume that the front wheel is free to roll at this moment.  
 (a) Draw the free body diagram for the system. Consider both the motorcycle and the rider as a single system. (6%)  
 (b) List the equations of motion. How many unknowns are there in these equations?  
 List the unknowns. (7%)

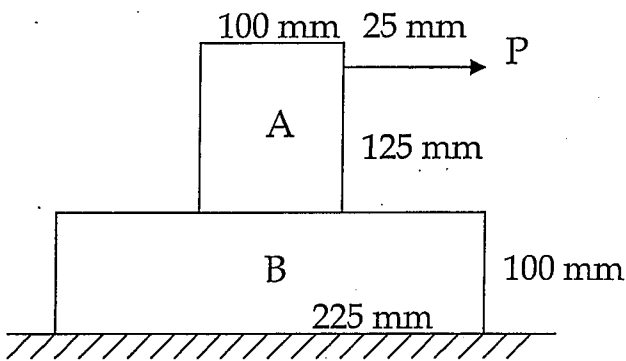




18. The gear shown in the figure below has a mass of 10 kg and is mounted at an angle of  $10^\circ$  with the rotating shaft having negligible mass. If  $I_z = 0.1 \text{ kg}\cdot\text{m}^2$ ,  $I_x = I_y = 0.05 \text{ kg}\cdot\text{m}^2$  for the gear, and the shaft is rotating with a constant angular velocity of  $\omega = 30 \text{ rad/s}$ .
- (a) Draw the free body diagram for the system including the gear and the shaft. (6%)
- (b) This is a three dimension problem. There are six equations of motion. How many unknowns are there in the equations of motion for this problem? List the unknowns. (4%)



19. Block A with mass 30 kg is located on the top of a block with mass 45 kg as shown in the figure below. The coefficient of friction between the block is  $1/3$ . The coefficient between the block B and the horizontal plane is  $1/10$ . There is a maximum value of  $P$  that will not cause block A to tip.
- (a) Draw the free body diagram for block A. (3%)
- (b) Draw the free body diagram for block B. (3%)
- (c) List the unknowns in your free body of diagrams. (4%)



## 科目：靜力學【機電系碩士班丁組、戊組】

請在答案卷作答

1. SINGLE OR MULTIPLE CHOICES. (Single: 2% each problem, multiple: 1% each choice. You have to determine by yourself which ones are single or multiple-choice problems. 35% in total)
  - (1) Mechanics is the branch of physical science that deals with the state of rest or motion of bodies under the influence of [(a) velocity, (b) rigidity, (c) forces, (d) and magnetism].
  - (2) Modern research and development in the fields of vibrations, stability of structures, [(a) robotics, (b) automatic control, (c) electrical machines, (d) and molecular behavior] are all highly dependent upon the basic principles of mechanics.
  - (3) Substantial contributions to the advancement of mechanics in the history were made collectively by [(a) Pythagoras, (b) da Vinci, (c) Laplace, (d) Renoir] and others.
  - (4) *Newton's laws* state that [(a) A particle remains at rest or continuous to move in a straight line with a uniform velocity if there is no unbalanced force acting on it. (b) The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force. (c) The forces of action and reaction between interacting bodies are equal in direction, opposite in magnitude, and collinear. (d) All of the above are true.]
  - (5) One useful principle of mechanics is *Varignon's theorem*, which states that the moment of a force about any point is equal to the [(a) inner product, (b) outer product, (c) difference, (d) sum] of the moments of the components of the force about the same point.
  - (6) In a system of forces and moments, when the resultant moment vector  $M$  is parallel to the resultant force  $R$ , the resultant is said to be a [(a) wrench, (b) twist, (c) screw, (d) couple].
  - (7) When discussing the [(a) rotation, (b) acceleration, (c) equilibrium, (d) deformation] of a body under the action of 3 forces, we see that the lines of action of the 3 forces must be concurrent.
  - (8) A body that possesses more external supports or constraints than are needed to maintain an equilibrium position is called [(a) statically determinate, (b) statically indeterminate, (c) statistically determinate, (d) statistically indeterminate].
  - (9) If the density is [(a) uniform, (b) isotropic, (c) variable, (d) isentropic] throughout a body, the positions of its centroid and center of mass are identical.
  - (10) Consider a beam extending in the  $x$ -direction with distributed loads. The loading  $w$  represents the force per unit length of beam. At the location  $x$  the shear force acting on the beam element is  $V$  and the internal moment is  $M$ . Equilibrium of such a beam element requires that [(a)  $w = -\frac{dV}{dx}$  (b)  $V = \frac{dM}{dx}$  (c) and  $\frac{d^2M}{dx^2} = -w$ ].
  - (11) By the principle of buoyancy, we see that the resultant force exerted on the surface of an object immersed in a fluid is equal and opposite to the weight of fluid displaced and passes through the center of mass of the [(a) immersed object, (b) displace fluid, (c) combined system].
  - (12) [(a) Coulomb, (b) Fluid, (c) Internal, (d) Dry] friction is encountered when the un-lubricated surfaces of two solids are in contact under a condition of sliding or tendency to slide.
  - (13) The principle of virtual work states that the virtual work done by external [(a) active and reactive, (b) reactive, (c) active, (d) and internal reactive] forces on an ideal mechanical system in equilibrium is zero for any and all virtual displacements consistent with the constraints.

2. As shown in Figure 1, a metal rod having a mass of 200 kg is to be supported by the cord which wraps over a 1.0 m pipe. Note that the pipe is fixed at both ends. Take  $\mu_s = 0.2$ .

- Determine the smallest vertical force  $F$  needed to support the load if the cord passed once over the pipe,  $\beta = 180^\circ$ . (5%)
- Determine the smallest and the largest bending force at the center of the pipe if the metal rod doesn't move.  $\beta = 180^\circ$ . (5%)
- Please draw the free body diagram, if the cord passes two times over the pipe,  $\beta = 540^\circ$ . (5%)
- Determine the largest vertical force  $F$  that can be applied to the cord without moving the cylinder,  $\beta = 540^\circ$ . (5%)

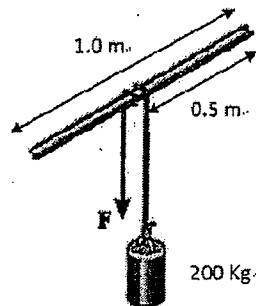


Figure 1

3. As shown in Figure 2, the rigid metal strip is used as part of an electromagnetic switch. The strip is originally horizontal when the springs are unstretched.

- If the stiffness of the springs at A and B is  $k = 5 \text{ N/m}$ , and the mass of the strip is negligible, determine the smallest vertical force needed to close the contract gap at C. (5%)
- If the stiffness of the springs at A and B is  $k = 5 \text{ N/m}$ , and the mass of the strip is 0.1 kg, determine the smallest vertical force needed to close the contract gap at C. (5%)
- Determine the maximum stiffness  $k$  of the springs at A and B so that the contact at C closed when the vertical force developed there is 0.8 N. The mass of the strip is negligible in this case. (5%)

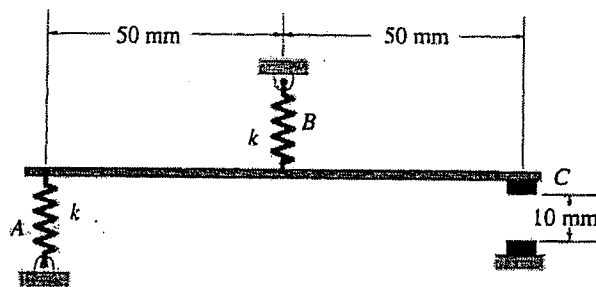


Figure 2

4. The device shown in Figure 3 is designed to exert a large force on the horizontal bar at A for a stamping operation. If the hydraulic cylinder, DE exert an axial force of 800 N and  $\alpha=80^\circ$ , what horizontal force is exerted on the horizontal bar. (15%)

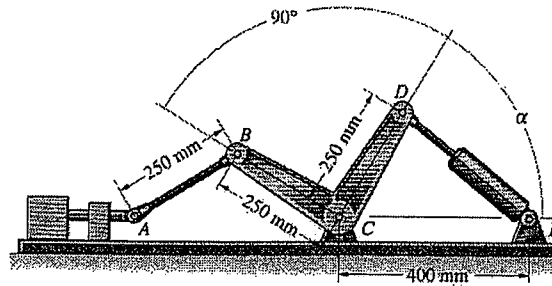


Figure 3

5. As shown in Figure 4, the hydraulic cylinder exerts an 8-kN force at B that is parallel to the cylinder and points from C toward B. Determine the moments of the force about points A and D. (15%)

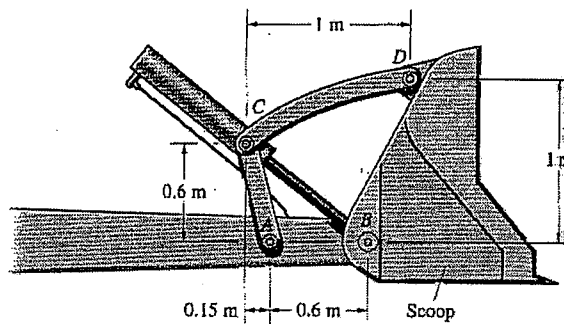


Figure 4