

1. Please solve the initial value problem. (10 %)

$$y''' + 3y'' + 3y' + y = e^{-x} \sin x, y(0) = 2, y'(0) = 0, y''(0) = -1 \quad (y^{(n)} = \frac{d^n y}{dx^n})$$

2. Please find the Laplace transform of the function $e^t u(t-2)$. (10 %)

3. Please find a basis of eigenvectors and diagonalize. (15 %)

$$A = \begin{bmatrix} 5 & 10 & -10 \\ 10 & 5 & -20 \\ 5 & -5 & -10 \end{bmatrix}$$

4. Find the Fourier transform of the function $f(x)$. (10 %)

$$f(x) = \begin{cases} x e^{-x} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$$

5. Show that when $0 < |z| < 1$, $\frac{1+2z^2}{z^3+z^5} = \frac{1}{z^3} + \frac{1}{z} + \sum_{n=1}^{\infty} (-1)^n z^{(2n-1)}$. (15 %)

6. Please integrate the function $\frac{e^{z^2}}{z^2(z-1-i)}$ over the given contour C, where C consists of

$$|z| = 2 \text{ (counterclockwise) and } |z| = 1 \text{ (clockwise)}. (10\%)$$

7. Evaluate the integrals. (15%)

$$(a) \int_1^5 \int_0^{x^2} (1+2x)e^{x+y} dy dx \quad (b) \int_{-\infty}^{\infty} \frac{\cos x dx}{(x^2+a^2)(x^2+b^2)} (a > b > 0)$$

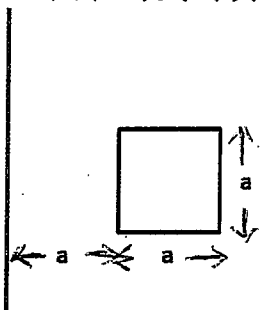
8. Find and plot the sample regression line of y on x and also find y at room temperature 66°F . (15%)

Temperature x [$^\circ\text{F}$]	32	50	100	150	212
Conductivity y [Btu/hr ft $^\circ\text{F}$]	0.337	0.345	0.365	0.380	0.395

研究所入學考-電磁學

填充題(每格5分,共60%)

1. 非均勻球體電荷密度為 ρ_v ,半徑為 a 的球性帶電體其半徑(R)內電荷分佈為 $\rho_0(1-R^2/a^2)$,則其球內電場(1)與球外電場(2)分別為?
2. 點電荷 $+Q$,置於內外半徑分別為 a 與 b 之球型介電殼(介電常數為 ϵ)中心位置,球殼內外之表面極化電荷密度分別為(3)與(4),另外,介電質之體電荷密度為(5)。
3. 圓柱導體內半徑為 a ,電位為 V_0 ,外半徑為 b ,電位為零,則內外導體間之封閉區域在半徑 r 的電位分佈為(6)。
4. 無限長之同軸電纜,其實心內導體半徑為 a ,外邊體之內外半徑各為 b 與 c ,假設內外導體之電流大小均為大小均勻分布且其值為 I ,維方向相反,請問在外導體中半徑 r 處之磁場強度 H 為(7)。
5. 半徑為 b 之磁性均勻分布球體其磁化向量大小 $M=M_0$,則在球心處之磁通密度 B_0 大小為(8)。
6. 如下圖,長導線與正方形環路之間之互感值為(9)。



7. 請寫下 source free、linear、isotropic 與 nonhomogeneous 條件下之波動方程式(10)。
8. 帶直流電流 I 且半徑與導電率分別為 b 與 σ 之常直導線,導線表面之 Poynting vector 之大小與方向分別為(11)、(12)。

問答與計算題(40%)

1. 請證明 Fresnel equation 中反射率與穿透率的描述中,在不考慮吸收條件下,能量將守恆(即 $R+T=1$)。(20%)
2. 表面鍍有金、銀或鋁之反射鏡已大量被使用在光學系統中做為反射鏡,請解釋原因(5%);其中相較於鍍銀鏡,鍍金鏡在紅外線的反射效果較好,其中原因為何?(5%)
3. 請說明何以電磁波是橫波而不是縱波。(10%)

- (1) An n-type Si bar is fabricated in a IC circuit, serving as a resistor. The donor concentration (N_d) is $7 \times 10^{14} \text{ cm}^{-3}$. And the resistor is 0.1mm long with a rectangular cross section of $20 \mu\text{m} \times 10 \mu\text{m}$. The measured material parameters of Si at 300°K are shown in table 1. Please answer the following questions:
- (a)(5%) Determine the conductivity of such n-type Si material. Why the conductivity can only be counted by the electron concentration?
- (b) (5%) Find the resistance of such n-type Si bar.
- (c) (5%) Use the parameters of table 1 to verify the Einstein relation.

Table 1

Properties (300°K)	Values
Relative permittivity (dielectric constant)	12
Electron mobility, μ_n	$1500 \text{ cm}^2/(\text{V} \cdot \text{s})$
Hole mobility, μ_p	$475 \text{ cm}^2/(\text{V} \cdot \text{s})$
Intrinsic concentration, n_i	$1.46 \times 10^{10} \text{ cm}^{-3}$
Electron diffusion constant, D_n	$34 \text{ cm}^2/\text{s}$
Hole diffusion constant, D_p	$13 \text{ cm}^2/\text{s}$
Boltzmann constant, k	$1.381 \times 10^{-23} \text{ J/K}$
Electron charge, q	$1.602 \times 10^{-19} \text{ coulomb}$

- (2) Figure 1 plots I-V characteristic of a diode (left) and a voltage-chipping circuit (right). In the circuit, $V_R=3\text{V}$, $R_1=1\text{k}\Omega$, and $R_2=11\Omega$.
- (a) (5%) Please find and explain the equivalent circuit models for the diode at the forward and reverse biases.
- (b) (5%) Describe the operational principle of the voltage-chipping circuit. When the input voltage is $V_i=A \cdot \sin(2 \cdot \pi \cdot f \cdot t)$, use the equivalent circuit model from (a) to find and plot the $V_o(t)$, where $A=5\text{V}$, $f=1\text{KHz}$.
- (c) (5%) If a two level of clipping voltage is needed in the circuit, please plot the circuit layout.

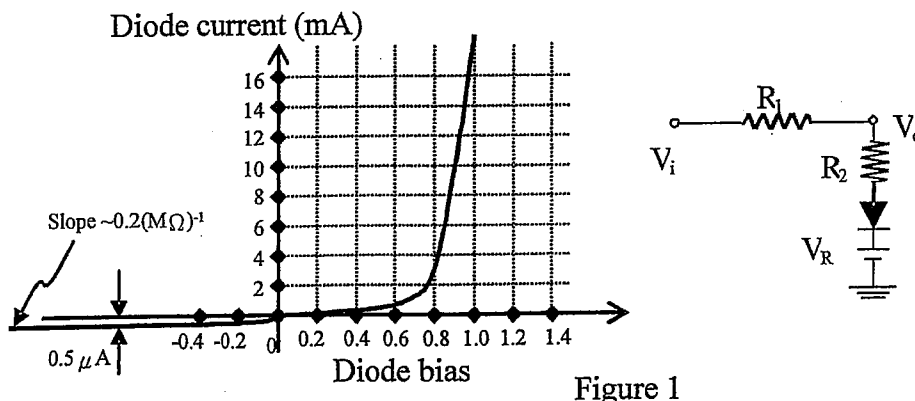


Figure 1

- (3) (a)(5%) In a Bipolar Junction Transistor (BJT), what are the common-base forward short-circuit current gain (α_F) and common-emitter forward short-circuit current gain (β_F). And derive their relation between α_F and β_F using Ebers-Moll model.
- (b)(5%) Draw and explain the equivalent circuit model of BJT at low-frequency and high-frequency regimes.
- (c)(5%) Define and explain the threshold voltages (V_T) of JFET (Junction Field-Effect Transistor) and MOSFET (Metal-Oxide-Semiconductor Field-Effect Transistor). And state their operational principles (you should include the Ohmic region and Saturation region).

- (4) Figure 2 shows a circuit based on a JFET which works as an amplifier. Also, the I-V characteristic of JFET is shown in the right. The circuit has $V_{DD}=30V$, $R_D=2k\Omega$, $V_{GG}=-1.5V$ and $R_G=20k\Omega$, $V_s(t)=0.2 \times \sin(\omega \cdot t)$.
- (5%) Define and explain the equivalent circuit of JFET and plot the overall circuit. What is the main factor leading to a finite output resistance (r_{ds})?
 - (10%) Please define and find the transconductance (g_m) and r_{ds} of JFET, if $V_{GG}=-0.5V$, $-1.5V$, and $-2.5V$. Are g_m and r_{ds} linearly related with V_{GG} ? Why?
 - (5%) Find the voltage gain of the circuit when $V_{GG}=-1.5V$.
 - (10%) Define the high-frequency equivalent circuit (you should explain all the parameters you assume). And derive the equation for the frequency response of the voltage gain in terms of the parameters you set.

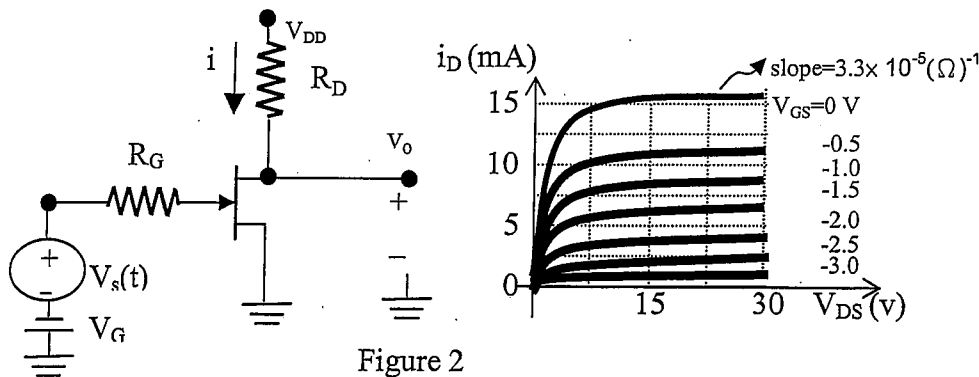


Figure 2

- (5) Figure 3 shows a logic circuit which includes two identical enhancement-mode NMOS-FETs and one depletion-mode NMOS-FET. The input ports are A and B. The output port is defined as C.
- (5%) What is the function of this circuit? Please verify it by truth table.
 - (5%) What is the function of the enhancement-mode NMOS-FET? Can it be replaced by a resistor?
- (6) Figure 4 shows a circuit with an ideal OP amplifier.
- (5%) Please define an ideal OP amplifier.
 - (5%) Find the relation for V_o and V_i ($i=1,2,3,\dots,n$) using the parameters of R , R_1 , R_2 , ... and R_n . If $R=R_i$ ($i=1,2,3,\dots,n$), then what will be the result?
 - (5%) If the electrical polarity connecting with the OP amplifier is reversed, the results of (b) will be the same? Why?

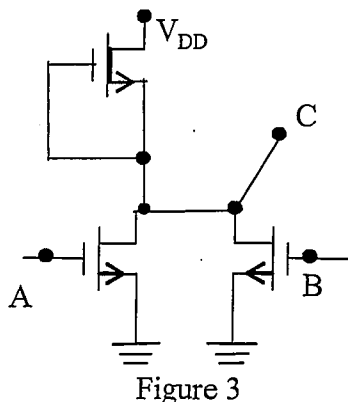


Figure 3

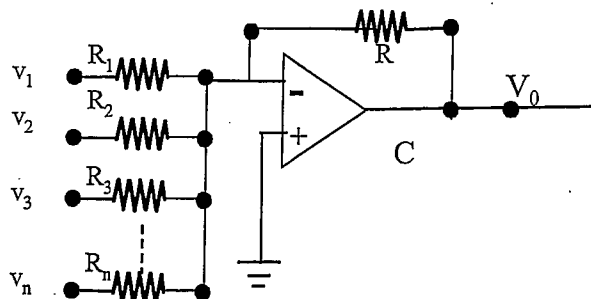


Figure 4

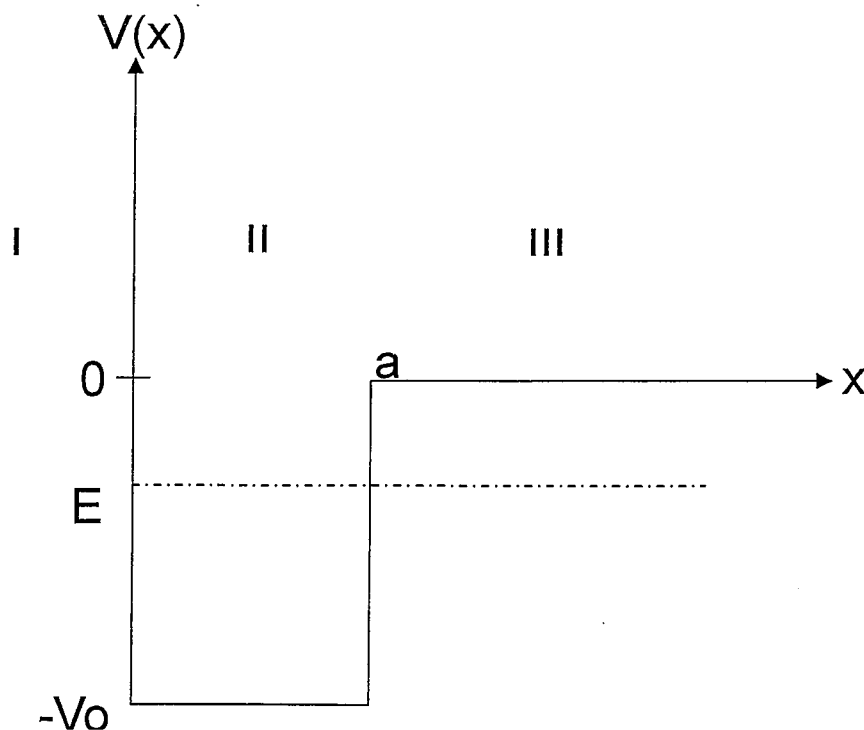
簡答題(40 分)

(請勿做過長回答)

- (共 7.5 分) 請問你知道 2009 年三位諾貝爾物理學獎得主中唯一的華人是誰? (2.5 分) 他得獎的主題與貢獻是什麼? (5 分)
- (共 15 分) 在近代發展的量子物理中, 粒子與波的觀念皆可以描述原子內部電子的行為, 請寫出 Schrodinger 波動方程式, (5 分) 並說明特徵解的物理義含(5 分), 所解出的波函數有何意義 (5 分)?
- (共 7.5 分) 何謂穿隧效應 (Tunneling Effect) (5 分)? 它是用粒子或是波動的概念所能夠推衍出來的 (2.5 分)?
- (共 10 分) 何謂測不準原理 (5 分)? 它所對應到的最小數量級是 10^{-N} , $N=?$ (5 分)

計算題(60 分)

- (共 25 分) 解特徵能量: (a) 請將 I, II, III 區域合理的波函數表示出來, 分為 $x < 0$, $0 < x < a$, $x > a$ (5 分) (b) 解 Schrodinger 方程式, 並考慮邊界條件, 討論有最低特徵能量解的條件為何(10 分)? (c) 若此系統最多有 n 個特徵解, 請寫出 n 與 V_0 的關係 (5 分) (d) 請說明為何 V_0 較大時, 系統能允許的特徵能量數目 n 比較高(5 分)?



2. (共 15 分) 簡諧振子 (harmonic oscillator) 的 Hamiltonian 可寫為:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2, \text{ (a) 若將其中 } \hat{p} = \frac{\hat{P}}{\sqrt{m\hbar\omega}}, \hat{q} = \hat{x}\sqrt{\frac{m\omega}{\hbar}}, \text{ 請重新用 } \hat{p}, \hat{q}$$

來表示上述的 Hamiltonian (5 分) (b) 引入 $\hat{a} = \frac{1}{\sqrt{2}}(\hat{q} + i^* \hat{p})$,

$$\hat{a}^+ = \frac{1}{\sqrt{2}}(\hat{q} - i^* \hat{p}), \text{ 請證明 } \hat{H} = \hbar\omega(\hat{a}^+ \hat{a} + \frac{1}{2}) \text{ (10 分)}$$

3. (共 20 分) (a) 若角動量算符 $\hat{L} = \hat{r} \times \hat{p} = \hat{L}_x + \hat{L}_y + \hat{L}_z$, 請寫出 L^2, L_x, L_y, L_z

之間 commutation 關係 (5 分) (b) 若 ψ_{lm} 是 L^2 及 L_z 的本徵解, 且

$$\hat{L}^2|\psi_{lm}\rangle = l(l+1)\hbar^2|\psi_{lm}\rangle, \hat{L}_z|\psi_{lm}\rangle = m\hbar|\psi_{lm}\rangle \text{ 若定義 } L_{\pm} = L_x \pm i^* L_y, \text{ 請利用}$$

$$\hat{L}_z(\hat{L}_{\pm}|l, m\rangle) \text{ 及 } \hat{L}^2(\hat{L}_{\pm}|l, m\rangle) \text{ 來證明: } \hat{L}_{\pm}|l, m\rangle = C_{lm}^{\pm}|l, m \pm 1\rangle \text{ (5 分), 並說明}$$

L_{\pm} 與 L^2 commute, 但不與 L_z commute (5 分) (c) 請說明若兩個算符互相 commute, 其物理上的意義 (5 分)