

# 國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：工程數學【機電系碩士班甲組、乙組、丙組、丁組、戊組】

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I. (35%)

1. Solve the following ODEs. Find the general solution or the particular solution by using the given initial conditions. Show the detail of your work.

- (a)  $yy'' = y'^2$ ,  $y(0) = 1, y'(0) = 2$ . (7%)
- (b)  $y' = -\frac{2e^y + 2y}{e^y + 1}$ . (8%)
- (c)  $y'' + 2y' + 2y = e^{-x} \cos x$ . (10%)
- (d)  $x^3 y''' - 3x^2 y'' + 6xy' - 6y = x$ . (10%)

II. (35%)

1. (20%)

Find an eigenbasis of matrix A and use similarity transformation to diagonalize it.

$$A = \begin{bmatrix} 1 & 6 \\ 1.5 & 1 \end{bmatrix}$$

2. (15%)

- (a) Given a curve C:  $r(t) = [5 \cos t, 5 \sin t, 0]$ , find a tangent vector  $r'(t)$ , and the tangent of C at P: (4, 3, 0). (10%)
- (b) Sketch the curve and the tangent. (5%)

III. (30%)

1. (10%) This happens if PDE involves derivatives with respect to one variable only, so that the other variables can be treated as parameters.

Solve for  $u = u(x, y)$  as list:

- (a)  $u_{yy} + 16u = 0$  (5%)
- (b)  $u_{xy} = u_x$  (5%)

2. (20%) The faces of the thin square plate in Figure 1 with side  $a = 24$  are perfectly insulated. The upper side is kept at  $20^\circ\text{C}$  and the other sides are kept at  $0^\circ\text{C}$ .

- (a) Find the steady-state temperature  $u(x, y)$  in the plate. (10%)
- (b) Find the steady-state temperature in the plate with the upper and lower sides perfectly insulated, the left side kept at  $0^\circ\text{C}$ , and the right side kept at  $f(y)^\circ\text{C}$ . (10%)

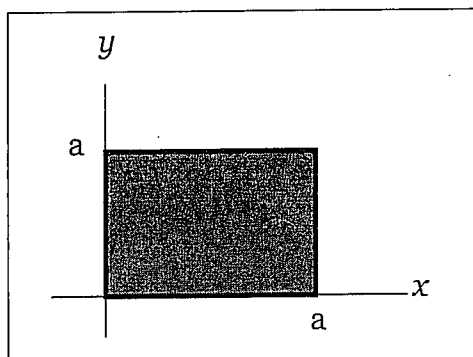


Figure 1: Square Plate

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1. (10%) Obtain the transfer function  $\theta(s)/T(s)$  of the system shown in Fig. 1. If torque  $T(t)$  is a unit step function with  $K = 1\text{N}\cdot\text{m}/\text{rad}$ ,  $D = 0.2\text{N}\cdot\text{m s}/\text{rad}$  and  $J = 0.4444\text{kg}\cdot\text{m}^2$ , find the output response  $\theta(t)$ .

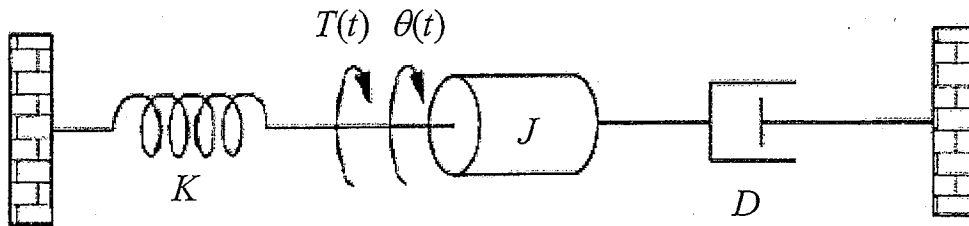


Fig. 1

2. (10%) Consider a unit-feedback control system with the closed-loop transfer function  $\frac{C(s)}{R(s)} = \frac{4s+3}{s^2+10s+3}$ , determine the open-loop transfer function  $G(s)$ . Also find the steady-state error in the unit-ramp response.
3. (10%) Consider a unit-feedback system with two different transfer functions  $G_1(s) = \frac{K(s-2)}{(s+3)(s+6)(s+9)}$  and  $G_2(s) = \frac{K(2-s)}{(s+3)(s+6)(s+9)}$ , plot the root loci for these two systems, respectively.
4. (20%) Consider the system shown in Fig. 2. Design a PD controller ( $K_P$  and  $K_D$ ) to satisfy the following performance specifications:  
 Steady state error due to a unit-ramp input  $\leq 0.001$   
 Maximum overshoot  $\leq 5\%$   
 Rise time  $\leq 0.005$  sec  
 Settling time  $\leq 0.005$  sec  
 In addition, find the range of  $K_P$  and  $K_D$  so that the system is stable.

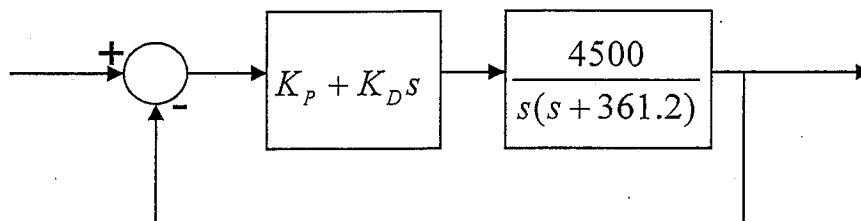


Fig. 2

5. (10%) Consider two dynamic systems with transfer functions  $\frac{1000(s^2+2s+100)}{(s+1)(s+100)(s+1000)}$  and  $\frac{100(s+1)e^{-0.01s}}{(s+10)(s+100)}$ , respectively. Please draw the Bode diagrams and polar plots of these two systems.
6. (10%) Please discuss the reasons why the relative stability of a closed-loop control system can be measured from the phase and gain margins of its open loop transfer function.
7. (10%) Consider a unit-feedback control system as shown in Fig. 3. Based on the characteristics of the M-circle and N-circle, draw a polar plot of  $G(s)C(s)$  that the closed-loop control system can have a bandwidth better than 50 rad/sec and own a satisfactory stability.

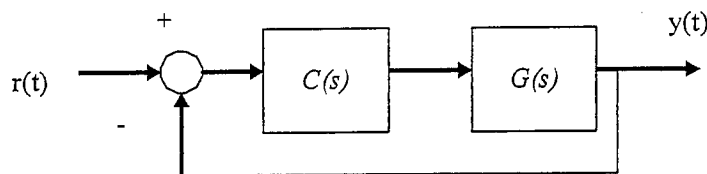


Fig. 3

8. (20%) Consider a dynamic system, whose Bode diagram and polar plot are shown in Fig.4 and 5, respectively. Please guess the possible transfer function of this system (discuss your answer in details). It is desired to control the system to have a bandwidth wider than 50 rad/sec. What kinds of control scheme and controller will you adopt to control the system? Why? In your viewpoint, can such a goal be achieved with a satisfactory stability? Why?

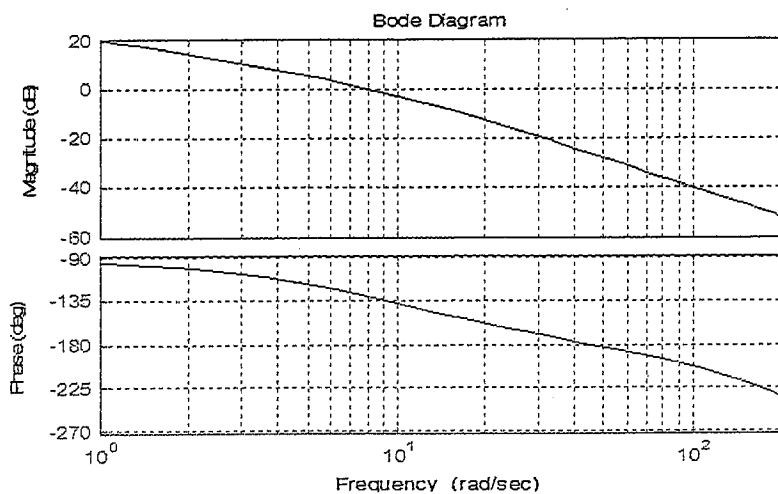


Fig 4

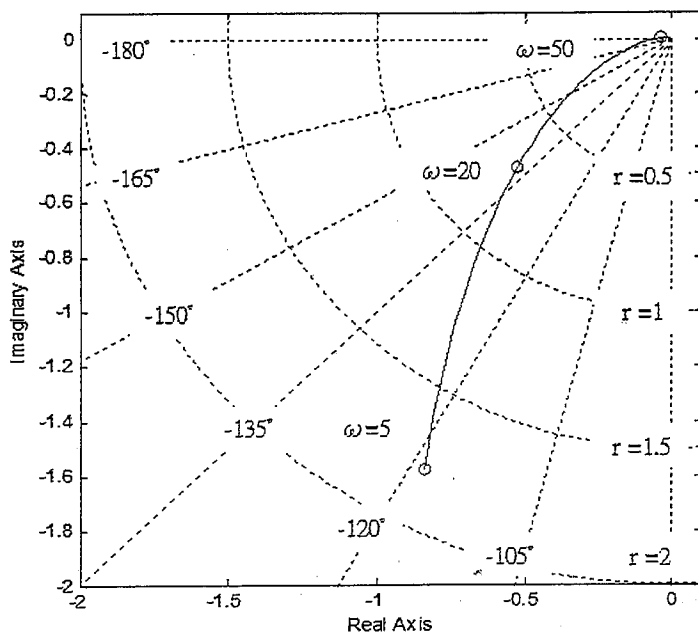


Fig 5

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一、選擇題 (單選題) (下列 20 題選擇題，每題 2.5 分，共計 50 分)  
(請於答案紙上作答，僅在試題上作答部份將不與計分)

- ( ) 在平面應力(Plane Stress)假設下，若一機件上某點之應力值，已知為  $\sigma_{xx} = 10 \text{ MPa}$ ， $\sigma_{yy} = 2 \text{ MPa}$ ，且  $\tau_{xy} = 4 \text{ MPa}$ ，下列結果何者為誤？  
(A) 最大主應力值為 11.7 MPa (B) 最小主應力值為零  
(C) 最大剪應力值為 4 MPa (D) 最大主應力值一定在 XY 平面上  
(E) 以上皆非
- ( ) 下列那一個公式係用於計算兩接觸曲面間之表面接觸應力？  
(A) Hertz Equation (B) Sodeberg's Equation  
(C) Modified Goodman's Equation (D) Miner's Rule  
(E) 以上皆非
- ( ) 下列那一個公式係用於計算工件承受疲勞負載時之殘留壽命？  
(A) Hertz Equation (B) Sodeberg's Equation  
(C) Modified Goodman's Equation (D) Miner's Rule  
(E) 以上皆非

(I) Following strains are derived from three strain gages which are located on the free surface of a loaded body.

$$\varepsilon_{xx} = 0.005; \varepsilon_{yy} = 0.01 \text{ and } \gamma_{xy} = 0$$

Consider the Young's modulus and the Poisson's ration of this body are  $E = 207 \text{ GPa}$  and  $\nu = 0.3$  respectively.

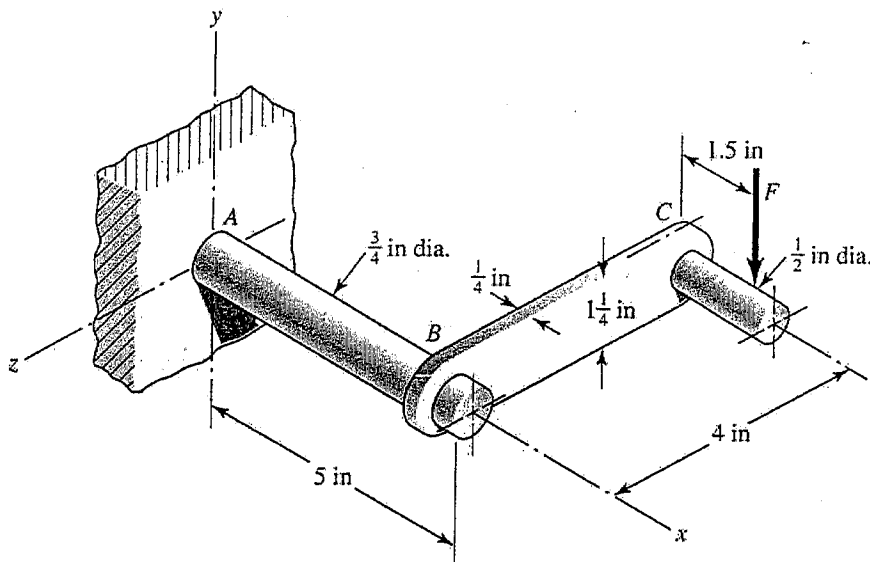
- ( ) The principal stress  $\sigma_1, \sigma_2, \sigma_3$  at this measured point are  
(A)  $\sigma_1 = 2.616; \sigma_2 = 1.820$  and  $\sigma_3 = 0 \text{ GPa}$   
(B)  $\sigma_1 = 12.36; \sigma_2 = 0$  and  $\sigma_3 = -32.36 \text{ GPa}$   
(C)  $\sigma_1 = 2.616; \sigma_2 = -2.30$  and  $\sigma_3 = -32.36 \text{ GPa}$   
(D)  $\sigma_1 = 32.36; \sigma_2 = 1.820$  and  $\sigma_3 = 0 \text{ GPa}$   
(E) None
- ( ) The maximum shear stress  $\tau_{\max}$  at this point.  
(A)  $\tau_{\max} = 1.308 \text{ GPa}$  (B)  $\tau_{\max} = 22.36 \text{ GPa}$   
(C)  $\tau_{\max} = 17.488 \text{ GPa}$  (D)  $\tau_{\max} = 0 \text{ GPa}$   
(E) None

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(II) A crank is loaded by a force  $F=300 \text{ lbf}$  that causes twisting and bending of a  $3/4$  inch diameter shaft fixed to a support at the origin of the reference system.



6. ( ) The bending stress at a stress element A is  
 (A)  $\sigma_x = 19400 \text{ psi}$  (B)  $\sigma_x = 47100 \text{ psi}$   
 (C)  $\sigma_x = 68900 \text{ psi}$  (D)  $\sigma_x = 94700 \text{ psi}$  (E) None
7. ( ) The torsional stress at a stress element A is  
 (A)  $\tau_{xz} = -14500 \text{ psi}$  (B)  $\tau_{xz} = -44500 \text{ psi}$   
 (C)  $\tau_{xz} = 14500 \text{ psi}$  (D)  $\tau_{xz} = 44500 \text{ psi}$  (E) None
8. ( ) Point A is in a state of  
 (A) Plane strain state in the xz plane  
 (B) Plane stress state in the xz plane  
 (C) Plane strain state in the xy plane  
 (D) Plane stress state in the xy plane (E) None
9. ( ) The maximum principal stress at the point A is  
 (A)  $\sigma_1 = 3.82 \text{ kpsi}$  (B)  $\sigma_1 = 47.1 \text{ kpsi}$   
 (C)  $\sigma_1 = 51.2 \text{ kpsi}$  (D)  $\sigma_1 = 93.87 \text{ kpsi}$  (E) None
10. ( ) The maximum shear stress at the point A is  
 (A)  $\tau_{\max} = 2.56 \text{ kpsi}$  (B)  $\tau_{\max} = 4.76 \text{ kpsi}$   
 (C)  $\tau_{\max} = 11.2 \text{ kpsi}$  (D)  $\tau_{\max} = 27.7 \text{ kpsi}$  (E) None

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(III) A hot-rolled steel has a yield strength of  $S_{yt} = S_{yc} = 400 \text{ MPa}$ . Estimate the factor of safety for the principal stress states of  $\sigma_1 = 100 \text{ MPa}$ ,  $\sigma_2 = 0 \text{ MPa}$ , and  $\sigma_3 = -100 \text{ MPa}$ .

11. ( ) The factor of safety based on the maximum shear stress theory is  
(A)  $SF = 4.0$  (B)  $SF = 2.30$  (C)  $SF = 2.0$   
(D)  $SF = \infty$  (E) None
12. ( ) The factor of safety based on the maximum distortion energy theory is  
(A)  $SF = 4.0$  (B)  $SF = 2.30$  (C)  $SF = 2.0$   
(D)  $SF = \infty$  (E) None
13. ( ) The factor of safety based on the maximum normal stress theory is  
(A)  $SF = 4.0$  (B)  $SF = 2.30$  (C)  $SF = 2.0$   
(D)  $SF = \infty$  (E) None

(III) Assume the stress tensor at a point is

$$[\sigma_{ij}] = \begin{bmatrix} -2 & 4 & 0 \\ 4 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ (MPa)}$$

14. ( ) The principal stresses at this point is  
(A)  $\sigma_1 = 0; \sigma_2 = 0$  and  $\sigma_3 = -10 \text{ MPa}$   
(B)  $\sigma_1 = 12.36; \sigma_2 = 0$  and  $\sigma_3 = -32.36 \text{ MPa}$   
(C)  $\sigma_1 = 0; \sigma_2 = -23.0$  and  $\sigma_3 = -32.36 \text{ MPa}$   
(D)  $\sigma_1 = 0; \sigma_2 = 0$  and  $\sigma_3 = -32.36 \text{ MPa}$   
(E) None
15. ( ) The maximum shear stress at this point is  
(A)  $\tau_{\max} = 5.0 \text{ MPa}$  (B)  $\tau_{\max} = 22.36 \text{ MPa}$  (C)  $\tau_{\max} = 16.18 \text{ MPa}$   
(D)  $\tau_{\max} = 32.36 \text{ MPa}$  (E) None
16. ( ) The stress state at this point is  
(A) a pure bending stress state (B) a pure torsion stress state  
(C) a pure transverse shear stress state (D) a plane stress state (E) None
17. ( ) The von-Mises stress at this point is  
(A)  $\tau_{\max} = 5.0 \text{ MPa}$  (B)  $\tau_{\max} = 22.36 \text{ MPa}$  (C)  $\tau_{\max} = 16.18 \text{ MPa}$   
(D)  $\tau_{\max} = 32.36 \text{ MPa}$  (E) None

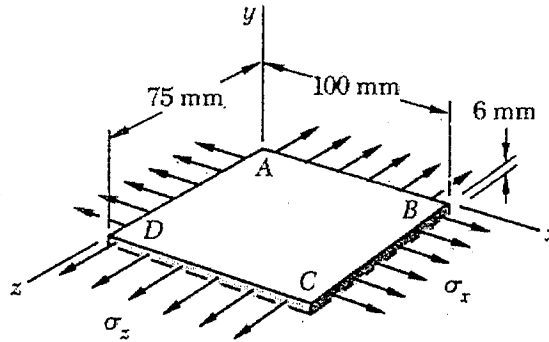
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(IV) A homogeneous plate ABCD is subjected to a biaxial loading which results in the normal stress  $\sigma_x = 150 \text{ MPa}$  and  $\sigma_z = 100 \text{ MPa}$ . Knowing that the plate is made of steel which  $E = 200 \text{ GPa}$  and Poisson's ratio  $\nu = 0.30$ .



18. ( ) The change in length of edge AB is  
 (A)  $+7.5 \mu\text{m}$  (B)  $+10.8 \mu\text{m}$  (C)  $+60.0 \mu\text{m}$   
 (D)  $+120 \mu\text{m}$  (E) None
19. ( ) The change in length of edge BC is  
 (A)  $+7.5 \mu\text{m}$  (B)  $+8.8 \mu\text{m}$  (C)  $+10.4 \mu\text{m}$   
 (D)  $+20.6 \mu\text{m}$  (E) None
20. ( ) The change in length of diagonal AC is  
 (A)  $+47.5 \mu\text{m}$  (B)  $+60.4 \mu\text{m}$  (C)  $+101.4 \mu\text{m}$   
 (D)  $+205.6 \mu\text{m}$  (E) None



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## 二、問答分析及計算題 (下列共四題，共計 50 分)

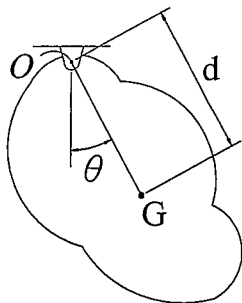
1. Please plot the stress-strain diagrams (not to scale) and denote three specified points (if necessary) for (a) brittle materials, (b) ductile materials, such as mild steel, and (c) nonlinearly elastic materials. (15%)
2. What two conditions must be valid if the principle of superposition is to be applied? (10%)
3. A solid shaft of radius  $c$  is subjected to a torque  $T$ . Please determine the fraction of  $T$  that is resisted by the material contained within the outer region of the shaft as a tube, which has (a) an inner radius of  $c/2$  and outer radius  $c$ , and (b) an inner radius  $3c/4$  and outer radius  $c$ . (10%)
4. In plane stress condition the normal stresses are  $-20$  MPa and  $90$  MPa in  $x$  and  $y$  axes, and shear stress  $60$  MPa, respectively. Please find the principal strains and maximum shear strain with Young's modulus  $10$  GPa and Poisson's ratio  $0.3$ . (15%)

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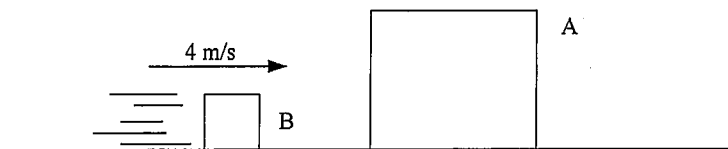
科目：動力學【機電系碩士班丁組】

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- (1) Show the natural frequency  $\omega_n$  of a 1-DOF system with mass  $m$  and stiffness  $k$ . (10%)
- (2) Determine the period of a harmonic motion with an amplitude of  $0.05 m$  and a frequency of 10 Hz. (10%)
- (3) The maximum amplitude and the maximum acceleration of the foundation of a centrifugal pump were found to be  $x_{\max} = 0.25 mm$  and  $\ddot{x}_{\max} = 0.4 g$  for its harmonic oscillation. Determine the operating speed of the pump ( $rpm$ ). (10%)
- (4) An automobile is found to have a natural frequency of  $20 rad/s$  without passengers and  $17.32 rad/s$  with passengers of mass  $500 kg$ . Determine the mass of the automobile by treating it as a single degree of freedom system. (10%)
- (5) The particular solution of the linear system  $m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$  is given by  $x_p(t) = X \cos(\omega t - \phi)$ . Determine the amplitude  $X$  of the particular solution. (10%)
- (6) The body of arbitrary shape (Fig. 1) has a mass  $m$ , mass center at  $G$ , and a radius of gyration about  $G$  of  $k_G$ . If it is displaced a slight amount  $\theta$  from its equilibrium position and released, determine the natural period of vibration. (10%)



(Fig. 1)



(Fig. 2)

- (7) If a particle's position is described by the polar coordinates  $r = (2 \sin 2\theta) m$  and  $\theta = (4t) rad.$ , where  $t$  is in seconds. Determine the radial and tangential components of its velocity and acceleration when  $t = 1s$ . (20%)
- (8) As shown in Fig. 2, Block B has a mass of  $0.75 kg$  and is sliding forward on the smooth surface with a velocity  $(v_B)_1 = 4 m/s$  when it strikes the  $2-kg$  block A, which is originally at rest. If the coefficient of restitution between the blocks is  $e = 0.5$ , compute the velocities of A and B just after collision. (20%)

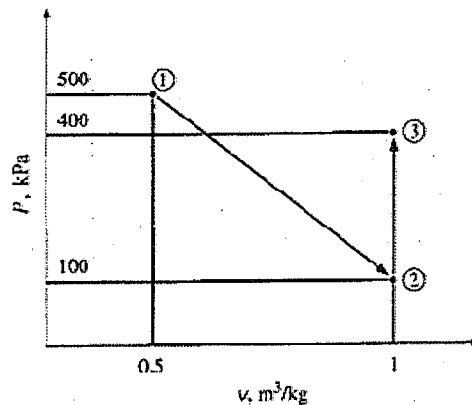
# 國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：熱力及熱傳導、熱輻射學【機電系碩士班甲組】

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## Thermodynamics: (65%)

1. (10%, 每個子題5%) Calculate the work produced, in kJ/kg, (a) for the reversible process from state 1 to state 3 of a closed system and (b) for the reversible steady-flow process 1-3.



2. (10%) An open system is interactive with its surroundings through the mechanisms of mass, heat or work transfer. Please describe above what mechanisms can cause the changes of the energy, the entropy, and the exergy of a control volume, respectively?
3. (8%, 每個子題 2%) A simple ideal Brayton cycle without regeneration is modified to incorporate multistage compression with intercooling and multistage expansion with reheating, without changing the pressure or temperature limits of the cycle. As a result of these two modifications,
- Does the net work output increase, decrease, or remain the same?
  - Does the back work ratio increase, decrease, or remain the same?
  - Does the thermal efficiency increase, decrease, or remain the same?
  - Does the heat rejected increase, decrease, or remain the same?
4. (12%, 每個子題 6%) Consider a simple ideal Rankine cycle (A) with fixed turbine inlet conditions. What is the effect of lowering the condenser pressure? (B) with fixed boiler and condenser pressure. What is the effect of superheating the steam to a higher temperature?

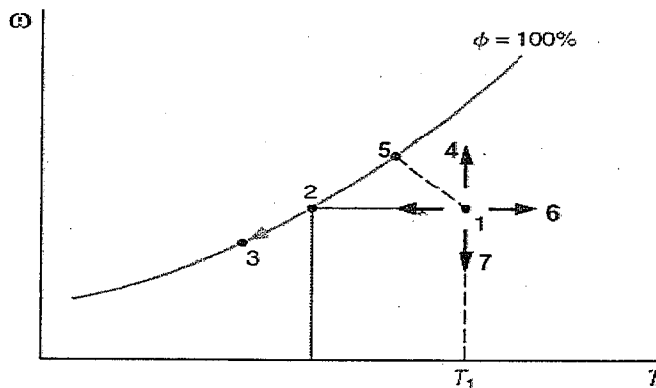
Pump work input:	(a) increases, (b) decreases, (c) remains the same
Turbine work output:	(a) increases, (b) decreases, (c) remains the same
Heat supplied:	(a) increases, (b) decreases, (c) remains the same
Heat rejected:	(a) increases, (b) decreases, (c) remains the same
Cycle efficiency:	(a) increases, (b) decreases, (c) remains the same
Moisture content at turbine exit:	(a) increases, (b) decreases, (c) remains the same

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5. (10%) Show that the Joule-Thomson coefficient of an ideal gas is zero.  
[ Note that  $dh = c_p dT + [v - T(\partial v/\partial T)_p] dP$  ]
6. (10%, 每個子題 2%) For an **ideal** vapor-compression refrigeration cycle:  
 (a) Draw the T-s diagram for the cycle.  
 (b) What are the four processes of the cycle?  
 (c) Write down the energy balance for each process to show how you can calculate heat or work for each process of the cycle.  
 (d) Which process provides refrigeration effect?  
 (e) What is the COP for the refrigeration cycle? If the cycle is used as a heat pump, what is the COP?
7. (5%, 每個子題 1%) In the psychrometric Chart, what are the paths for air, starting at state 1, go through the following devices  
 (a) hair dryer  
 (b) cooling tower  
 (c) evaporator of an air-conditioner  
 (d) condenser of an air-conditioner  
 (e) electric heater



## Heat Conduction and Radiation: (35%)

8. (6%, 每個子題 3%) Write down the basic law for heat conduction? Why there is a negative sign in the law?
9. (8%, 每個子題 4%) What is the Lumped-Capacitance-Method (LCM)? When the LCM is valid?
10. (6%) 冬天晚上發現地上的積水上結了一層薄冰, 但大氣的溫度略高於  $0^\circ\text{C}$ , 試以熱傳學解釋該現象之原因。

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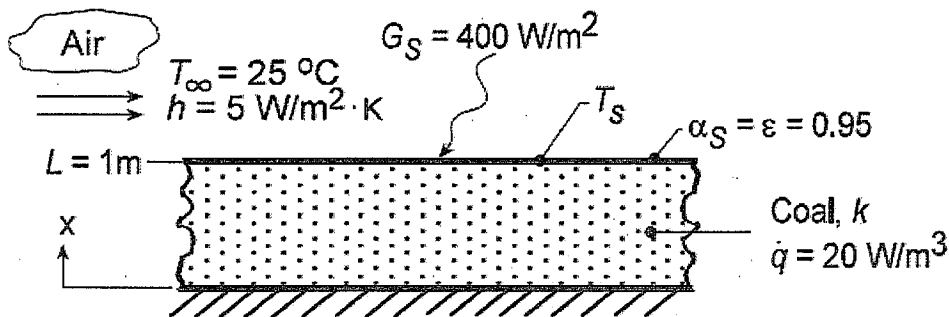
科目：熱力及熱傳導、熱輻射學【機電系碩士班甲組】

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11. (15%, 每個子題 5%) Volumetric heat generation rate in a plane layer of coal (煤) of thickness  $L = 1 \text{ m}$  is  $\dot{q} = 25 \text{ W/m}^3$ . The top surface of the layer transfers heat by convection to ambient air for which  $h = 5 \text{ W/m}^2 \text{ K}$  and  $T_\infty = 25 \text{ }^\circ\text{C}$  while receiving solar irradiation in the amount  $G_s = 400 \text{ W/m}^2$ . The solar absorptivity and emissivity of the surface are each  $\alpha_s = \varepsilon = 0.95$ . The bottom surface at  $x = 0$  is insulated.

- Write the steady-state form of the one dimensional (1-D) heat conduction equation for the layer of coal.
- If the top surface temperature is  $T_s$ , derive the temperature distribution  $T(x)$  for the layer of coal by integrating the equation of (a).
- Write the 1-D energy balance equation per unit area for the top surface of the coal and derive the equation ready for solving  $T_s$ . (Note that the equation should contain only  $T_s$  and numbers, no other symbols or variables. You don't need to solve out the value of  $T_s$ . The Stefan-Boltzmann constant  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$ )



# 國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：流體力學及熱對流【機電系碩士班甲組】

題號：4097  
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## Fluid Mechanics: (65%)

1. (7%) Consider a balloon filled with gas of density  $\rho_b$  ( $\rho_b < \rho_{air}$ ) in a stationary car, and then the car accelerates suddenly in the  $x$  direction with a constant acceleration  $a_x$ .

(a). Neglect the viscous effect and relative fluid motion, and then draw a suitable control volume to derive the equation of motion in the  $x$  direction:

$$\frac{dp}{dx} = -\rho_{air} \cdot a_x$$

where  $p$  is the static pressure and  $\rho_{air}$  is the density of air in the car. (3%)

(b). When the car suddenly accelerates in  $x$  direction, your body will move backward. Will the balloon move forward or backward? Why? (4%)

2. (13%) The tank shown in the Figure problem 2 rolls with negligible resistance along a horizontal track. It is to be driven from rest by a liquid jet of velocity  $V$  and cross-sectional area  $A$  that strikes the vane at the edge of the tank and is deflected into the tank. Assume the mass of the tank is  $M(t)$ , and the velocity of the tank is  $U(t)$ . Use Reynolds transport equation to derive the (a) continuity (5%) and (b) momentum equation (8%) as  $t \geq 0$ . (You don't need to solve the equations)

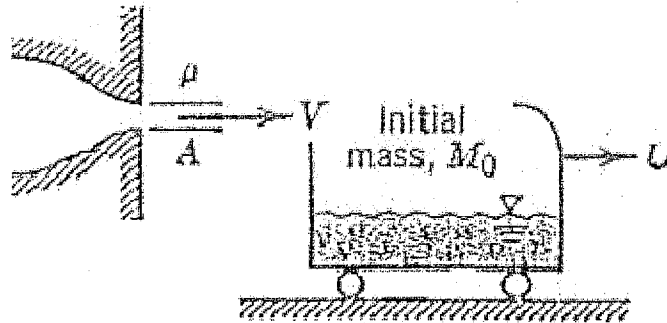


Figure Problem 2

3. (15%) Consider a uniform airflow  $U=5 \text{ m/s}$  past a horizontal flat plate. Assume the critical Reynolds number  $= 1.25 \times 10^6$ , and the kinetic viscosity of air  $\nu = 1.6 \times 10^{-5} \text{ m}^2/\text{s}$ . You will also need the following equation for boundary layer thickness:

For laminar :  $\delta_L = 5x / \sqrt{Re_x}$

For turbulent:  $\delta_T = 0.37x Re_x^{-0.2}$

(a)(5%) Decide the transition location  $x_{crit}$  that separates the laminar and turbulence on the flat plate.

(b)(10%) Compute the boundary layer thickness at  $x = 4 \text{ m}$  on the flat plate.

# 國立中山大學 101 學年度碩士暨碩士專班招生考試試題

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4. (15%) Many flying and swimming animals rely on an oscillating motion for propulsion through air or water. Assume the average travel speed,  $U$ , depends on the oscillation frequency  $f$ , the amplitude of the motion  $a$  (unit of length), the characteristic length scale of the animal  $L$ , the gravity  $g$ , the density of the animal  $\rho_a$ , the density of the fluid  $\rho$ , and the viscosity of the fluid  $\mu$ .

- (6%) Formulate a dimensionless scaling law for  $U$  involving all the other parameters by using  $\rho$ ,  $g$  and  $L$  as the repeating parameters
- (3%) Simplify your answer for (a) for high Reynolds numbers where  $\mu$  is no longer a parameter. (Hint: drop the dimensionless parameter regarding viscosity  $\mu$  directly from (a))
- (6%) While swimming submerged, fish and marine mammals are usually neutrally buoyant or very nearly so. Thus, simplify your answer for (b) so that  $g$  drops out. For this situation, determine how the speed  $U$  depends on the frequency  $f$ . (Hint: try to cancel the gravity  $g$  by manipulating the dimensionless parameters regarding  $g$  from (b))

5. (15%)

- (3%) Write down the Navier-Stokes equation in vector form. (Which is a momentum equation including body force, viscous force, pressure force and inertia force.)
- (3%) What are the conditions for the Navier-Stokes equations valid?
- (3%) What are the major difficulties in solving the Navier-Stokes equations.
- (3%) Write the Navier-Stokes equations in Cartesian coordinates for  $x$ ,  $y$  and  $z$  components.
- (3%) Simplify the equation in (d) for solving steady laminar flow between fixed parallel plates, and solve the velocity profile  $u(y)$  corresponding to the figure given below.

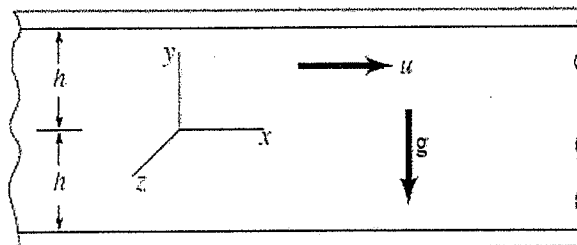


Figure Problem 5

國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：流體力學及熱對流【機電系碩士班甲組】

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**Convection: (35%)**

6. (10%, 2% each)

- (a) Define the convection heat transfer coefficient,  $h$ , and write down its unit in SI.
- (b) Define the Nusselt number and write its physical interpretation.
- (c) Define the Prandtl number and write its physical interpretation.
- (d) Define the Grashof number and write its physical interpretation.
- (e) How to determine if a convection problem is forced convection dominant, free convection dominant.

7. (10%) Typical values for the convection heat transfer coefficient are given in the following table for processes of free convection, forced convection, and convection with phase change. Write possible process for (a) to (e). (2% each)

$h$ ( $W/m^2 \cdot K$ )	
2-25	← (a)
50-1000	← (b)
25-250	← (c)
100-20,000	← (d)
2500-100,000	← (e)

8. (9%, 3% each) For a uniform flow over a flat plate,

- (a) draw a diagram to show the profile of local convection heat transfer coefficient,  $h(x)$ , varies along the flow direction  $x$  and explain the profile.
- (b) write down the boundary layer momentum and energy equations for steady laminar flow situation.
- (c) non-dimensionalize the equations of (b) and gives all dimensionless dependent variables for dimensionless temperature.

9. (6%, 2% each) For laminar flow in a circular tube with constant fluid properties :

- (a) What is a thermally fully-developed flow?
- (b) What are the values of  $Nu$  for fully-developed flows with constant wall temperature and constant wall heat flux, respectively?
- (c) Draw a diagram to show how the heat transfer coefficient ( $h$ ) varying along the flow direction ( $x$ ) from the tube entrance to fully-developed.



# 國立中山大學 101 學年度碩士暨碩士專班招生考試試題

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1. Locate the centroid ( $x_c, y_c$ ) of the composite area as shown in Figure 1. (15%)
2. Determine the force at pin A in Figure 2. You must draw the free-body diagram and state the magnitude and direction of force or its components. (15%)

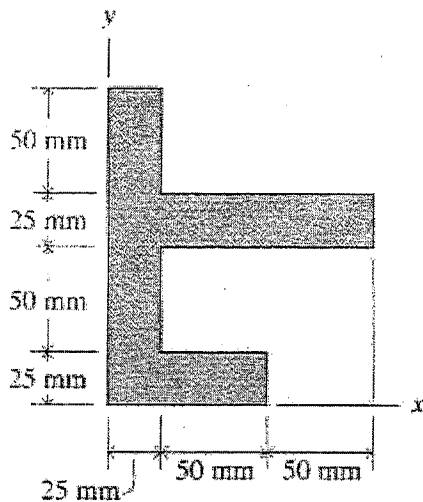


Figure 1

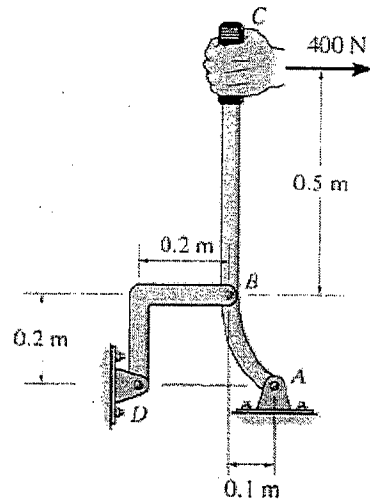


Figure 2

3. The traffic light shown in Figure 3 is supported by a system of cables. Determine the tensions in cables A, B, and C if the traffic light has a mass of 100kg. (10%)
4. Please refer to Figure 4. Determine the scalar component of the moment of the 100-lb force about the axis of the hinges (line AB). (10%)

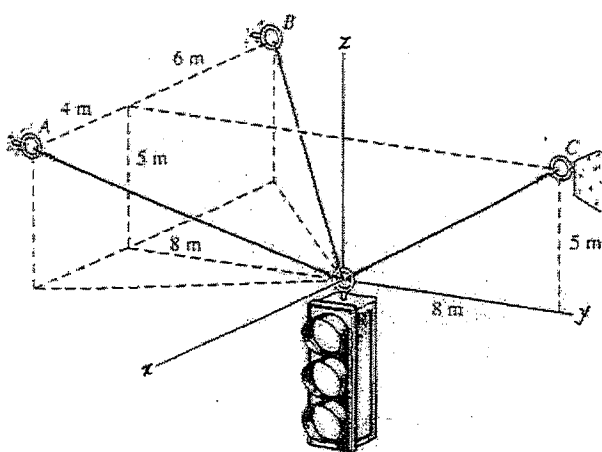


Figure 3

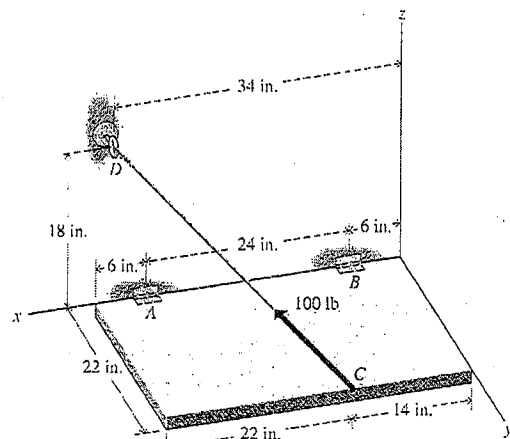


Figure 4

# 國立中山大學 101 學年度碩士暨碩士專班招生考試試題

科目：靜力學【機電系碩士班丁組】

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5. Refer to Figure 5. A speaker system is suspended from the truss by cables attached at D and E. The mass of the speaker system is 130 kg, and its weight acts at G. Determine the axial forces in members BC and CD. (20%)

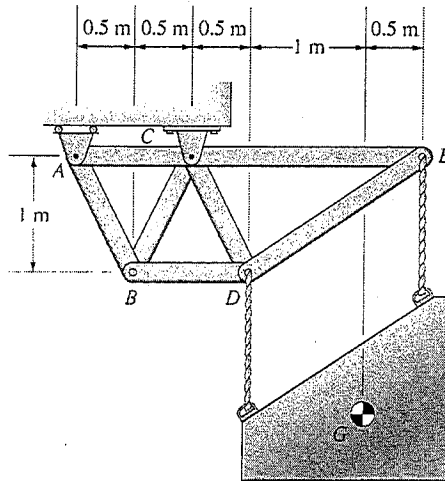


Figure 5

6. Refer to Figure 6. The cylinder has weight  $W$ . The coefficient of static friction between the cylinder and the floor and between the cylinder and the wall is  $\mu_s$ . What is the largest couple  $M$  that can be applied to the stationary cylinder without causing it to rotate? (15%)
7. Refer to Figure 7. The bar AB has mass  $m$  and length  $L$ . The spring is unstretched when the bar is vertical ( $\alpha = 0$ ). The light collar C slides on the smooth vertical bar so that the spring remains horizontal. Show that the equilibrium position  $\alpha = 0$  is stable only if  $2kL > mg$ . (15%)

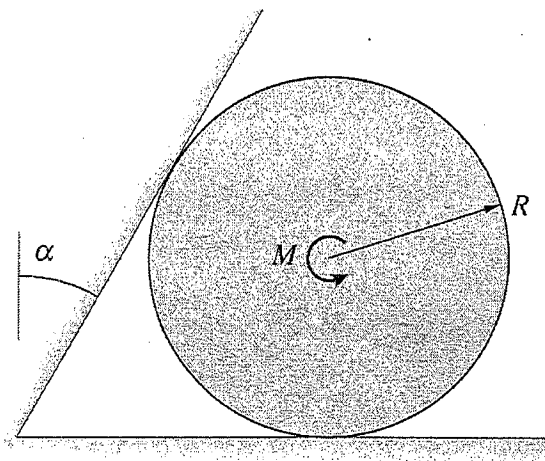


Figure 6

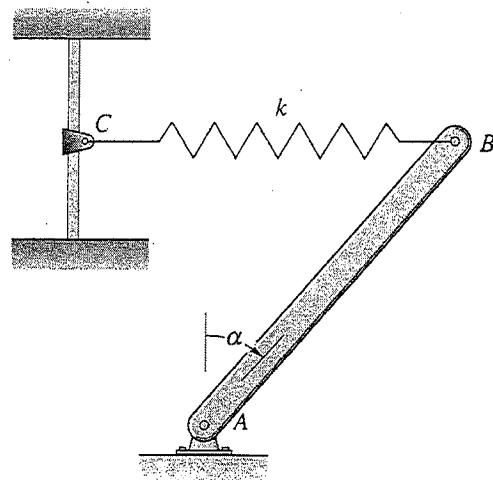


Figure 7

# 國立中山大學 101 學年度碩士暨碩士專班招生考試試題

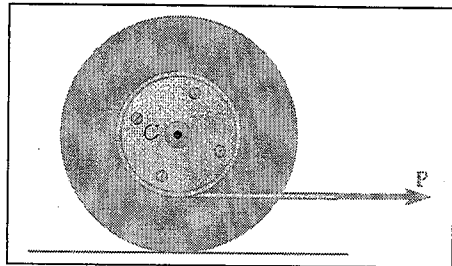
科目：應用力學【機電系碩士班乙組、丙組】

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Please choose the correct answers for problem 1 to problem 3  
Please be noted that the correct answers for each problem may be more than one

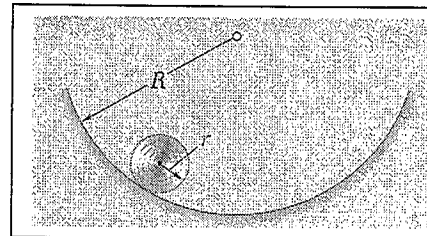
1. Consider a particle P moves along a space curve. The Cartesian coordinates and the cylindrical coordinates of the position of the particle P is  $(x, y, z)$  and  $(r, \theta, z)$ , respectively. Let  $[\mathbf{i}, \mathbf{j}, \mathbf{k}]$  and  $[\mathbf{e}_r, \mathbf{e}_\theta, \mathbf{k}]$  be the base unit vectors of the Cartesian coordinate system and the cylindrical coordinate system, respectively. If the velocity of P at an instant is  $2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$  [unit: m/s], then which of the following statements are correct? (15 %)
- (A)  $(d\mathbf{e}_r/d\theta) = -\mathbf{e}_\theta, (d\mathbf{e}_\theta/d\theta) = \mathbf{e}_r$ .
  - (B)  $\mathbf{e}_r = \cos\theta \mathbf{i} + \sin\theta \mathbf{j}, \mathbf{e}_\theta = \sin\theta \mathbf{i} - \cos\theta \mathbf{j}$ .
  - (C)  $\mathbf{i} = \cos\theta \mathbf{e}_r + \sin\theta \mathbf{e}_\theta, \mathbf{j} = \sin\theta \mathbf{e}_r - \cos\theta \mathbf{e}_\theta$ .
  - (D) At this instant,  $(dr/dt) = 2\cos\theta + 4\sin\theta$ , where  $t$  is the time.
  - (E) At this instant,  $(d\theta/dt) = (4\cos\theta - 2\sin\theta)/(x^2 + y^2 + z^2)^{1/2}$ , where  $t$  is the time.
  - (F) None of the previous statements is correct.

2. A drum of radius  $r$  is attached to a disk of radius  $R$ . The disk and drum have a total mass of  $M$  and a combined radius of gyration of  $k$ . A cord is attached to the drum as shown and pulled with a force of magnitude  $P$ . Knowing that the disk rolls without sliding, then which of the following statements are correct? (15 %)



- (A) The angular acceleration of the disk =  $(Pr)/(Mk^2)$ .
- (B) The acceleration of  $G = (PRr)/(Mk^2)$ .
- (C) The friction force existed between the disk and the ground =  $P[1 - (Rr)/(k^2 + R^2)]$ .
- (D) the minimum value of the coefficient of static friction compatible with this motion =  $P(k^2 + r^2 - Rr)/[(k^2 + r^2)(Mg)]$ , where  $g$  is the acceleration of gravity.
- (E) At any instant, the friction force existed between the disk and the ground is less than or equal to (the coefficient of static friction)(normal force on the contact point).
- (F) None of the previous statements is correct.

3. Consider the small oscillations of a cylinder of radius  $r$  with center  $G$  which rolls without slipping inside a curved surface of radius  $R$  with center  $O$ . Let  $\theta$  be the angle which line  $OG$  forms with the vertical, and  $\theta_m$  is the maximum value of  $\theta$ . If  $\theta_m \ll 1$ , then which of the following statements are correct? (20 %)



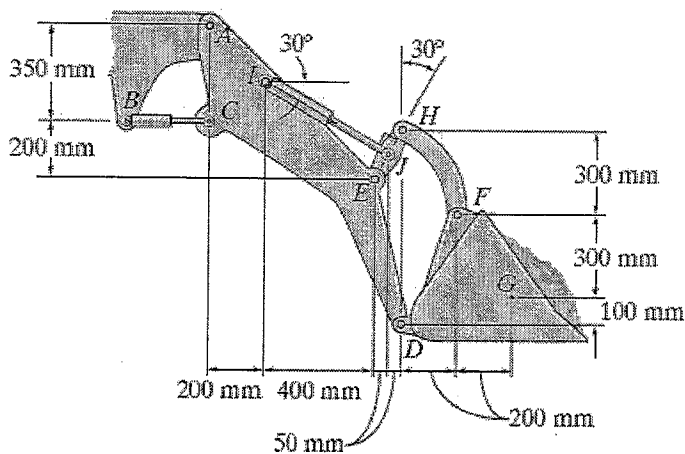
- (A) At any instant, the velocity of  $G = (r)$ (angular velocity of the cylinder)
- (B) By letting the potential energy at  $\theta = 0$  is zero, then at  $\theta = \theta_m$ , the corresponding potential energy =  $W(R - r)(\theta_m)^2/2$ .
- (C) At  $\theta = 0$ , the angular velocity of the cylinder equals to zero.
- (D) The natural frequency of those oscillations =  $[(g/3)/(R - r)]^{1/2}$ , where  $g$  is the acceleration of gravity.
- (E) The corresponding period =  $(2\pi)[3(R - r)/g]^{1/2}$ , where  $g$  is the acceleration of gravity.
- (F) None of the previous statements is correct.

# 國立中山大學 101 學年度碩士暨碩士專班招生考試試題

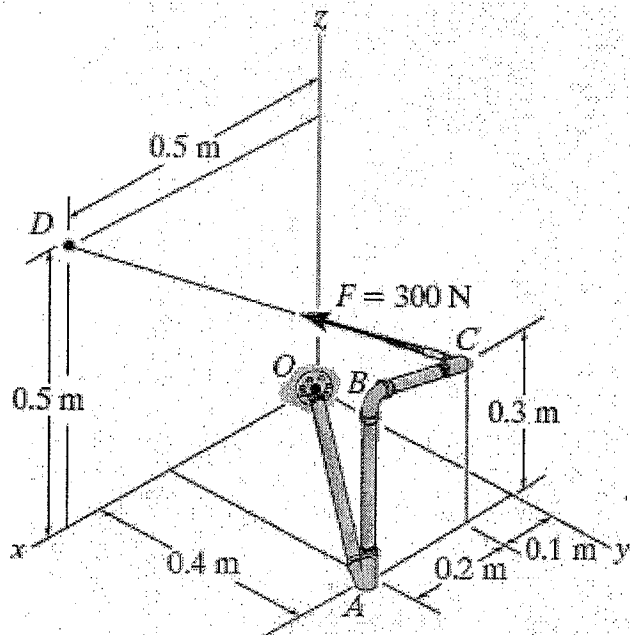
科目：應用力學【機電系碩士班乙組、丙組】

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4. The tractor shovel carries a 500-kg load of soil, having a center of mass at G. Compute the forces developed in the hydraulic cylinders IJ and BC due to this loading. (20%)



5. Determine the magnitude of the moment of force F about segment OA of the pipe assembly. (15%)



# 國立中山大學 101 學年度碩士暨碩士專班招生考試試題

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6. The crankshaft AB of an engine turns with a clockwise angular acceleration of  $20 \text{ rad/s}^2$ . Determine the acceleration of the piston C at this instant as shown in the figure. At this instant,  $\omega_{AB} = 10 \text{ rad/s}$  and  $\omega_{BC} = 2.43 \text{ rad/s}$ . (15 %)

