[I] Please choose the correct answers for the following problems.

(1) Let **F** = 2x**i** + 3y**j** + z**k** , S is the surface of the cylinder $x^2 + y^2 \leq 4, 0 \leq z \leq 2$, and **n** is the outward normal of the surface. Then the value of the surface integral ($\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$) is

(A) 36π
(B) 48π (5%)
(C) 60π
(D) zero
(E) neither one

(2) The directional derivative of $f(x, y, z) = 2x^2 - y^2 + z^2$ at (1, 2, 3) in the direction from (1, 2, 3) to (3 ,5, 0) is

(A) 22
(B) −22 (5%)
(C) $\sqrt{22}$
(D) $-\sqrt{22}$
(E) neither one

(3) Let **A**, **B**, & **C** be vector fields , P & Q be scalar fields, **R** is a constant vector, **F** = x**i** + y**j** + z**k** and ∇is the del operator. Then which of the following statements are true ? (NOTE: the answers may be more than one)

(A) **A** X **B** is orthogonal to both **A** and **B**.
(B) If **A** X **B** = 0, then **A** and **B** are parallel
(C) ∇ · (**R** − **F**) = 3 (15%)
(D) ∇ X (**R** − **F**) =**R**
(E) **A** X (**B** X **C**) = (**A** · **C**)**B** − (**A** · **B**)**C**
(F) (**A** X **B**) X **C** = (**A** · **C**)**B** − (**B** · **C**)**A**
(G) ∇X (P**A**) = (∇P) X **A** + P (∇X **A**)
(H) ∇(PQ) = P(∇Q) + Q(∇P)
(I) neither one

[II] Evaluate the following real integral for any positive constant α,

$$\int_{-\infty}^{\infty} \frac{\cos(\alpha x)}{x^2+1} dx. \qquad (10\%)$$

[III] Solve the boundary value problem for any constant A. (15 %)

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} + Ax \qquad (L > x > 0,\ t > 0),$$

$$y(0,t) = y(L,t) = 0 \qquad (t > 0),$$

$$y(x,0) = 0, \quad \frac{\partial y}{\partial t}(x,0) = 0 \qquad (L > x > 0).$$

[IV] Find the inverse of the following matrix A by using determinants. (10%)

$$A = \begin{bmatrix} 8 & 0 & 1 \\ 3 & -2 & 1 \\ 1 & 4 & 0 \end{bmatrix}$$

[V] For the following matrix A, find (a) eigenvalues, (b) unit eigenvectors , and (c) an orthogonal matrix Q which diagonalizes A. (15%)

$$A = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{bmatrix}$$

[VI] Following is a linear constant coefficient ordinary differential equation,

$$4\ddot{y} + 2\dot{y} + y = u$$

with initial conditions of $y(0) = 0$ and $\dot{y}(0) = 0$.
The equation could be describing the motion of a mass-spring-damper system or the current of a serial R-C-L circuit. Please answer the following two questions:

(i) 15%

Find the complete solution of $y(t)$ when u is a unit-step input.

(ii) 10%

If u is a unit-impulse input, find the impulse response $y(t)$.

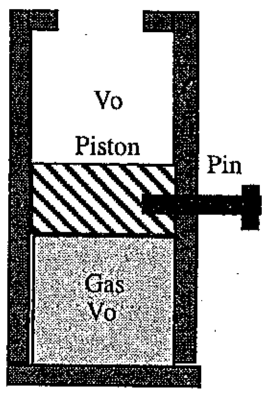
1. Explain the differences between ideal Rankine cycle and ideal Brayton cycle for power plant. Why do we prefer to use Rankine cycle rather than use Brayton cycle in generating electrical power? 5%

2. Internal energy, work, and heat are the energies most often to present in thermodynamics. Please tell me what type of energy can be stored, and what type of energy is a property. What type of energy can only occur in transient state? 5%

3. Please explain how to obtain the latent heat of a pure substance from the following measurable properties :temperature, pressure, and specific volume? 5%

4. The entropy of a steam increases in actual steam turbines as a result of irreversibilities. In an effort to control entropy increase, it is proposed to cool the steam in the turbine by running cooling water around the turbine casing. It is argued that this will reduce the entropy and the enthalpy of the steam at the turbine exit and thus increase the work output. How would you evaluate this proposal? 5%

5. An adiabatical cylinder has a thick piston initially held by a pin as shown in following figure. The cylinder contains ideal gas with constant specific heat c_V. The metal piston has a weight W, and the cross area A_o. The initial gas pressure P_o is $4W/A_o$. The initial volume V_o and the initial temperature T_o of gas are known. The pin is now removed, allowing the piston to move adiabatically until the piston reaches the top where the volume occupied by gas increase to double of the initial volume. Please evaluate the final temperature and pressure in terms of R, c_V, W, A_o, V_o, and T_o. 15%



6. A certain gas which obeys Nobel-Abel equation of state

$$\mathbf{P} = \frac{RT}{v-b}$$

where b is covolume coefficient. Please find the changes of enthalpy and entropy from state 1 to state 2 in terms of measurable properties c_p, pressure, temperature, and specific volume. 15%

7. (10%)

You are given that the ratio of specific heats γ for a thermally perfect gas varies according to

$$\frac{\gamma}{\gamma-1} = A_o - A_1 T \quad (T\text{=absolute temperature})$$

where A_o and A_1 are constants. Find an equation relating pressure p and temperature T at two stations, (1) and (2), for an isentropic process occurring between (1) and (2). Note that the expression you obtain should be analogous to the relation for a calorically perfect gas (a perfect gas with constant specific heats)

$$\frac{p_2}{p_1} = \left[\frac{T_2}{T_1}\right]^{\frac{\gamma}{\gamma-1}}$$

8. (9%)

Explain the following terms of radiation heat transfer

(a) black surface

(b) diffuse surface

(c) gray surface

9. (8%)

(a) Consider two large, black parallel walls, 1 and 2, separated by two thin black plates as show below at steady state, find the radiation energy exchange per unit area,

$\dot{Q}_{12}/A = ?$

(b) Repeat the problem if the walls and the plates are gray surfaces. Use $\varepsilon_1, \varepsilon_2, \varepsilon_3$ as the emittances of the wall 1, wall 2 and the plates, respectively.

10. (8%)

To determine the effect of the temperature dependence of the thermal conductivity on the temperature distribution in a solid, consider a material for which this dependence may be represented as $k = k_o + aT$, where k_o is a positive constant and <u>a is negative</u> coefficient. Sketch the steady-state temperature distribution associated with heat transfer in a plane wall. You must support your answer with an adequate explanation.

11. (15%)

An experimental device that produces excess heat is passively cooled. The addition of pin fins to the casing of this device is being considered augment the rate of cooling. Consider a copper pin fin 0.25 cm in diameter that protrudes from a wall at 95℃ into ambient air at 25℃ as shown in the figure. The heat transfer is mainly by natural convection with a coefficient equal to 10W/m²K. Assuming that the fin is "infinitely long". (Thermal conductivity of the copper k=396W/mK)

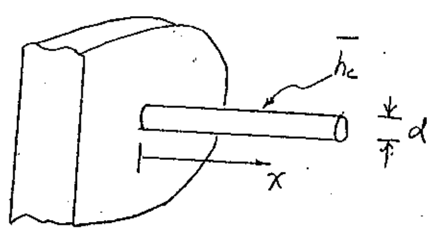
(a) under what condition you can assume T=T(x),

(b) derive the governing equation for T(x),

(c) boundary conditions for the equation in (b),

(d) solve for T(x),

(e) calculate the heat loss.

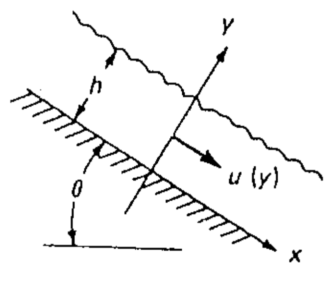


1. (25%)下面是有關於圓管內層流(laminar flow)流場的問題(假設流體之性質為常數)：
   a. 何謂全展流(fully developed flow)？(3%)
   b. 何謂熱全展流(thermally fully developed flow)？(3%)
   c. 證明熱全展流時之 Nusselt number(Nu)為常數。(6%)
   d. 熱全展流時，等壁溫之 Nu=？等壁面熱通量之 Nu=？(4%)
   e. 自管入口至熱全展流之管長為何？(3%)
   f. 畫圖示出熱對流係數(h)沿著流向(x)自管入口至熱全展流之變化。(3%)
   g. 畫圖示出等壁面熱通量時，壁溫（T_s）及流體平均溫度(T_m)，沿著流向(x)自管入口至熱全展流之變化情形。(3%)

2. (25%)假設流體之性質為常數，下面是有關於邊界層（boundary layer，簡稱 B. L.）的問題：
   a. 流場邊界層（flow B. L.）之定義為何？熱邊界層(thermal B. L.)之定義為何？(3%)
   b. 寫出二維穩定流之 flow B. L. equation 及 thermal B. L. equation。(4%)
   c. 無因次化上題兩 B. L.式。要列出無因次變數及參數之定義。(4%)
   d. 定義摩擦係數（friction coefficient）及熱傳係數，再以無因次變數表示出摩擦係數及無因次熱傳係數（Nu）。(3%)
   e. 在壁面接近於平板時，c 中之 flow B. L. equation 有何特殊之處（例如有那一項變成特大、特小或零）？這時候 flow B. L. 和 thermal B. L. 何者發展較快，由什麼參數決定？該參數的物理意義為何？(4%)
   f. 何謂 Reynolds analogy？列出它的條件。(3%)
   g. 何謂 modified Reynolds (or Chilton-Colburn) analogy？列出它層流適用的條件及紊流適用的條件。(4%)

3. (15%) Consider a wide liquid film of constant thickness h flowing steadily due to gravity down an inclined plane at angle θ, as shown in Fig. P3-15. The atmosphere exerts constant pressure and negligible shear on the free surface. Show that the velocity distribution is given by

$$u = \frac{\rho g \sin\theta}{2\mu} y(2h - y)$$



and that the volume flow rate per unit width is $Q = \rho g h^3 \sin\theta / 3\mu$.

4. (15%) A two-dimensional unsteady flow has the velocity components:

$$u = \frac{x}{1+t} \qquad v = \frac{y}{1+2t}$$

Find the equation of the streamlines of this flow which pass through the point (x_0, y_0) at time $t = 0$.

5. (20%) Consider turbulent flow past an isothermal flat plate of width b and length L with constant (ρ, μ, c_p, k). Assume $\delta_u \approx \delta_T$, that is, $Pr \approx 1$. At $x = 0$ the flow has uniform velocity U and temperature T_e. At $x = L$, the mean flow may be approximated by one-seventh power-law profiles:

$$\frac{u}{U} \approx \frac{T - T_w}{T_e - T_w} \approx \left(\frac{y}{\delta}\right)^{1/7}$$

There is no information about the flow structure between the leading and trailing edges. Use a control-volume analysis to estimate, on one side of the plate, (*a*) the total friction drag and (*b*) the total heat transfer, in terms of the boundary-layer thickness.

1. 解釋名詞（20%）

    (1) homogeneous isotropic material
    (2) engineering strain 及 true strain
    (3) 試繪出 mild steel 的 stress-strain curve（不必依比率繪製，只要繪出整體曲線的走勢即可）
    (4) 試繪出 aluminum alloy 的 stress-strain curve（方法同上）
    (5) modulus of resilience
    (6) modulus of toughness
    (7) Saint-Venant's principle
    (8) Plane stress
    (9) Plane strain
    (10) Principal stress

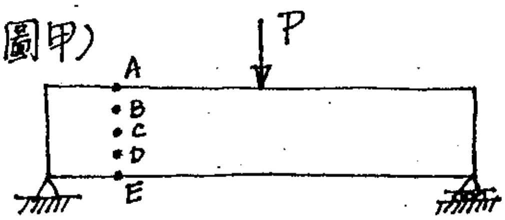
2. 長方形截面之樑，中間受一集中力作用（見圖甲），會在 A 點處造成如圖乙之(1) normal and shear stresses, (2) principal stresses, (3) maximum shear stresses，試分別對 B, C, D, E 點，畫此三圖。（20%）

3. 簡支撐樑受一集中力作用，如圖所示。試求 y 方向之位移（需要的物理量請自行假設）。（20%）

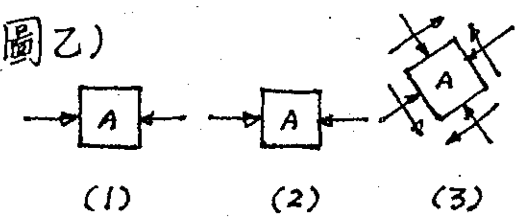
4. The assembly consists of a 6061-T6-aluminum member and a C83400-red-brass member that rest on the rigid plates. Determine the distance *d* where the vertical load **P** should be placed on the plates so that the plates remain horizontal when the materials deform. Each member has a width of 8 in. and they are not bonded together. （20%）

5. The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of T=50lb-ft is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear stress and maximum shear strain in the brass and steel. Take $G_{st}=11.5(10^3)$ ksi, $G_{br}=5.6(10^3)$ ksi. （20%）

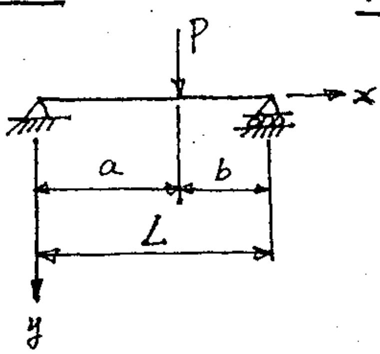
Prob.2 (圖甲)



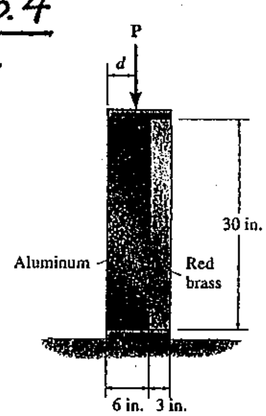
(圖乙)



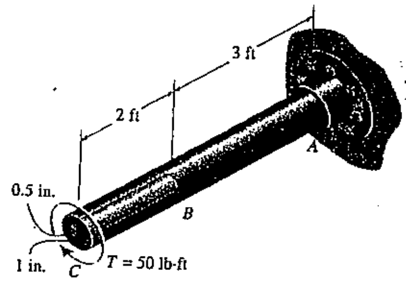
Prob.3



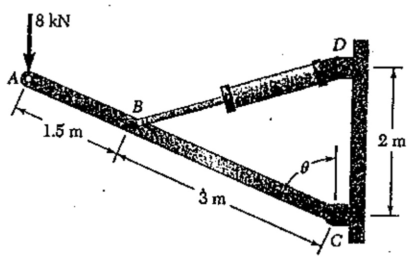
Prob.4



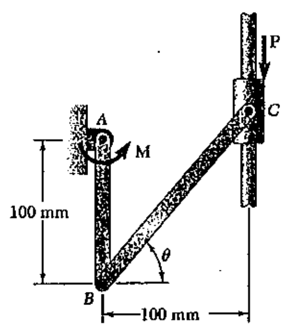
Prob.5



1. The position of boom ABC is controlled by the hydraulic cylinder BD. For the loading shown, determine the force exerted by the hydraulic cylinder on pin B when $\theta = 70°$. (20%)



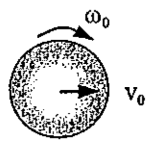
2. Knowing that the coefficient of static friction between the collar and the rod is 0.35, determine the range of values of P for which equilibrium is maintained when $\theta = 50°$ and $M = 20$ N·m. (20%)



3. A sphere of mass m and radius r is projected along a rough horizontal surface with a linear velocity v_0 and an angular velocity $\omega_0 = 2v_0/r$. The sphere will reach a uniform motion after a period of time. The coefficient of friction is denoted by μ and the mass moment of inertia of the sphere with respect to its center is $2mr^2/5$. Determine

(a) the linear and angular accelerations of the sphere before it reaches the uniform motion, (10%)
(b) the time required for the motion to become uniform, and (10%)
(c) the final linear and angular velocities of the sphere. (10%)

Note: Be sure of indicating directions of all accelerations and velocities.



4 Consider a rigid body (with arbitrary shape) moving in the three dimensional space. Please derive the kinetic energy of the rigid body. (15%)

5 For a system of n particles, show that the total external moment about an arbitrary (moving) point P can be written as

$$\sum \mathbf{M}_P = \dot{\mathbf{H}}_G + \mathbf{r}_{G/P} \times m\,\mathbf{a}_G$$

where

$$\sum \mathbf{M}_P = \sum \mathbf{r}_{i/P} \times \mathbf{F}_i$$

$$\mathbf{H}_G = \sum \mathbf{r}_{i/G} \times m_i\,\mathbf{v}_i$$

$$m = \sum m_i$$

and $\mathbf{a}_G$ is the absolute acceleration of the mass center G of the system of particles. $\mathbf{r}_{i/P}$ denotes the position of i relative to P and $\mathbf{r}_{i/G}$ is the position of i relative to G. (15%)

1. Answer the following questions in **short** sentences and diagrams. Each 5%

(1) How to determine a system is stable or not without knowing the governing equations of the system and the eigenvalue position in S domain.

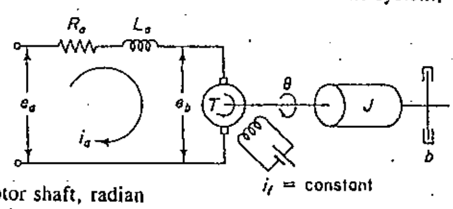
(2) In designing a control system, what is the purpose of step input response?

(3) Draw the step input response of a third order system, the eigenvalue positions of which are -2 + 4i, -2 – 4i and –20 in S domain

(4) Draw the bode diagram for a second order system with damping ratio = 0.3 and nature frequency = 100.

2. **Armature control of dc servomotors.** Consider the armature-controlled dc servomotor shown , where the field current is held constant. In this system,

R_a = armature resistance, ohm
L_a = armature inductance, henry
i_a = armature current, ampere
i_f = field current, ampere
e_a = applied armature voltage, volt
e_b = back emf, volt
θ = angular displacement of the motor shaft, radian
T = torque developed by the motor, N-m
J = equivalent moment of inertia of the motor and load referred to the motor shaft, kg-m^2
b = equivalent viscous-friction coefficient of the motor and load referred to the motor shaft, N-m/rad/sec



The torque T developed by the motor is proportional to the product of the armature current

$$T = Ki_a$$

where K is a motor-torque constant.

(1) Derive the two governing equation in terms of current and angular velocity. 5%

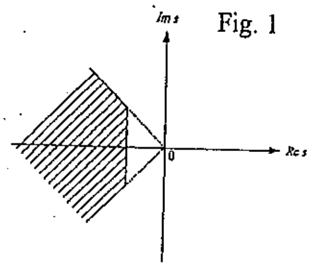
(2) Draw the block diagram from the above equations, considering Ea(s) as input and θ(s) as output. 5%

(3) Find the transfer function, when the inductance is neglected. 10%

(4) If you use the motor to control the angular displacement, show the block diagram of your control method where θr is the reference angular displacement. 10%

3. (10%) In designing a control system, one of senior engineers in your company recommended you to place the poles of the closed-loop system in the shaded region of Fig. 1. Why do you think he made such a recommendation.

Fig. 1



4. (10%) In designing a control system, we often require that

$$-M \le u(t) \le M$$

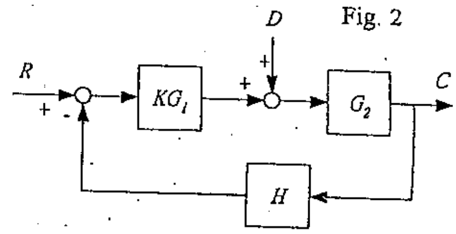
where $u(t)$ is the controller output and M is a constant. Describe the impacts of such constraints on the performance of the closed loop system.

5. (10%) If the input of a derivative controller with derivative constant k_d is $e(t)$, then its output is $k_d\, de(t)/dt$ or, in the S domain, $k_d\, sE(s)$ where $E(s)$ is the Laplace transform of $e(t)$. Therefore, the transfer function of the derivative controller is $k_d s$. Explain why then, in practice, the derivative controller is built as

$$\frac{k_d s}{1+{k_d s}/{N}}$$

6. (10%) This problem considers the system shown in Fig. 2. Note that, in Fig. 2, R is the reference input, C is the output and D is the unwanted disturbance. What is the transfer function of this system? Also, give us your ideas for designing the closed-loop system such that the unwanted disturbance can be effectively rejected.

Fig. 2



7. (10%) Comment on the importance of convolution integral for control system design.

國立中山大學 八十九學年度碩博士班招生考試試題

科　目：動力學　機械工程學系碩士班 丁　

一、簡答下列各題：

(a)在動力學中，在何狀況下可將物體（如汽車）視為質點 (Particle), 又在何狀況下要將它視為剛體 (Rigid-body)? (10%)

(b) 讓一旋轉軸達於靜態平衡 (Static balance), 和達於動態的平衡 (Dynamic balance) 分別是什麼意思? (10%)

二、如圖 P2 所示，桿 AB 以等速 20 rad/s 逆時針旋轉，計算下列各項：

(a) 桿 BDH 的速度 (5%)

(b) 點 H 的速度 (5%)

(c) 桿 BDH 的加速度 (5%)

(d) 點 H 的加速度 (5%)

(e) 如桿 BDH 可無限延伸，求此桿上速度為零的點 (5%)

(f) 同上，求 BDH 桿上加速度為零的點 (5%)

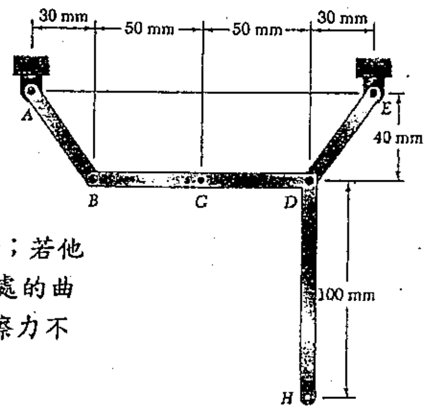


圖 P2

三、如圖 P3 所示，一男孩騎自行車，總重為 125 lb 且質心為點 G；若他自點 A 處以 10ft/s 速率向下滑行，當他到達點 B 處時，點 B 處的曲率半徑為 ρ=50ft，試求作用於自行車車輪的正向力。其中摩擦力不計。(15%)

四、如圖 P4 所示，一根 5kg 的細長桿繞點 O 迴轉，細長桿最初為靜止；若有一顆 4g 子彈以 400m/s 速度射向該桿，如圖中所示，試求在子彈剛射入桿內後細長桿之角速度。(15%)

五、如圖 P5 所示之單擺由桿 AB 與桿 BD 所組成，桿 AB 在 Y-Z 平面內繞固定軸 A 旋轉，桿 BD 以軸承 B 繞桿 AB 做旋轉運動。如圖中所示之瞬時，桿 AB 與桿 BD 的角速度與角加速度分別如圖中所示；若有一個套筒 C 在點 B 的 0.2m 處以速度為 3m/s、加速度為 $2m/s^2$ 沿 BD 方向運動，試求該瞬時套筒 C 的絕對速度與絕對加速度。(20%)

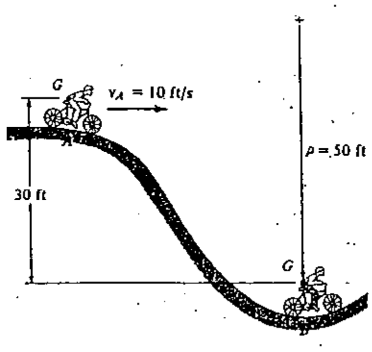


圖 P3

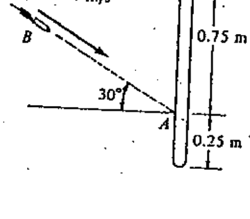


圖 P4

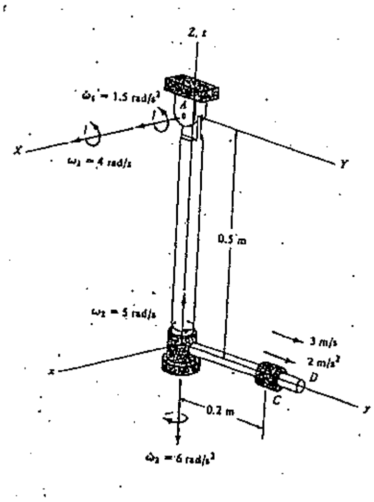


圖 P5

1. Identify the zero-force members present when the truss shown in Figure 1 is subjected to the loadings indicated. (10%)

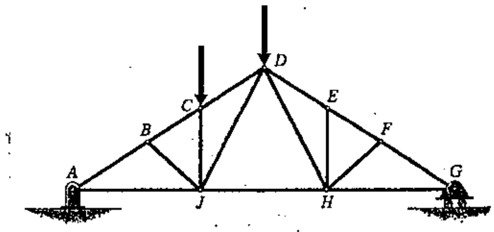


Figure 1

2. The static coefficient of friction between the brake pad and the brake drum of Figure 2 is 0.40. When a force of 350 N is being applied to the brake arm, determine the couple required to initiate rotation of the drum if the direction of rotation is (a) clockwise and (b) counterclockwise. (20%)

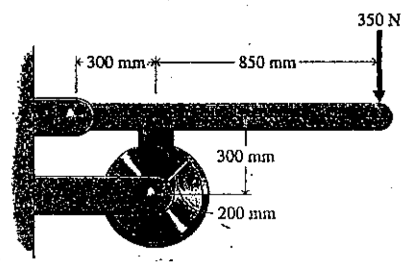


Figure 2

3. Determine an expression for the torque M required to rotate a shaft carrying an axial load P and supported by a conical end bearing as shown in Figure 3. The coefficient of friction is μ. (20%)

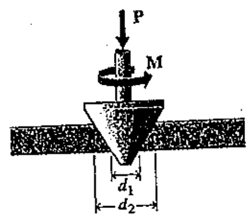


Figure 3

4. A bracket is subjected to the force-couple system as shown in Figure 4. Determine the magnitude and the direction of the resultant force. (15%)

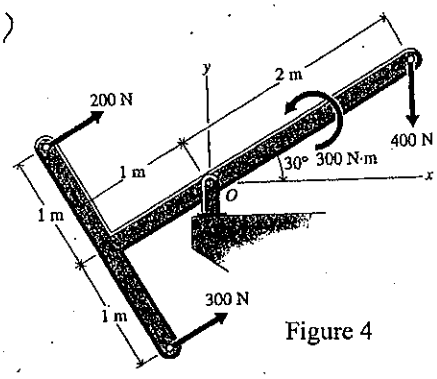


Figure 4

5. The width of the rectangular gate shown in Figure 5 is 4m. Determine the magnitude of the resultant force **R** exerted on the gate by the water (ρ= 1000 kg/m3) pressure and the location of the center of pressure with respect to the hinge at the bottom of the gate. (10%)

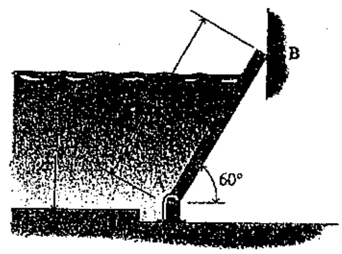


Figure 5

6. A curved slender bar AB is loaded and supported as shown in Figure 6. Draw the Free-Body-Diagram of the curved bar AB and determine the reactions at supports A and B and the tension T in the cable. (25%)

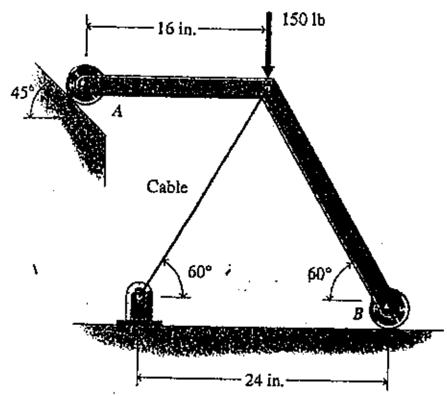


Figure 6