國立中山大學 110 學年度
碩士暨碩士專班招生考試試題

科目名稱：工程數學【材光甲組聯合招生碩士班選考、材光系碩士班甲組、材料前瞻應材碩士班甲組】

1. Find the general solutions of y_1 and y_2 (10 分)

$y_1' = -y_1 - y_2 + 8t + 5$
$y_2' = -4y_1 + 2y_2 + 3e^{-t} - 15t - 2$

2. The tank in Fig. 1 contains 80 lb of salt dissolved in 500 gal of water. The inflow per minute is 20 lb of salt dissolved in 20 gal of water. The outflow is 20 gal/min of the uniform mixture. Find the time when the salt content y(t) in the tank reaches 95% of its limiting value (as $t \to \infty$). (10 分)
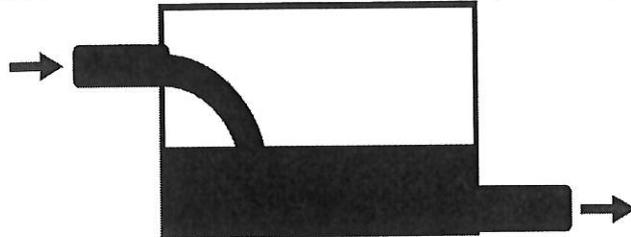

Fig.1

3. Find out the particular solution of the ODE (10 分)

$y'' + 4y = -12\sin 2x$, $y(0) = 1.8$, $y'(0) = 5.0$

4. Use the Laplace transform to solve the following ODE (10 分)

$y'' + y' - 2y = \begin{cases} 3\sin t - \cos t, (0 < t < 2\pi) \\ 3\sin 2t - \cos 2t, (t > 2\pi) \end{cases}$
$y(0) = 0, \qquad y'(0) = -1$

5. Use the Laplace transform to solve the following ODE (10 分)

$y'' + 2y' + 5y = 25t - 100\delta(t - \pi)$, $y(0) = -2$, $y'(0) = 5$

6. Let

$A = \begin{bmatrix} 2 & -1 & 3 \\ -2 & 1 & 4 \\ 1 & 1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, $a = \begin{bmatrix} -1 & -2 & 0 \end{bmatrix}$.

Find the resultant matrices of the following expressions or give reasons why they are undefined. Each calculation result should be given in a step-by-step way.

(a) $B^T A$; **(b)** $(3A - 2B)^T$; **(c)** aBa^T; **(d)** $3A^T - 2B^T$　　　(10 分)

7.

$A = \begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 0 \\ 3 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 2 & 1 & 3 \end{bmatrix}$　　$B = \begin{bmatrix} 2 & 4 & 6 & 0 & 0 & 0 \\ 24 & 36 & 12 & 0 & 0 & 0 \\ 9 & 3 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 21 & 35 \\ 0 & 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 10 & 5 & 15 \end{bmatrix}$

Find the det(A) = ?, det($A \times B$) = ?, and A^{-1} = ?.　　(10 分)

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※本科目依簡章規定「不可以」使用計算機(問答申論題)　　　　　　共 2 頁第 2 頁

8. $f = 7x_1^2 + 6x_1x_2 + 7x_2^2 = 200$, plot the curve.

   Hint. Through the quadratic form, transform it to principal axes, and express $[x_1\ x_2]$ in terms of the new coordinate vector $[y_1\ y_2]$. Please try specifying each dimension of your plotted conic section. (10 分)

9. Find the complex Fourier Series of $f(x) = e^x$ if $-p < x < p$ and $f(x+2p) = f(x)$. (10 分)

10. Find the trigonometric polynomial $F(x)$ of the form:

$$F(x) = A_0 + \sum_{n=1}^{N} (A_0 \cos nx + B_0 \sin nx)$$

   for which the <u>square error</u> with respect to the given $f(x) = x^2$ on the interval $-\pi < x < \pi$ is <u>minimum</u>. Compute the minimum value for $N = 1, 2, \ldots, 5$. (10 分)

試題請隨卷繳回，請留意背面是否有題

國立中山大學 110 學年度
碩士暨碩士專班招生考試試題

科目名稱：基礎熱力學【材光甲組聯合招生碩士班選考、材光系碩士班甲組、材料前瞻應材碩士班甲組】

Section 1 – For questions 1 – 3, Select the correct phrase to fill in the blanks for the following statements. Mark on the answer sheet with the correct letter choice. (15%, at 5% each)

1, The first law of thermodynamics states that energy _____ be generated or destroyed in an isolated system.
   A, "can"
   B, "cannot"

2, The second law of thermodynamics indicates that the total entropy of an isolated system must _____ when undergoing an irreversible process.
   A, "increase"
   B, "decrease"
   C, "stay the same"

3, According to the third law of thermodynamics, the entropy of a system reaches a minimum at the_____.
   A, "absolute zero"
   B, "melting temperature"
   C, "below freezing temperature"

Section 2 – For questions 4 – 7 Answer "True" or false for the following questions. Mark on the answer sheet with the correct letter choice. (20%, at 5% each)

4, Collisions between ideal gas molecules are completely elastic.
   A, True
   B, False

5, It is possible to create a perpetual motion machine that can do work indefinitely without relying on an external source of energy.
   A, True
   B, False

6, A material's specific heat capacity describes the amount of heat required to raise the temperature of 1.0 mole of material by 1.0 degree Celsius.
   A, True
   B, False

7, A process with a negative change in Gibbs free energy is thermodynamically favorable.
   A, True
   B, False

Section 3 – For questions 8-12 – Select the one correct choice from the list of potential answers. Mark on the answer sheet with the correct letter choice. (25%, at 5% each)

8, An ideal gas has a higher specific heat at constant pressure (C_p) than its specific heat at constant volume (C_v). This is because ____?
   A, Gas expanding at constant pressure does work on the environment when its temperature is increased.
   B, Gas's internal energy increases more when held at constant pressure than when held at constant volume.
   C, Pressure of the gas remains constant if the temperature is held constant.
   D, More heat is required to heat up gas at increasing pressure than at increasing volumes.
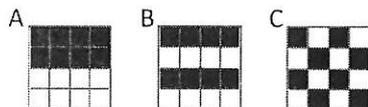
9, A 2D system consisting of 16 black and white components arranged in various configurations in a 4 by 4 grid. Among the following 3 configurations below, which one has the highest entropy?

10, A system undergoing a perfectly reversible thermodynamic process must have the following characteristic?

A, The internal energy of the system needs to be constant throughout the process.

B, Entropy of the system and its surrounding does not change.

C, The work done by the system must be equal to the work done to the system.

D, Temperature of the system must stay constant throughout the process.

11, Sam, a typical college student, needs to bake a mini cubical cake of 512.0 cm^3 for an upcoming party. The cake dough needs to be heated from 25 °C to 190 °C and baked for 1.5 hours. However, the kitchen oven is broken, and the only heat source available is a large old electric stove that heats up to 120 °C. If Sam puts the cake onto the 120 °C electric stove, is it possible to concentrate the heat from electric stove and heat the cake to 190 °C?

A, Yes, it is possible if the stove is at least 3 times as large as the cake.

B, Yes, if the cake dough is placed at the exact center of the stove, heat would flow in from all sides to heat it up.

C, No, heat cannot flow from a colder object to a hotter object.

D, No, because heating the cake dough like this would violate the first law of thermodynamics.

12, In a big box, two ideal gas species were separated by a stainless steel divider. Each species is maintained at 1atm pressure and 25 °C temperature. When the steel divider is removed, the two gas species are allowed to mix. Among the following, which condition is not true?

A, $\Delta G^{mix} < 0$

B, $\Delta S^{mix} > 0$

C, $\Delta H^{mix} < 0$

D, $\Delta G^{mix} = \Delta H^{mix} - T * \Delta S^{mix}$

Section 4 – For questions 13 and 14 – Short answer. (40% at 10% each sub-question)

13, A little boy, named Johnny, got a beautiful red balloon at his friend's birthday party on a snowy day. The balloon was filled with helium up to 0.25 MPa. After having a good time indoors, Johnny and his father decided to walk home in the -5.0 °C weather. As they walked, Johnny's balloon shriveled up and flopped to the ground. It seems that the balloon cooled down and shrank.

i) Why did Johnny's balloon shrink? Use the ideal gas law to explain this phenomenon.

ii) Can the balloon recover? Explain your reasoning.

14, An engine has a gas piston chamber that contains 1.0 mole of a proprietary gas. At rest, the entire engine is maintained at 25.0 °C at 1.0 atm pressure. Bob the engineer needs the engine to do some work, so he heats the system to cause the gas to expand. Bob noted that he added 5 kJ of heat to this system, but he only got the piston to do 1.43 kJ of work. The gas can be assumed to behave like an ideal gas and the process is done reversibly.

i) What are the final volume of the gas piston chamber and the temperature of the gas?

ii) What are the gas's specific heat capacity at constant pressure (C_p) and the gas's specific heat capacity at constant volume(C_v)?

Ideal gas constant (R) = 0.08205 (L*atm)/(mol*K) = 8.3145 J / (mol*K)

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國立中山大學 110 學年度
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科目名稱：材料科學導論【材光乙組聯合招生碩士班、材光系碩士班乙組、材料前瞻應材碩士班乙組】

－作答注意事項－

考試時間：100 分鐘

● **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。

● **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，不得另攜帶紙張，請衡酌作答。

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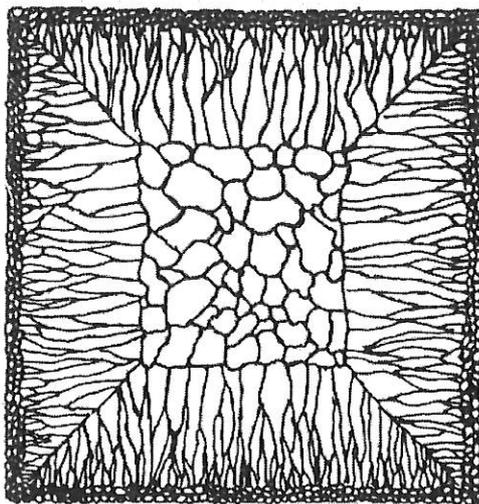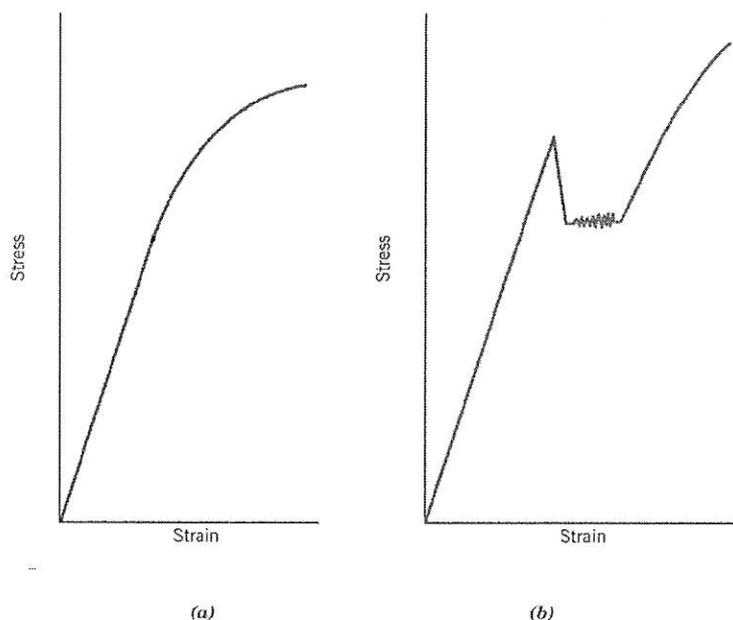(1) 如果你要在維基百科撰寫有關下列名詞之內容你會如何撰寫？(a) Dislocation, (b) Fatigue, (c) Shape memory alloy, (d) Heterogeneous nucleation, and (e) Bragg diffraction. 5 points each, 25 points.

(2) A schematic solidification grain structure is shown below. Explain why this structure is formed. 10 points

(3) Use schematic drawings to show the locations (include point coordinates) of tetrahedral and octahedral interstitial sites within (a) BCC and (b) FCC unit cells. 10 points

(4) Give schematic magnetization curves for soft and hard magnetic materials. Briefly explain your drawing. 8 points

(5) The engineering stress-strain curves shown below represent two different types of yielding behaviors of metals. Discuss these two types of yielding deformation. 8 points.

(6) Discuss how the dopant content and temperature affect charge carriers' mobility in a semiconducting material.　　　　　　　8 points

(7) Calculate the indices of the intersection line direction of (112) and $(0\,\bar{2}\,1)$ crystal planes in cubic crystal structure.　　　　　　　5 points.

(8) Give all your knowledge about vacancies in crystals.　　　　　　　8 points.

(9) How to define the primary and cross slip systems of a crystal when it is sheared?　　　　　　　6 points

(10) Can fatigue occur in a material without external applied mechanical stresses? If yes, give your reason.　　　　　　　6 points

(11) Discuss the effect of the following factors on the recrystallization behavior of deformed metals. (a) Amount of strain, (b) Initial grain size, and (c) Material purity.　　　　　　　6 points

國立中山大學 110 學年度
碩士暨碩士專班招生考試試題

科目名稱：普通物理【材光乙組聯合招生碩士班、材光甲組聯合招生碩士班選考、材光系碩士班甲組、材料前瞻應材碩士班甲組、材光系碩士班乙組、材料前瞻應材碩士班乙組】

Problem 1. [Mechanics: 40 points]

The atoms can bind with each other with various kinds of chemical bonds, and form molecules, and crystals. Around the equilibrium positions of all atoms, the binding force can be modeled as a spring with elastic constant k. The normal modes of these coupled motion play important roles for materials properties, and their frequencies can be observed experimentally.

(a) [5 points] Which forces are responsible for the binding forces among atoms? (A) gravity, (B) electromagnetic force, (C) weak force, (D) strong force.

(b) [5 points] If two particles with the same mass m is linked with a spring. The elastic constant of the spring is k. If one of the particle is fixed, meaning not be able to move. Solve the equation of motion to obtain the vibrational angular frequency ω of the particle (express your answer ω with m, and k).

(c) [5 points] If two atoms with the same mass m is linked to form a linear molecule. The binding force is approximated to be a spring with the elastic constant k. Assume the center of mass is at rest, solve the equation of motion to obtain the vibrational angular frequency ω of this molecule (express your answer ω with m, and k). Is your answer the same with (b)?

(d) [5 points] The elastic force is a conservative force, wherein a potential can be defined. If the potential $U(x)$ of the spring in the above two-atom molecule is:

$$U(x) = -\frac{1}{2}k(x - x_0)^2,$$

calculate the bond length of the molecule.

(e) [5 points] Following the above problem, if there is a correction term for the potential $U(x)$ of the spring such that:

$$U(x) = -\frac{1}{2}k(x - x_0)^2 + qx^3$$

(q is a positive constant), calculate the bond length of the molecule. Is the bond length longer or shorter than the answer in (d)?

(f) [15 points] If three atoms with the mass m, M, and m are linked to form a linear molecule, as shown in the figure below. The binding force is approximated to be a spring with the elastic constant k. Assume the center of mass is at rest, solve the equation of motion to obtain the vibrational angular frequencies of this molecule (express your answer ω with m, M, and k).
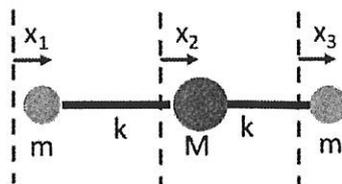

Figure 1

Problem 2. [Electromagnetism: 30 points]

(a) [5 points] If there are two charges with opposite signs separated with a distance d: $-q$ at $(0, 0, -d/2)$ and q at $(0, 0, +d/2)$. Find (i) the magnitude and (ii) the direction of the electric field at the point $(0, 0, s)$.

(b) [5 points] If there is an infinite line (at z axis), carrying a uniform line charge density λ. Find (i) the magnitude and (ii) the direction, of the electric field a distance s away from the line (You can use Gauss's law).

(c) [10 points] If there is a ring of radius R at the $x - y$ plane ($x^2 + y^2 = R^2$), carrying a uniform line charge density λ. Find (i) the magnitude and (ii) the direction of the electric field at the point $(0, 0, s)$.

(d) [5 points] Calculate the electric potential at the origin for the ring in the previous problem.

(e) [5 points] Write down (i) the electric potential $V(\mathbf{r})$ of a point charge, (ii) how to calculate $\mathbf{E}(\mathbf{r})$ using $V(\mathbf{r})$.

Problem 3. [Thermodynamics: 20 points]

(a) [10 points] Considering an ideal gas at equilibrium, the probability of the particles with the energy E to $E + dE$ follows the Maxwell-Boltzmann energy distribution:

$$P(E) \propto e^{-\frac{E}{k_B T}},$$

where k_B is the Boltzmann's constant, and T is the temperature. Calculate the average energy:

$$\langle E \rangle \equiv \frac{\int_0^\infty E P(E) dE}{\int_0^\infty P(E) dE}.$$

(b) [5 points] Which of the following types of particles that Maxwell-Boltzmann distribution is suitable to describe: (A) Bosons, (B) Fermions, (C) classical particles.

(c) [5 points] The entropy S can be expressed as:

$$S = k_B \ln W,$$

where W is the multiplicity of configuration, or the number of microstates. Suppose that there are 10 molecules in a box. Calculate how many microstates for the following two configuration: (i) 5 molecules in the left side of the box, and 5 molecules in the right side; (ii) all 10 molecules in the left side. Which configuration then has the greater entropy?

Problem 4. [Waves/Optics/Modern physics: 10 points]

(a) [5 points] Write down the standing wave condition for a circular string with the radius R, and the wave length λ.

(b) [5 points] In Young's double-slits experiments, assume that the distance between the two slits is a, the distance from the slits to the screen is D, and the wave length of the light is λ. If the center of the screen is set to be $x = 0$, express the position x of the bright fringe using a, D, and λ under the condition $x \ll D$ (that is, the bright fringe near the center).

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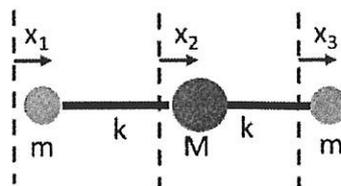

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(c) [5 points] The entropy S can be expressed as:

$$S = k_B \ln W,$$

where W is the multiplicity of configuration, or the number of microstates. Suppose that there are 10 molecules in a box. Calculate how many microstates for the following two configuration: (i) 5 molecules in the left side of the box, and 5 molecules in the right side; (ii) all 10 molecules in the left side. Which configuration then has the greater entropy?

Problem 4. [Waves/Optics/Modern physics: 10 points]

(a) [5 points] Write down the standing wave condition for a circular string with the radius R, and the wave length λ.

(b) [5 points] In Young's double-slits experiments, assume that the distance between the two slits is a, the distance from the slits to the screen is D, and the wave length of the light is λ. If the center of the screen is set to be $x = 0$, express the position x of the bright fringe using a, D, and λ under the condition $x \ll D$ (that is, the bright fringe near the center).