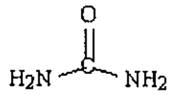
Multiple-Choice Questions (2 pt / ea)

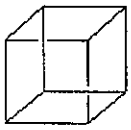
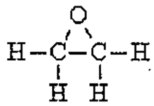
1. What is the hybridization of the carbon atom in Urea?

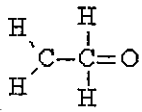


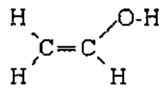
Urea

(A) sp (B) sp^2 (C) sp^3 (D) sp^4

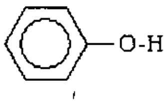
2. Which of the following compounds are *not* consistent with valence rules?

(A)  (B) 

(C) 

(D) 

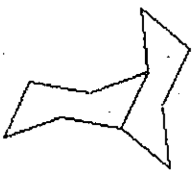
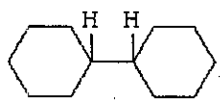
3. Which of the following acids will be almost completely deprotonated by NaOH?

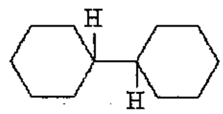
(A) Phenol 

pKa = 10.0 (B) Ethanol CH_3-CH_2-OH pKa = 16.0

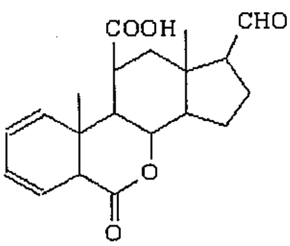
(C) Water HOH pKa = 15.7 (D) Both Ethanol and Water

4. What is the structure of trans-decalin?

(A)  (B)  (C) 

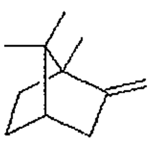
(D) 

5. This compound has the following functional groups:



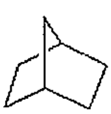
(A) ketone, alkene, carboxylic acid, ester
(B) alkyne, ester, carboxylic acid, aldehyde
(C) carboxylic acid, alkene, ketone, ester
(D) ester, aldehyde, carboxylic acid, alkene

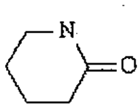
6. How many quaternary carbons are on the following molecule:



(A) 1 (B) 2 (C) 3 (D) 4

7. What is the IUPAC name of the following molecule:

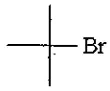
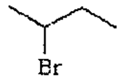
 (A) Bycyclo [2,2,1] heptane (B) Bycyclo [2,2,2] heptane (C) Bycyclo [1,2,1] heptane (D) Bycyclo [2,2,1] pentane

8. The Following compound is: 

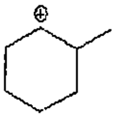
(A) A ketone (B) An amine (C) A ketone and an amine (D) A Lactam

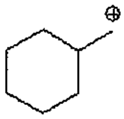
9. Of the carbocations in question 2, which ones are prone to rearrangement?
(A) 1 and 2 (B) 1 only (C) 2 only (D) 2 and 3 (E) None of them will rearrange

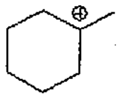
10. Which of the following alkyl bromides will undergo the S_N2 reaction the fastest?

(A)  (B)  (C)  (D) 

11. Rank the following carbocations in order of increasing stability (least stable to most stable)

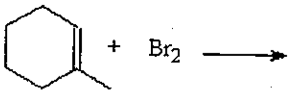

1

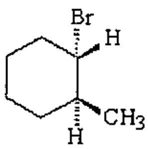
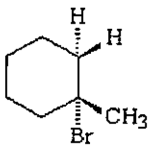
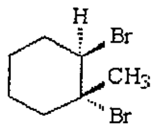
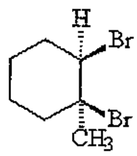

2


3

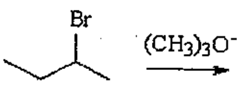
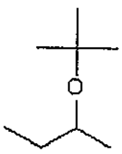
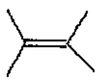
(A) 1, 3, 2 (B) 1, 2, 3 (C) 3, 2, 1
(D) 2, 1, 3 (E) 2, 3, 1

12. What is the product of the following reaction:

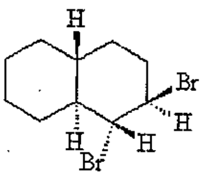


(A)  (B)  (C)  (D) 

13. What is the major product of the following reaction?

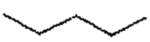
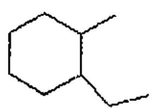
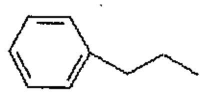
 (A)  (B)  (C)  (D) None of the Above

14. How many potential elimination products can form from the addition of NaOH to the following:

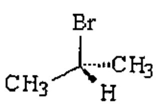


(A) 1 (B) 2 (C) 3 (D) No Elimination Products are Possible

15. Free radical chlorination (Cl_2, hv) is not very selective. Which of the following compounds will give the *most* selective chlorination?

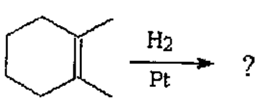
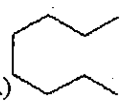
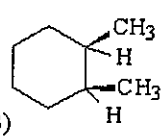
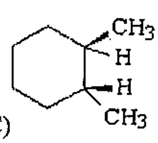
(A)  (B)  (C)  (D) 

16. What is the stereochemistry of the following compound:



(A) R (B) S (C) Meso (D) Not Chiral

17. Calculate the degree of unsaturation in Testosterone, $C_{19}H_{26}O_2$ (A) 5 (B) 6 (C) 7 (D) 8 (E) None of these

18.  (A)  (B)  (C)  (D) Both B *and* C (E) None of the Above

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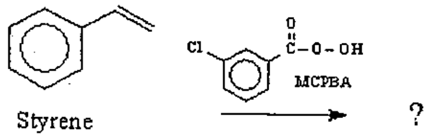
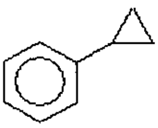
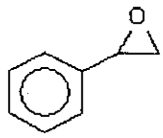
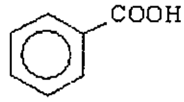
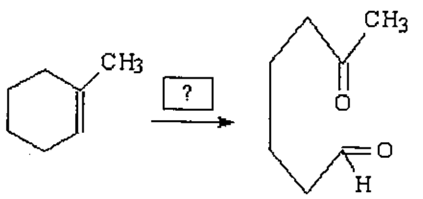
19. What is the product of the addition of MCPBA to styrene?



(A)  (B)  (C) 

(D) None of the Above

20. Choose the best reagent from the list below to do the following conversion:



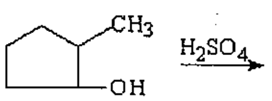
(A) 1. O_3 2. Zn, H_3O^+ (B) $KMnO_4$, Acid (C) 1. OsO_4 2. $NaHSO_3$, H_2O (D) CH_2I_2, Zn(Cu)

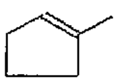
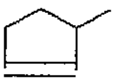
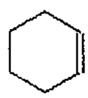
21. Rank the following substituents by the Cahn-Ingold-Prelog sequence rules and put them in decreasing order (from the highest priority substituent to the lowest priority substituent)

1	2	3	4
—C(=O)-H	—CH=CHCH$_3$	—CH=NCH$_3$	—C≡C-CH$_3$

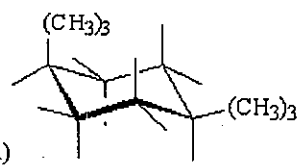
(A) 1,3,4,2 (B) 4,1,2,3 (C) 1,3,2,4 (D) 3,1,2,4 (E) None of the Above

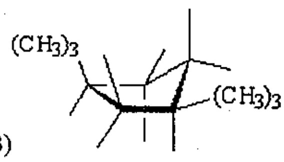
22. What would probably not be a product of the following reaction:

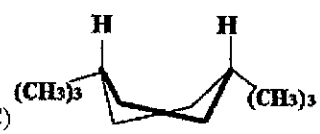


(A)  (B)  (C)  (D) 

23. What is the most stable conformation of *cis* 1,4 T-butylcyclohexane?

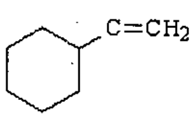
(A) 

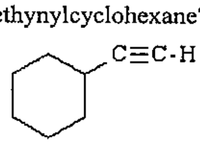
(B) 

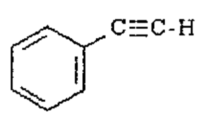
(C) 

(D) None of these are the most stable

24. What is the structure of ethynylcyclohexane?

(A) 

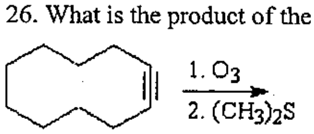
(B) 

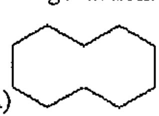
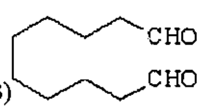
(C) 

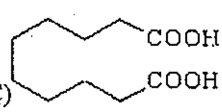
(D) None of the Above

25. What is the common name of ethyne? (A) Acetone (B) Acetylene (C) Angstrom (D) Acrylic

26. What is the product of the following reaction:



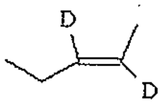
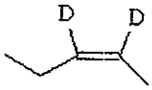
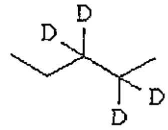
(A)  (B) 

(C) 

(D) None of the Above

27. What is the product of the following reaction:

$CH_3CH_2-C\equiv C-CH_3 \xrightarrow[\text{Lindlar catalyst}]{D_2}$

(A)  (B)  (C)  (D) None of the Above

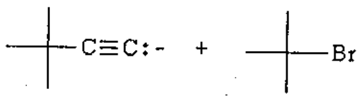
28. Which of the following bases are strong enough to deprotonate acetylene?

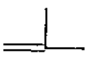
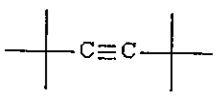
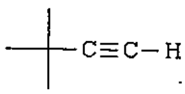
(A) Concentrated H_2SO_4 (B) $NaNH_2$ (C) $CH_3\text{-}CH_2\text{:}^- Li^+$ (D) Both B and C

29. Which of the following reagents will best convert methyl acetylene to acetone?

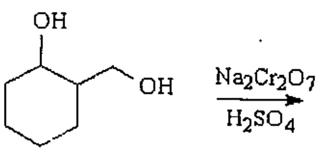
(A) H_2SO_4, H_2O, $HgSO_4$ (B) 1. BH_3, THF 2. HOOH (C) H_2, Lindlar (D) 1. HCl 2. $NaNH_2$, NH_3

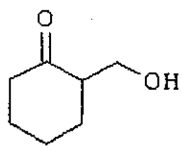
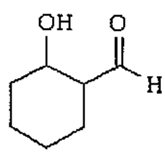
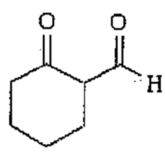
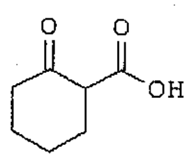
30. What is the major product of the following reaction:



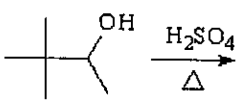
(A)  (B)  (C)  (D) Both A *and* C (E) No Reaction

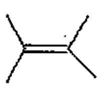
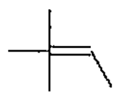
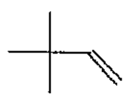
31. What is the product of the following reaction:



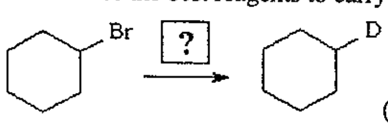
(A)  (B)  (C)  (D) 

32. What is the major product of the following reaction:



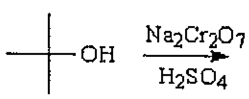
(A)  (B)  (C)  (D) None of the Above

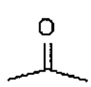
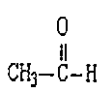
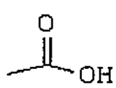
33. Choose the best reagents to carry out the following reaction:



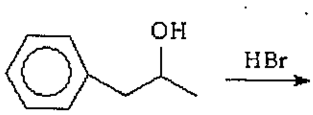
(A) D_2, Pt (B) 1. Mg, Ether 2. D_2O (C) D_2SO_4, D_2O, Δ (D) D_3O^+, Acetone, Δ

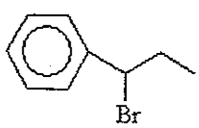
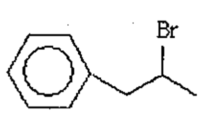
34. What is the major product of the following reaction:



(A)  (B)  (C)  (D) No reaction

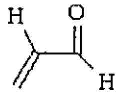
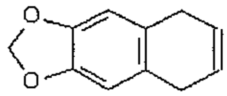
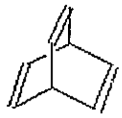
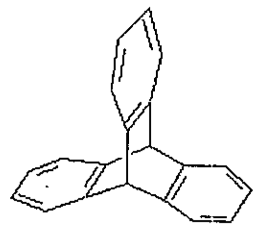
35. What is the major product of the following reaction:



(A)  (B)  (C) 

(D) None of the Above

36. Which of the following compounds have no conjugated portions?

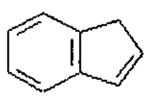
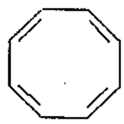
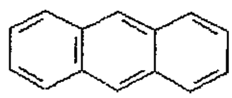
(A)  (B)  (C)  (D) 

37. For a diene to undergo Diels-Alder reaction it must: (A) be substituted with electron-withdawing groups. (B) be able to adopt an s-trans conformation (C) be substituted with electron-donating groups (D) be able to adopt the s-cis conformation

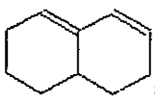
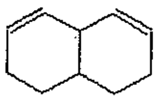
38. Which of the following would show the longest wavelength (λ max) in its UV spectrum.

(A)  (B)  (C)  (D) 

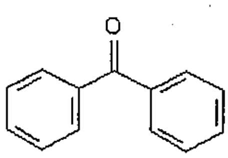
39. Which of the following are aromatic:

(A)  (B)  (C)  (D)  (E) Both A and D

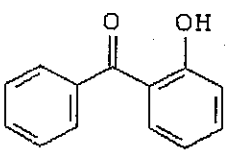
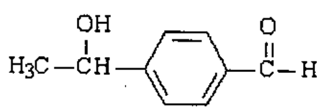
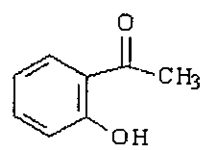
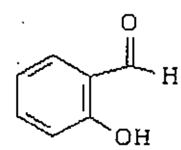
40. Indicate which spectral technique would best be used to separate the following compounds:

 and  (A) ^{1}H NMR (B) IR Spectroscopy (C) Mass Spectrometry (D) UV Spectroscopy

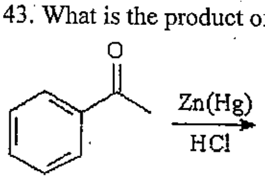
41. What is the IUPAC name of the following compound:

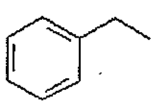
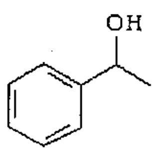
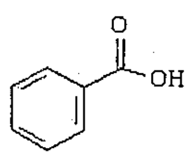
 (A) Diphenyl Ketone (B) Benzophenone (C) Dibenzyl Ketone (D) 1,1-Diphenylmethanone

42. What is the correct structure for 2-Hydroxyacetophenone:

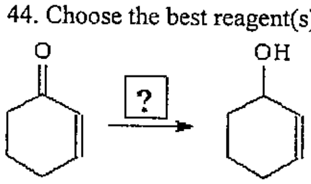
(A)  (B)  (C)  (D) 

43. What is the product of the following reaction:



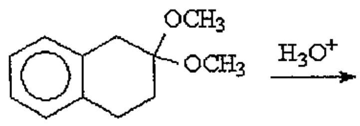
(A)  (B)  (C)  (D) None of the Above

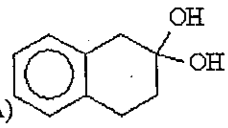
44. Choose the best reagent(s) from the list to do the following conversion:

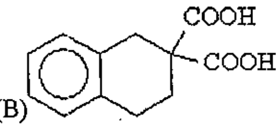


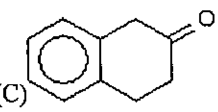
(A) H_2/Ni (R) (B) $Hg(OAc)_2$, H_3O^+ (C) 1. BH_3, THF 2. H_2O_2, NaOH (D) $NaBH_4$, THF

45. What is the product of the following reaction:



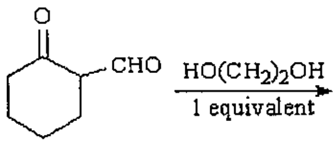
(A) 

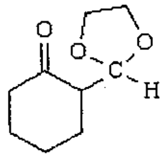
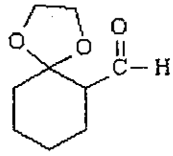
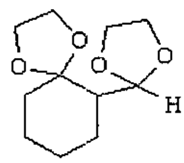
(B) 

(C) 

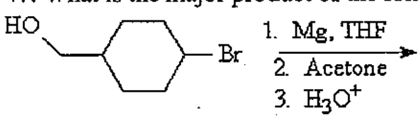
(D) None of the Above

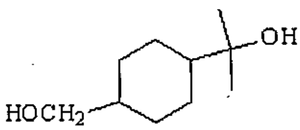
46. What is the major product of the following reaction:

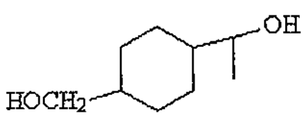


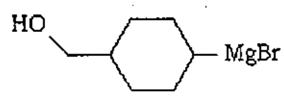
(A) 
(B) 
(C) 
(D) None of the Above

47. What is the major product of the following reaction:



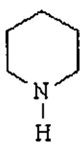
(A) 

(B) 

(C) 

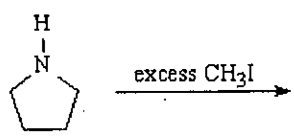
(D) No Reaction

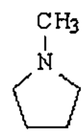
48. What is the name of the following amine:

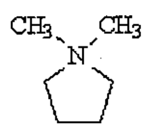


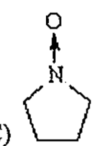
(A) Pyrrole (B) Pyridine (C) Pyramidine (D) Piperdine

49. What is the major product of the following reaction:



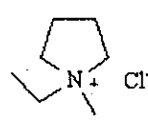
(A) 

(B) 

(C) 

(D) None of the Above

50. Which of the following amines can be resolved into enantiomers:

(A) ethyl amine (B) N, N-Dimethylaniline (C) 1-Methylpiperdine (D) 

科目：物理化學【材料所碩士班甲組】

1. Explain the following terms.
   (1) Einstein-de Broglie postulate [5%]
   (2) Born-Oppenheimer approximation [5%]
   (3) Pauli principle [5%]
   (4) Spin-orbit coupling [5%]
   (5) Spin-spin correlation [5%]
   (6) Frank-Condon principle [5%]

2. There are 3 types of statistics frequently adopted in physical sciences: the Bose-Einstein, the Fermi-Dirac, and the Boltzmann distributions.
   (1) Give the basic functional forms of the 3 types of distribution functions. [6%]
   (2) What are the intrinsic physical differences embedded in these distribution functions? [6%]
   (3) Show that, at high temperatures, all the 3 functions converge into one. [4%]
   (4) Provide physical interpretation to the observation in (3). [4%]

3. Vibrational spectroscopy mainly concerns with the absorption spectrum in the infrared region (IR spectroscopy) and scattering spectrum of a monochomated (typically in the visible range) light (Raman spectroscopy). The observed peaks in absorbance at certain wavenumbers or in scattering intensity at inelastically shifted wavenumbers are then related to respective "normal modes" which are often localized in character and may be assigned to groups of a small number of atoms (i.e., functional groups).
   (1) What is the physical meaning of the "normal modes"? [4%]
   (2) Show that there are generally 3N-6 normal modes of vibration for a molecule composed of N atoms but only 3N-5 normal modes in the special case of linear molecules. [4%]
   (3) Quantum mechanical treatment of molecular vibration results in quantized energy levels with non-zero energy even in the ground (lowest-energy) state. What is the physical meaning of this peculiarity? [3%]
   (4) These normal modes are often referred to as IR-active or Raman-active, depending on whether the corresponding peaks appear in the IR or Raman spectrum. What factors determine the "activity" of a normal mode towards IR or Raman spectroscopy? [6%]
   (5) The Beer-Lambert law that $A = \varepsilon bc$ (where A is the absorbance, ε is the extinction coefficient, b is the path length, and c is the concentration) allows for quantitative analysis of the absorbing species. Derive this equation and specify simplifying assumptions or approximations adopted. [8%]

4. Using an energy diagram, explain the differences between fluorescence and phosphorescence. [5%]
   (1) What are the typical time-scales involved in the two processes? [5%]
   (2) It is typically observed that the characteristic lifetime in the phosphorescence process is much longer than that in the fluorescence process. In your opinion, what is the physical origin of this prolonged lifetime in phosphorescence? [5%]

5. Assuming the presence of a metastable transition-state (AB)*, derive a rate expression for the idealized reaction $A + B \Leftrightarrow (AB)^* \rightarrow C$. Compare this rate expression with the empirical "Arrhenius law" that $k = k_o \exp(-E_a/RT)$ and comment on physical meanings of the activation energy E_a and the pre-exponential factor k_o. [10%]

科目：工程數學甲【材料所碩士班乙組】

1. ***(Differential Equations)*** (48%)

Please solve for *y(t)* from the following problems. Show the details of your work.

(a) $\frac{d^3y}{dt^3} - 4\frac{dy}{dt} = \cos t + \sin t$,

(b) $\frac{d^2y}{dt^2} + \frac{1}{t}\frac{dy}{dt} + y = 0$, and

(c) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = r(t)$, where $r(t) = 4t$ if $0 < t < 1$ and 1 if $t > 1$.

2. ***(Fourier Integral)*** (16%)

The Fourier integral is represented as $f(x) = \int_0^\infty [A(w)\cos(wx) + B(w)\sin(wx)]dw$, where $A(w) = \frac{1}{\pi}\int_0^\infty f(v)\cos(wx)dv$, and $B(w) = \frac{1}{\pi}\int_0^\infty f(v)\sin(wx)dv$. Please using the above notations, show that $\int_0^\infty \frac{\cos(wx)\cdot \sin w}{w}dw = \begin{cases} \frac{\pi}{2} & \text{if } 0 \le x < 1 \\ \frac{\pi}{4} & \text{if } x = 1 \\ 0 & \text{if } x > 1 \end{cases}$.

3. ***(Linear Algebra)*** (16%)

Please find a basis of eigenvectors and diagonalize the matrix: $\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & -4 \\ 1 & -1 & -2 \end{bmatrix}$.

4. ***(Heat Equation)*** (20%)

Please find the steady-state solution of the heat equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1$ in a thin square copper plate (Fig. 1). The temperature $u(x, y)$ satisfies the following boundary condition:

$$u(x,0) = u(x,a) = u(0,y) = u(a,y) = 1.$$

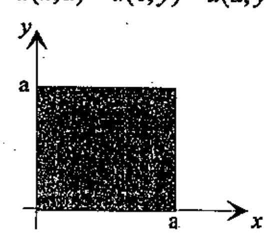


Fig. 1. Square copper plate

科目：材料力學【材料所碩士班乙組】

20% for each problem

1. Explain the following terminologies.
(a) Bauschinger effect
(b) Principal stresses
(c) Statically indeterminate problems
(d) Stress tensor at a point

2. Rods ABC consists of two cylindrical portions AB and BC; it is made of a mild steel which is assumed to be elastoplastic with E = 200 GPa and $\sigma_Y = 250$ MPa. A force P is applied to the rod and then removed to give it a permanent set $\delta_p = 2$ mm. Determine the value of the force P and the amount δ_m by which the rod should be stretched to give it the desired permanent set.

3. The steel block shown in Fig. P3 is subjected to a uniform pressure on all its faces. Knowing that the change in length of edge AB is -1.2 μm, determine (a) the change of the other two edges. (b) The pressure p applied to the faces of the block. Assume E = 210 GPa, and $\nu = 0.29$.

4. Shaft AB has a 30 mm diameter and is made of steel with an allowable shearing stress of 90 MPa, while shaft BC has a 50 mm diameter and is made of an aluminum alloy with an allowable shearing stress of 60 MPa. Neglecting the effect of stress concentration, determine the largest torque T which may be applied at A.

5. Using a 60° rosette, the following strains have been determined at point Q on the surface of a steel machine base: $\varepsilon_1 = 40\ \mu$, $\varepsilon_2 = 980\ \mu$, $\varepsilon_3 = 330\ \mu$. Using the coordinate axes shown, determine at point Q, (a) the strain components ε_x, ε_y, and γ_{xy}, (b) the principal strains, (c) the maximum shearing strain. (Use ν=0.29)

科目：材料力學【材料所碩士班乙組】

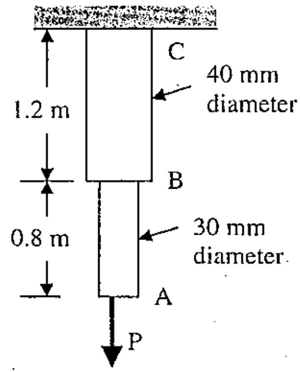


Fig. P2

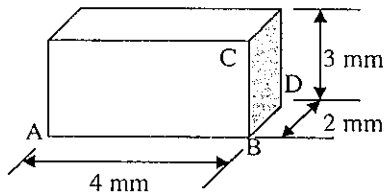


Fig. P3

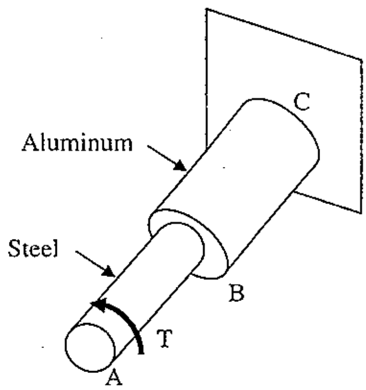


Fig. P4

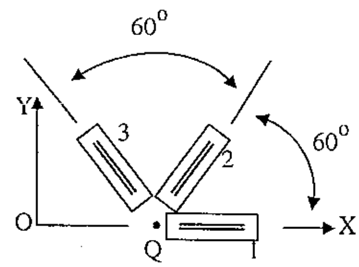


Fig. P5

科目：工程數學乙【材料所碩士班丙組】

1. ***(Differential Equations)*** (48%)

   Please solve for $y(t)$ from the following problems. Show the details of your work.

   (a) $\frac{d^3y}{dt^3} - 4\frac{dy}{dt} = \cos t + \sin t$,

   (b) $\frac{d^2y}{dt^2} + \frac{1}{t}\frac{dy}{dt} + y = 0$, and

   (c) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = r(t)$, where $r(t) = 4t$ if $0 < t < 1$ and 1 if $t > 1$.

2. ***(Fourier Integral)*** (16%)

   The Fourier integral is represented as $f(x) = \int_0^\infty [A(w)\cos(wx) + B(w)\sin(wx)]dw$, where $A(w) = \frac{1}{\pi}\int_0^\infty f(v)\cos(wx)dv$, and $B(w) = \frac{1}{\pi}\int_0^\infty f(v)\sin(wx)dv$. Please using the above notations, show that $\int_0^\infty \frac{\cos(wx)\cdot\sin w}{w}dw = \begin{cases} \frac{\pi}{2} & \text{if} \quad 0 \le x < 1 \\ \frac{\pi}{4} & \text{if} \quad x = 1 \\ 0 & \text{if} \quad x > 1 \end{cases}$.

3. ***(Linear Algebra)*** (16%)

   Please find a basis of eigenvectors and diagonalize the matrix: $\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & -4 \\ 1 & -1 & -2 \end{bmatrix}$.

4. ***(Heat Equation)*** (20%)

   Please find the steady-state solution of the heat equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1$ in a thin square copper plate (Fig. 1). The temperature $u(x,y)$ satisfies the following boundary condition:

   $$u(x,0) = u(x,a) = u(0,y) = u(a,y) = 1.$$

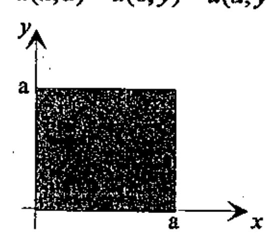


Fig. 1. Square copper plate

國立中山大學九十四學年度碩士班招生考試試題

科目：熱力學【材料所碩士班丙組】

1. Answer the following questions. 〔每小題 3 分，答錯倒扣〕
   - (I) Ice floating in water, thermally insulated from its surroundings, eventually reaches a stable ratio of volumes of the two phases. (equilibrium state or steady state ?)
   - (II) A foil birthday balloon brought into a hot room will first expand to its fullest, then, at a fixed volume, increase its internal pressure until it reaches a value. (equilibrium state or steady state ?)
   - (III) A hose supplies water to a bucket with a hole in it. The flow rate from hose is adjusted until the water level maintains a constant level. (equilibrium state or steady state ?)
   - (IV) Current passing through a tungsten wire heats it to incandescence〈白熱化〉. After a short transient the Joule heat generation and heat losses come into balance and the temperature distribution and illumination achieve constant values. (equilibrium state or steady state ?)
   - (V) The potential energy of a system in a gravitational field. (extensive property or intensive property?)
   - (VI) The molar concentration of NaCl in a salt solution. (extensive property or intensive property?)

2. Consider a model in which the available energy levels are linearly spaced along the energy axis: (20%)

   $En = (n + 1/2)^* Eo$ (n= 0,1,2,3,4,....9)

   The system contains ten particles. Consider two macrostates:

   { 0,0,1,2,4,2,1,0,0,0} ---- state I

   and { 0,1,1,2,2,2,1,1,0,0} ---- state II

   - (I) Which macrostate has the higher energy and derive the value?
   - (II) Which macrostate has the higher entropy and derive the value?
   - (III) Which macrostate is more likely to be observed?

3. A bimetallic strip of total thickness x is straight at temperature T. What is the radius of curvature of the strip, R, when it is heated to temperature $T+\Delta T$? The coefficients of linear expansion of the two metals are $\alpha 1$ and $\alpha 2$, respectively, when $\alpha 2 > \alpha 1$. You may assume that each metal has thickness x/2, and you may assume that $x << R$. (22%)

4. Calculate the change in the enthalpy and the change in entropy when one mole of SiC is heated from 25 ^{0}C to 1000 ^{0}C. The constant pressure molar heat capacity of SiC varies with temperature as, (20%)

   $Cp = 50.79 + 1.97^*10^{-3}T - 4.92^*10^{6}T^{-2} + 8.20^*10^{8}T^{-3}$ J/mole.K

5. (a) How much heat is required to raise the temperature of 1000 grams of nitrogen from –20 ^{0}C to 100 ^{0}C at constant pressure? (5%)

   (b) How much has the internal energy of the nitrogen increased? (5%)

   (c) How much external work was done? (5%)

   (d)How much heat is required if the volume is kept constant? (5%)

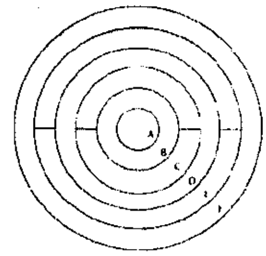
Take the specific heat at constant volume Cv = 5 cal/mole.^{0}C and R =2 cal/mole.^{0}C

科目：普通物理【材料所碩士班丁組】

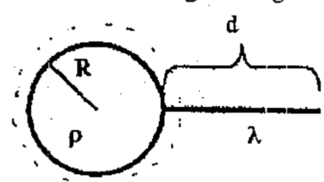
General Physics

1. A weighing scale measures the net downward force of an object placed on its top surface. A ball of lead is submerged in water at T °C and rests on a weighing scale. If the water temperature increases to $(T+\Delta T)$ °C, what is the fractional change $(\delta W/W)$ in weight as shown on the scale? Let the linear expansion coefficient of lead to be α and the volume expansion coefficient of water to be β. Also, the density of lead at initial temperature T is ρ_L. (15 points)

2. The figure shows six concentric conducting spheres, A, B, C, D, E and F, having radii R, 2R, 3R, 4R, 5R and 6R, respectively. Spheres B and C are connected by a conducting wire as are spheres D and E. Determine the equivalent capacitance C_{eff} of this system. (15 points)



3. A uniformly charged solid sphere of radius R carrying volume charge density ρ is centered at the origin. Find the total force acting on a uniform line charge having a density λ and a total charge Q. The line is oriented radially with respect to the sphere with its ends at R and $R+d$. (15 points)



4. Consider a thin disk of radius R mounted to rotate about the x axis in the yz plane. The disk has a positive uniform surface charge density σ and an angular velocity ω. What is the magnetic field at the center of the disk? (15 points)

5. A satellite is in a circular orbit about Earth at a height above Earth equal to Earth's mean radius.
(a) Find the satellite's centripetal acceleration in units of g (= 9.8 m/s^2). (10 points)
(b) Kepler's third law states that T^2, the square of satellite's period, is proportional to R^3, the cube of the satellite's orbital radius. If this satellite's height above the Earth had been three Earth's radii instead of one Earth radius, by what factor would the period have changed? (15 points)

6. A container holds a mixture of three non-reacting gases: n_1 moles of the first gas with molar specific heat at constant volume C_1, and so on (n_2, C_2; n_3, C_3). Find the molar specific heat at constant volume of the mixture, in terms of the molar specific heats and quantities of the separate gases. (15 points)

1. Two electrons ($m = 0.511\ MeV/c^2$) with a momenta of (a) 2.000 MeV/c and (b) 0.2eV/c, respectively. Find the total energy of each case. Now we have photons (m = 0) with identical momenta as the electrons. Find also the ratio $Energy(electron)/Energy(photon)$ in each case. (16 points)

2. The Compton effect states an x-ray strikes an electron. Show that $\lambda' - \lambda = \lambda_C \cdot (1-\cos\phi)$, where $\lambda_C = \frac{h}{mc}$. λ is the incident light wavelength, and λ' is the scattered light wavelength. ϕ is the angle between the directions of the incident and scattered photons (assumed electron initially at rest). (16 points)

3. Show that the expectation values $\langle px \rangle$ and $\langle xp \rangle$ are related by $\langle px \rangle - \langle xp \rangle = \frac{\hbar}{i}$. Derive the uncertainty principle by the relation. (14 points)

4. An electron gun emits a 200 keV energy electron beam, find (a) the velocity, and (b) the wavelength of the emitting electrons. (6 points)

5. Phosphorus is present in a germanium sample. Assume that one of its five valence electrons revolves in a Bohr orbit around each P^+ ion in the germanium lattice. (a) If the effective mass of the electron is 0.17 m_e and the dielectric constant of germanium is 16, find the radius of the first Bohr orbit of the electron. (b) The electron gap between the valence and conduction bands in germanium is 0.65 eV. How does the ionization energy of the above electron compared with this energy and with kT at room temperature? (16 points)

6. A particle with mass m locates in a potential $V(x) = -g[\delta(x+a) + \delta(x-a)]$, where g>0, and $\delta(x)$ is the Dirac δ-function, which satisfies $\delta(x) = \begin{cases} 0, & \text{for } x \neq 0 \\ \infty, & \text{for } x = 0 \end{cases}$ and $\int_{-\infty}^{\infty} \delta(x) = 1$. Find the eigenfunctions and eigenvalues. (16 points).

7. Express the corresponding experiments or the findings, and point out the significance of the findings.

   (a). Einstein's photoelectric effect : $h\nu = KE_{max} + \phi$

   (b). Rutherford scattering.

   (c). The Stern-Gerlach experiment.

   (d) Planck radiation law: $u(\nu)d\nu = \frac{8\pi h}{c} \cdot \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1}$.

   (16 points)

Physical Constants and Conversion Factors

Atomic mass unit	u	1.66054×10^{-27} kg
		931.49 MeV/c^2
Avogadro's number	N_0	6.022×10^{26} kmol^{-1}
Bohr magneton	μ_B	9.274×10^{-24} J/T
		5.788×10^{-5} eV/T
Bohr radius	a_0	5.292×10^{-11} m
Boltzmann's constant	k	1.381×10^{-23} J/K
		8.617×10^{-5} eV/K
Compton wavelength of electron	λ_C	2.426×10^{-12} m
Electron charge	e	1.602×10^{-19} C
Electron rest mass	m_e	9.1095×10^{-31} kg
		5.486×10^{-4} u
		0.5110 MeV/c^2
Electronvolt	eV	1.602×10^{-19} J
	eV/c	5.344×10^{-28} kg · m/s
	eV/c^2	1.783×10^{-30} kg
Hydrogen atom, ground-state energy	E_1	-2.179×10^{-18} J
		-13.61 eV
rest mass	m_H	1.6736×10^{-27} kg
		1.007825 u
		938.79 MeV/c^2
Joule	J	6.242×10^{18} eV
Kelvin	K	°C + 273.15
Neutron rest mass	m_n	1.6750×10^{-27} kg
		1.008665 u
		939.57 MeV/c^2
Nuclear magneton	μ_N	5.051×10^{-27} J/T
		3.152×10^{-8} eV/T
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T · m/A
Permittivity of free space	ε_0	8.854×10^{-12} $C^2/N \cdot m^2$
	$1/4\pi\varepsilon_0$	8.988×10^{9} $N \cdot m^2/C^2$
Planck's constant	h	6.626×10^{-34} J · s
		4.136×10^{-15} eV · s
	$\hbar = h/2\pi$	1.055×10^{-34} J · s
		6.582×10^{-16} eV · s
Proton rest mass	m_p	1.6726×10^{-27} kg
		1.007276 u
		938.28 MeV/c^2
Rydberg constant	R	1.097×10^{7} m^{-1}
Speed of light in free space	c	2.998×10^{8} m/s
Stefan's constant	σ	5.670×10^{-8} $W/m^2 \cdot K^4$