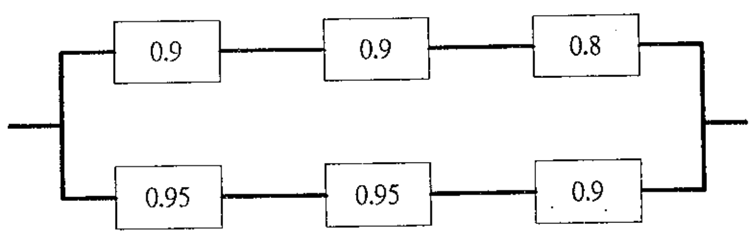
1. (15%) Students have recorded the temperature and resistance measurements as shown in the given table. Careful observation suggests a linear relationship. With R denoting resistance and T representing temperature, find the constants a and b in the following equation to predict R for any given T.

$$R = aT + b$$

Note that in the constants a and b should be chosen in such a way that the sum of the mean squares prediction errors is minimized.

T, °C	R, ohms
20.5	765
32.7	826
51.0	873
73.2	942
95.7	1032

2. (10%) The circuit below operates if and only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



3. (10%) For the quadratic function $f(x,y) = 5x^2 - 2xy + 5y^2$, find a transformation matrix $\boldsymbol{P}$ for

$$\begin{bmatrix} x \\ y \end{bmatrix} = \boldsymbol{P} \begin{bmatrix} X \\ Y \end{bmatrix}$$

so that the given quadratic function can be rewritten as $f(x,y) = f(X,Y) = aX^2 + bY^2$. Also, determine the values for a and b.

4. Try to solve the following simultaneous differential equations

$$\ddot{y}_1 = -3y_1 + 2(y_2 - y_1)$$
$$\ddot{y}_2 = -2(y_2 - y_1)$$

by (1) method of solution by elimination (15%)

(2) eigensolution (15%)

The initial conditions are $y_1(0) = 1$, $y_2(0) = 2$, $\dot{y}_1(0) = -2\sqrt{6}$, $\dot{y}_2(0) = \sqrt{6}$.

5. (12%) Evaluate the following real integrals.

$$\int_{-\infty}^{\infty} \frac{\cos x}{x^2+x+1}dx$$

6. (15%) Find the displacement $w(x, t)$ of an elastic string subject to the following conditions.

   (i) The string is initially at rest on the x-axis from $x = 0$ to ∞ ("semi-infinite string")

   (ii) For time $t > 0$ the left end of the string is moved in a given fashion

$$w(0, t) = f(t) = \begin{cases} \sin t & \text{if } 0 \le t \le 2\pi \\ 0 & \text{otherwise} \end{cases}$$

   (iii) $\lim\limits_{x\to\infty} w(x,t) = 0$ for $t \geqq 0$.

Hint: We have to solve the wave equation

$$\frac{\partial^2 w}{\partial t^2} = a^2 \frac{\partial^2 w}{\partial x^2} \qquad (x > 0, t > 0), \qquad a^2 = \frac{T}{\rho},$$

where T is the tension and ρ is the mass of the undeflected string per unit length.

7. (8%) Find a unit normal vector **n** of the cone of revolution $z^2 = 4(x^2 + y^2)$ at the point P: (1, 0, 2).

15%

1.

(a) Under what condition the pressure at a point is independent of direction.

(b) What is the net pressure force per unit volume at a point for a given pressure distribution p=p(x,y,z)? Explain by using a small rectangular element of fluid, (dx) × (dy) × (dz)

(c) If we have a pressure distribution in a fluid given as

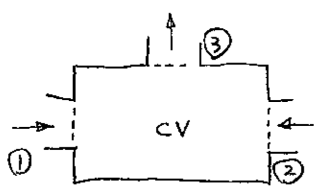
$$p = xy + x + y + z^2 + 20 \text{ kPa}$$

what is the pressure force per unit volume on a element of the medium at position x=5m, y=6m, z=7m?

10%

2. A fixed control volume has three one-dimensional boundary sections, as shown in the following Figure. The flow within the control volume is steady. The flow properties as each section are tabulated below. Find the rate of change of energy of the system which occupies the control volume at this instant.

Section	Type	$\rho, kg/m^3$	$V, m/s$	A, m^2	$e, J/kg$
1	Inlet	800	5.0	2.0	300
2	Inlet	800	8.0	3.0	100
3	Outlet	800	17.0	2.0	150



25%

3. A viscous fluid flows between two parallel plates (the bottom one is fixed and the upper one is moving with velocity U), as shown below. The flow is laminar, two-dimensional, steady, and incompressible. Assume that the velocity in the y-direction v=0, and the pressure drops linearly, p=-Cx+D.

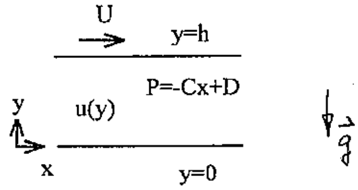
(a) Show the velocity in the x-direction is function of y only, i.e. u=u(y).

(b) Reduce the momentum equation $\rho \frac{d\vec{V}}{dt} = -\nabla p + \rho \vec{g} + \mu \nabla^2 \vec{V}$ for this case, write down the differential equation for u(y)

(c) Write down the boundary condition for u(y)

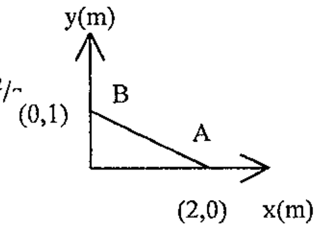
(d) Solve for u=u(y)

(e) Calculate the shear stress at each wall



15% 4. The velocity potential for a certain flow field is

$$\varphi = -2\,(2x + y)$$



where the velocity potential has the unit of m^2/s

(a)Determine the velocity field u = ? v = ?

where u is the velocity in the x-direction

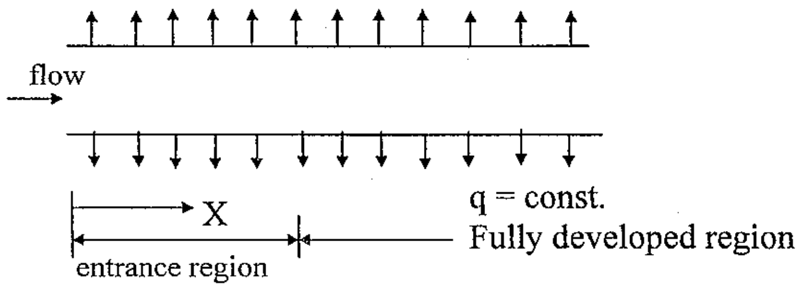
v is the velocity in the y-direction

(b)Determine the corresponding stream function

(c)Determine the rate of flow, per unit width perpendicular to the x-y plane, across the straight path AB as shown

10% 5. Sketch the distribution of bulk temperature T_b and surface temperature T_s of a tube as a function of X

(a) in case the heat flux out of the tube is constant.



(b) in case the surface temperature is constant and the entrance temperature of the flow is higher than the surface temperature.

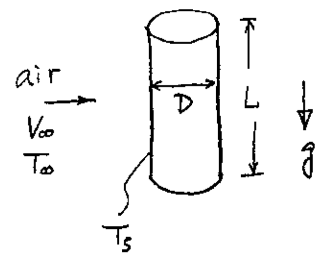
15% 6. Define the following numbers for a vertical cylinder in air as shown below, with kinematic viscosity ν, thermal diffusivity α, volumetric coefficient of thermal expansion β.

(a) Reynolds number Re

(b) Prandtl number Pr

(c) Grashof number Gr

(d) Under what condition natural convection is more important than forced convection in this problem?



10% 7. For external laminar flow over a flat plat, compare the hydrodynamics boundary-layer thickness δ_h and the thermal boundary-layer thickness δ_t for (a) an oil, (b) a gas, (c) a liquid mental (i.e. $\delta_h \ll \delta_t$? $\delta_h \sim \delta_t$? or $\delta_h \gg \delta_t$? for each case)

Part I. 熱力學 (65%)

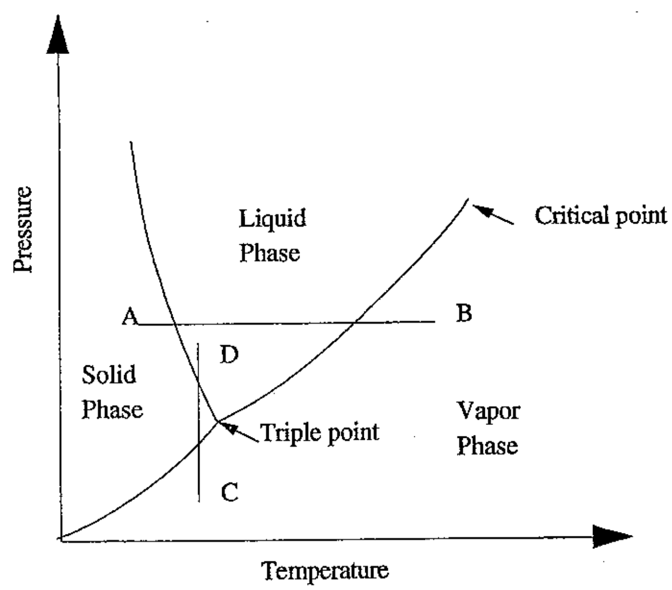
1. Explain the assumptions of steady-state, steady-flow process and uniform-state, uniform-flow process, respectively. 5%

2. What are the difference among heat engines, refrigerators, and heat pumps? What are their applications, respectively? Discuss them from their working principles and the definitions of the thermal efficiencies? 10%

3. What strategies can we generally use to increase the overall thermal efficiencies in vapor or gas power plants? Discuss its restraints also. 5%

4. What is the difference between spark-ignition engines and compression-ignition engines? 5%

5. Complete the following phase diagram (mark the name of each line), and explain the variation of the phases from A to B and C to D. By using this diagram, explain the reason of liquid water recovers to solid ice when a heavy wire pasts an ice block. 10%

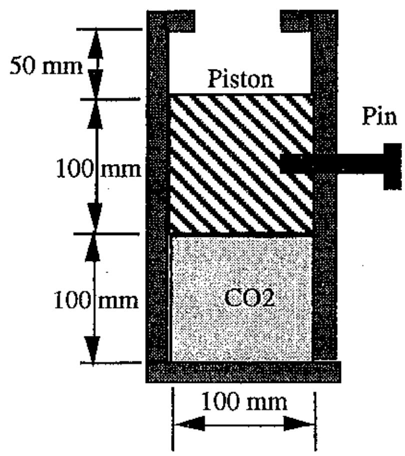


6. A certain gas which obeys Nobel-Abel equation of state

$$\mathbf{P} = \frac{RT}{v-b}$$

where b is covolume coefficient. Please find the deviations of enthalpy h and entropy s from the ideal gas at given temperature T and pressure P. 15%

7. A cylinder has a thick piston initially held by a pin as shown in following figure. The cylinder contains carbon dioxide at 150 kPa and ambient temperature of 290 K. The metal piston has a density of 8000 kg/m3 and the atmospheric pressure is 101 kPa. The pin is now removed, allowing the piston to move and after a while the gas return to ambient temperature. Is the piston against the stops? 15%



Part II. 熱傳學及熱輻射學(35%)

8. Solve the following differential equation :

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0.$$

with boundary conditions

at $x = 0$: $T = T_c$,

$x = L$: $T = T_c + T_m \sin\frac{\pi y}{W}$,

$y = 0$: $T = T_c$,

$y = W$: $T = T_c + T_m \sin\frac{\pi x}{L}$,

where $T_c = 300$ K and $T_m = 100$ K.

(i) Find the temperature distribution T(x,y) within the rectangular with width L and height W.
(ii) If L = W, determine the center temperature.
20%

9. Liquid oxygen is stored in a thin-walled spherical container, 96 cm in diameter, which in turn is enclosed in a concentric container 100 cm in diameter. The surfarces facing each other are plated and have an emittance of 0.05, and the space in between is evacuated. The inner surface is at 95 K, and the outer surface is at 280 K. (i) What is the oxygen boil-off rate? (ii) If a thin radiation shield also of emittance 0.05 is placed midway between the containers, what is the new boil-off rate? (Stefan-Boltzmann is $5.6697 * 10^{-8}$ W/m^2 K^4. The enthalpy of vaporation of oxygen is $0.213*10^6$ J/kg) 15%

Problem #1 (25%)

(a) Draw the engineering stress-strain curve for a low-carbon steel. Please also properly name each region in the curve.

(b) Discuss the differences between the engineering stress/strain and the true stress/strain. Also draw the true stress-strain curve for the same low-carbon steel mentioned above.

Problem #2 (25%)

The solid cylinders AB and BC are bounded together at B and attached to fixed supports at A and C. Knowing that AB is made of aluminum (G = 26GPa) and BC of brass (G = 39GPa), determine for the loading shown (a) the reaction at each support, (b) the maximum shearing stress in AB, (c) the maximum shearing stress in BC.

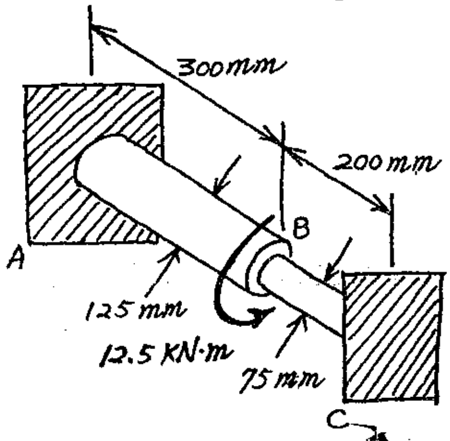


Fig. P2

Problem #3 (20%)

Given (see Fig.P3) $\sigma_{xx} = -1.2\times10^8$ Pa, $\sigma_{yy} = 2\times10^8$ Pa, $\tau_{xy} = -1\times10^8$ Pa, find

(a) $\sigma_{x'x'}, \sigma_{y'y'}, \tau_{x'y'}$.

(b) Principal axes.

(c) Principal stresses.

(d) Extreme shear.

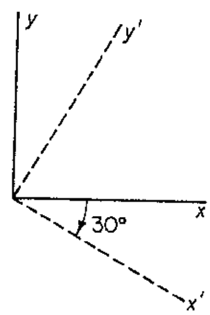


Fig. P3

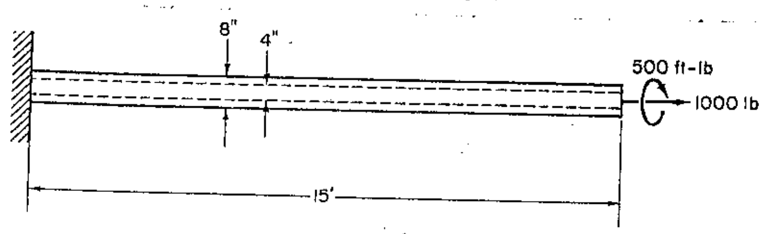


Fig. P4

Problem #4 (15%)

What is the strain energy for the shaft shown in Fig.P4 under the action of a twisting couple and an axial load at the end? Take E=30×10^6 psi and G=15×10^6 psi.

Problem #5 (15%)

What is the deflection (displacement) of joint C due to the 1000-lb load shown in Fig.P5? Both members are linear elastic and have same values of E and A.

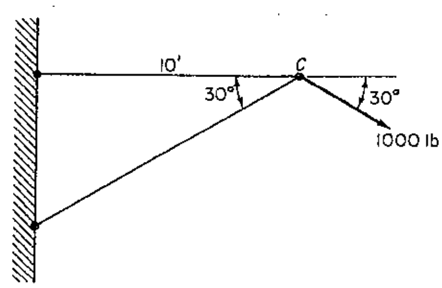


Fig. P5

[I] The man has a weight of 200 lb, and the coefficient of static friction between his shoes and the floor is $\mu_s = 0.5$. Determine where he should position his center of gravity G at d in order to exert the maximum horizontal force on the door. What is this force? (25%)

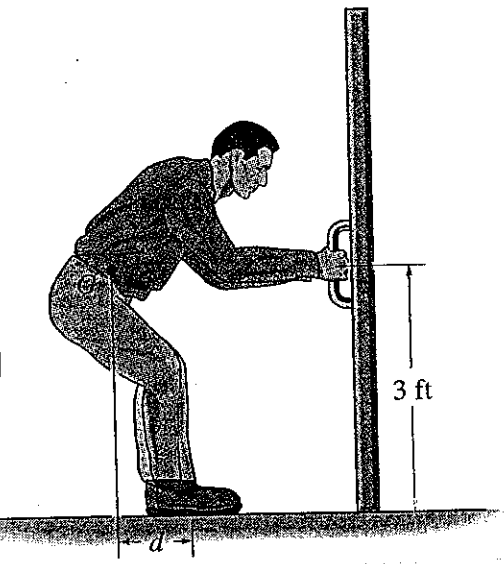


Fig.[I]

[II] The collar has a weight of 5 lb, and the attached spring has an unstretched length of 3 ft. If at the instant $\theta = 30°$ the collar has a speed $v = 4$ ft/s, determine the normal force on the collar and the magnitude of the collar's acceleration. Neglect friction. (25%)

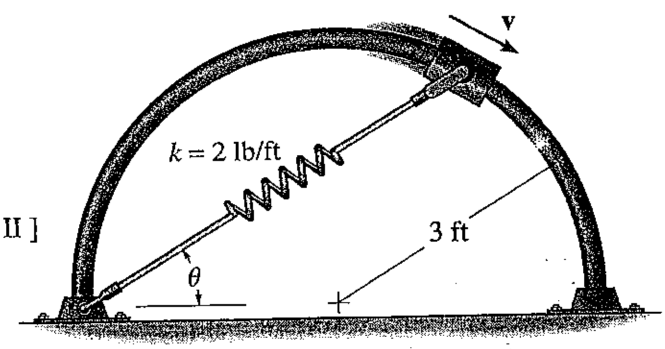
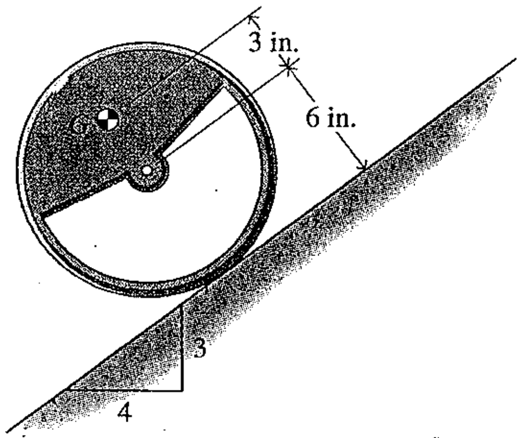


Fig. [II]

[III] The unbalanced wheel shown in Figure. [III] rolls without slipping down the inclined surface. The 32-lb wheel has a radius of gyration of 4.25 in. with respect to its axis of rotation. When the wheel is in the position shown, it has a counterclockwise angular velocity of 5 rad/s. At this instant, determine the angular acceleration of the wheel and the normal and frictional forces exerted on the wheel by the inclined surface. (25%)

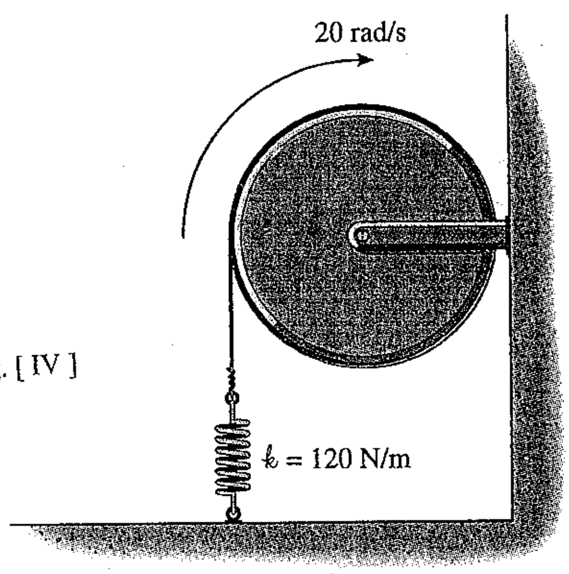
Fig. [III]



[IV] The 5-kg flywheel of Figure. [IV] has a diameter of 200 mm and a radius of gyration of 90 mm. A flexible rope is wrapped around the flywheel and attached to a spring that has a modulus $k = 120$ N/m. Initially, the flywheel is rotating clockwise at 20 rad/s and the spring is stretched by 800 mm. For the ensuing motion, determine

a. The maximum stretch in the spring.
b. The angular velocity of the flywheel when the rope becomes slack. (25%)

Fig. [IV]



1. When the system shown in Figure 1.1 is subjected to a unit-step input, the system output responds as shown in Figure 1.2. The percent overshoot vs. damping ratio is also shown as Figure 1.3. Determine the values of K and T from the response curve. 20%

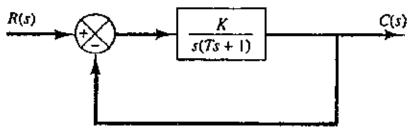


Figure 1.1

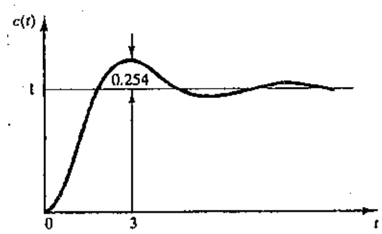


Figure 1.2

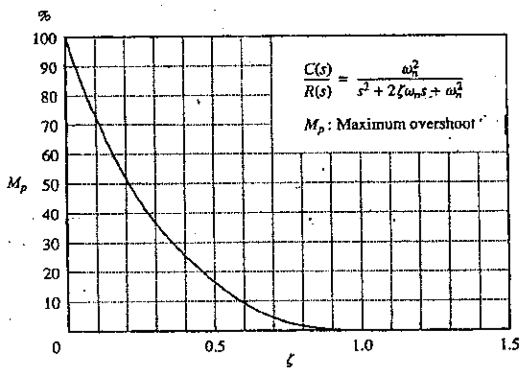


Figure 1.3

2. Figure 2 shows a linear closed-loop system subjected to two inputs (the reference input and disturbance input). Complete the following questions:

(a). What is the meaning of superposition principle for this problem? And, find the output.

(b). Suggest a way to suppress the effect of disturbance.

(c). Make an illustrative example for industrial implementations. 30%

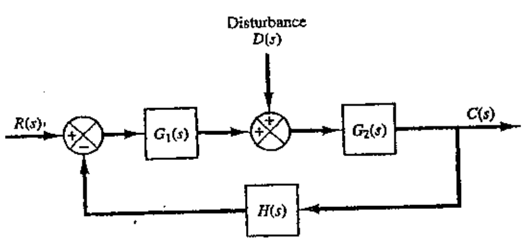


Figure 2

3. Please discuss the advantages and the disadvantages of applying the Bode diagram or polar plot of a dynamic system in modeling (or understanding) its dynamic characteristics. 15%

4. To obtain a satisfactory control performance, it is stated that the phase margin of the feedforward transfer function (assuming to be minimum phase) should be between 30° and 60° and the gain margin should be great than 6 db. In your viewpoint, is this statement proper? Please discuss your points in details. 15%

5. It is given a dynamic system whose polar plot is shown below (Figure 5.1). This system is to be controlled by a feedback control scheme (Figure 5.2). It is desired that the system output can well follow a periodic reference input r(t) with a specific line spectrum (Figure 5.3). Please design a controller to meet the desired performance. 20%

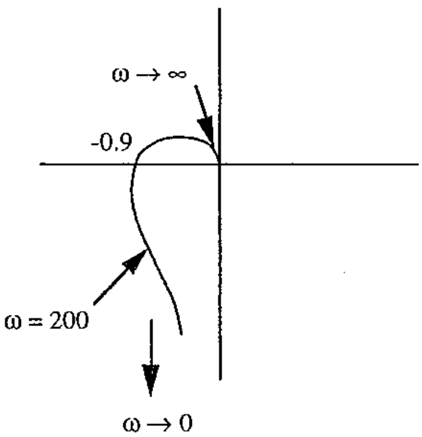


Figure 5.1 The polar plot of dynamic system

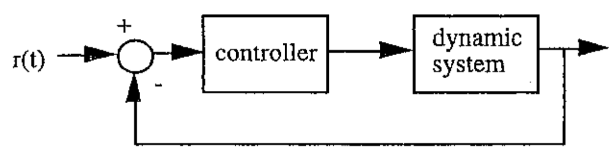


Figure 5.2 The feedback control scheme

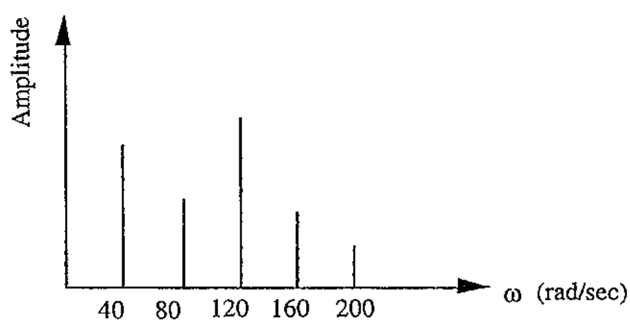
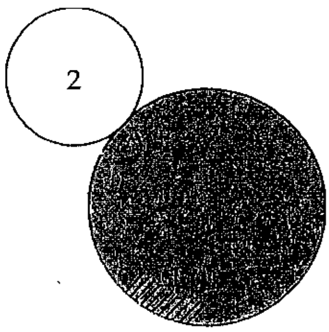


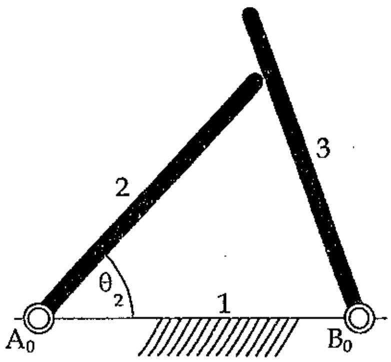
Figure 5.3 The line spectrum of r(t)

國立中山大學八十八學年度碩博士班招生考試試題

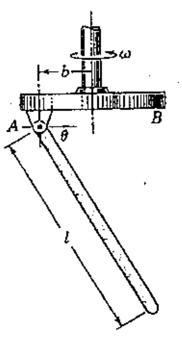
科　目：動力學　　機械系|丁組|　　　　共 1 頁 第　頁



1. 一半徑為 r 之圓柱狀剛體 2 在半徑為 R 之固定圓筒 1 外滾動，此時二物體之接觸點為 A，剛體 2 之角速度為 ω_2 等速。(a)求在此瞬間圓柱剛體 2 上速度 $V=0$ 點之位置。(10%)　(b)同樣的求在此瞬間，剛體 2 上加速度 $a=0$ 點的位置。(10%)



2. 左圖桿 2 與桿 3 在保持接觸之狀況下動作，桿 2 桿 3 以及 A_0B_0 間之距離均為等長。今桿 2 以等角速度之旋轉 (ω_2=const) 使 θ_2 自 50° 增加到 70°。請繪圖說明桿 3 之角加速度 α_3 在此運動範圍中之變化情形，假設二桿之寬度可以忽略。(20%)



3. 圓盤 B 以等速ω繞一鉛直軸轉動，其下方有迴轉接頭 A 懸吊一長度為 ℓ 的均質細長桿。如左圖中之尺寸 $b=\ell/4$，試求可使細長桿維持傾斜 $\theta=60°$ 角度之角速度ω值。(20%)

4. 如地球之轉速為ω，則在離地 h 高度落下的自由落體，將會因科氏加速度(Coriolis acceleration)的作用，落在距離其鉛直位置東方 $x=\frac{2}{3}\omega\sqrt{\frac{2h^3}{g}}\cos\gamma$ 之處。前式中 g 為重力加速度，γ 為該地之緯度。試證明此論述。(20%)

5. 已知地球之轉速 $\omega=0.729\times10^{-4}$ rad/s，現將一物從北緯 45° 某地上空 200 m 的高度落下，如不計算空氣的阻力，此物體將會落於距其鉛直位置多遠的地方？(20%)

國立中山大學八十八學年度碩博士班招生考試試題

科　目：靜力學　　機械系 丁組　

1. A man weighing 70 kg sits on a sling and supports himself by a rope wound 1.5 turns around fixed pulley (Fig. 1). Given that the coefficient of friction between the rope and pulley is 0.3, what is the minimum force he can exert to maintain his position? (10%)

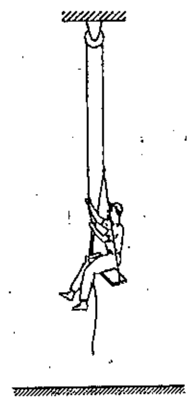


Fig. 1

2. Truss ABCDE is supported at the left end A by a pin-joint support, and at the right end D by a roller support, as shown in Fig. 2. Neglecting the weight of each member of the truss, draw the free-body diagrams of (i) the whole truss, (ii) part of the truss cut by mn, (iii) the joints A, B, C, D, and E. (12%)

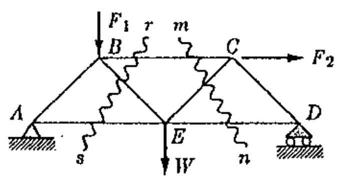


Fig. 2

3. Draw free-body diagrams of sphere A, sphere B, and spheres A and B of Fig. 3. Sphere A is hanging from a string CD and shere B is resting on A and against the wall. Assume that all contact surfaces are smooth. (8%)

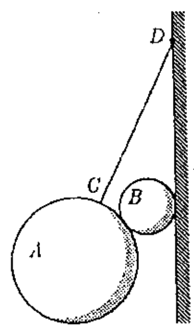


Fig. 3

4. The rod shown in Fig. 4 is supported by two brackets at A and B. Determine the moment produced by force F, which tends to rotate the rod about the AB axis. (12%)

5. Determine the shear force V and bending moment M at the section a distance x from the left end. The beam and its loading are shown in Fig. 5. (12%)

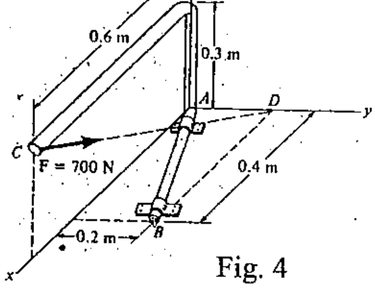


Fig. 4

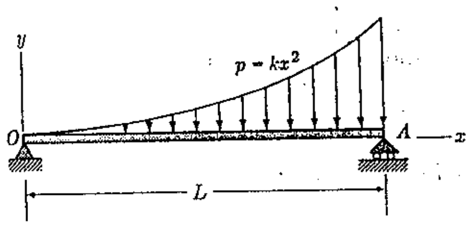


Fig. 5

6. The coefficient of friction between the end A of a scaffold hook and the horizontal surface is 0.50. Given that the vertical surface is smooth, determine whether or not the hook is safe for the two cases as shown in Fig. 6. (12%)

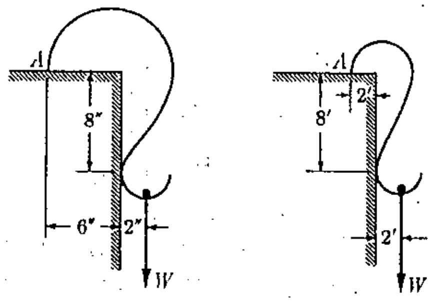


Fig. 6

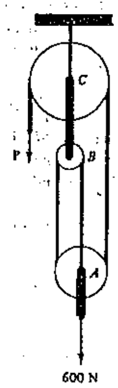


Fig. 7

7. Determine the tension in the cables and also the force **P** required to support the 600-N force using the frictionless pulley system shown in Fig. 7. (10%)

8. Determine the tension in cables BC and BD and the reactions at the ball-and-socket joint A for the mast shown in Fig. 8. (12%)

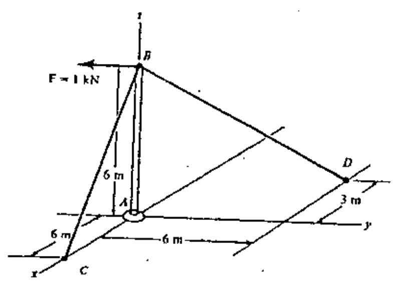


Fig. 8

9. Determine all the members of the Fink truss shown in Fig. 9, which are subjected to zero force. (12%)

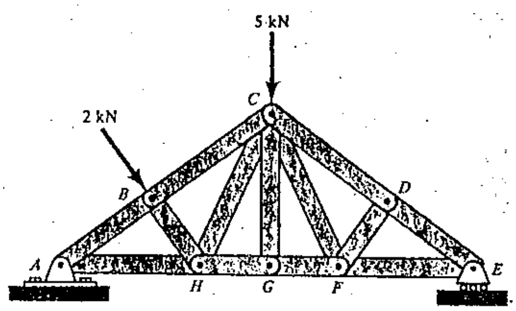


Fig. 9