科目：工程數學【光電所碩士班】

(答案請務必精準及簡潔,並列出演算的重要步驟,最後答案不需是數字但必需是最簡化的結果)

1. (*a*) $z^5 = -32$, 求出所有 z 值 。(5%)
   (*b*) $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & -1 & 2 \end{pmatrix}$, 求矩陣 A 的所有特徵值及所有特徵向量。(5%)

2. 下列脈沖函數 f(t) *(a)*求出它的 Fourier Transform, F(ω), *(b)* F(ω)會趨近什麼, 當ω→0, *(c)* ω會等於什麼(全部可能解答), 當 F(ω)=0, *(d)* F(ω)會趨近什麼, 當 T→∞, 及*(e)* 如何判斷此 f(t)可作有效 Fourier Transform。 $\mathbf{f(t)=1}$, 當 $-\frac{T}{2} < t < \frac{T}{2}$; $\mathbf{f(t)=0}$, 當 $\mathbf{t}$ = 其它值。(10%, 每一子題 2%)

3. 積分 $\frac{\tan z}{z^2-1}$, 反時間方向沿著一圓形(circle), $|z| = 3/2$ 。(10%)

4. 如果 $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $r = \sqrt[2]{x^2+y^2+z^2}$, *(a)*$\nabla r$ =?, *(b)*$\nabla X\vec{r}$ =?, *(c)* $\nabla \cdot \vec{r}$ =?, 及*(d)* 簡略解釋 ∇, ∇X, ∇· 三個運算所代表的涵意。(10%, 每一子題 2.5%)

5. 證明 $\cos(\theta) + \cos(\theta+\alpha) + \cdots + \cos(\theta + n\alpha) = \frac{\sin\frac{1}{2}(n+1)\alpha}{\sin\frac{1}{2}\alpha} \cos\left(\theta + \frac{1}{2}n\alpha\right)$。(10%)

6. 解下列方程式 $2y\frac{d^3y}{dx^3} + 2\left(y + 3\frac{dy}{dx}\right)\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = \sin x$。(10%)

7. 用<u>兩種</u>不同方法導出下列公式: $L(\cos\omega t) = \frac{s}{s^2+\omega^2}$, $L(\sin\omega t) = \frac{\omega}{s^2+\omega^2}$, L 是 Laplace Transform。(10%, 每一子題 5%)

8. *(a)*<u>解出</u>及*(b)*<u>大略劃出</u>下列阻尼的(damped) mass-spring 系統, 在 t=1 時, 一個 unit impulse $(\delta(t-1))$脈衝的作用下, 所產生振幅對時間(y(t))的變化。 $\mathbf{y'' + 3y' + 2y = \delta(t-1)}$
   (提示:利用 $L(\delta(t-1))$, $L(f')$及 $L(f'')$公式, $\delta(t-1)$是 Dirac Delta Function, f', f''是微分函數, L 是 Laplace Transform)。(10%, *(a)*子題 8%, *(b)*子題 2%)

9. 質量是 m 的自由粒子之 Schrodinger 是 $i\hbar\frac{\partial\varphi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial x^2}$, *(a)*請證明波函數(φ)經過
   $\phi \to \phi' = e^{i\alpha}\phi$ (α是任何實數) 的轉換後具有不變(invariant), 也就是守恆(conservation)的特性,
   (b) $\int_{-\infty}^{\infty} |\varphi(x,t)|^2 dx$是和時間無關。(10%, 每一子題 5%)

10. 請舉一個例子, 解釋利用微分方程式的觀念與方法在科學上的應用。(任何例子(idea)都可以。分數是根據妳(你)所列例子的創造性, 微分方程式的完整性及解此方程式的邏輯而定)。(10%)

科目：電磁學【光電所碩士班】

1. (30%) (a)Write down and explain the time-dependent and time-harmonic (sinusoidal steady-state) Maxwell's equation in differential form.
(b)Derive and explain the physical meaning of Poynting vector.
(c)The electric feld component of an electromagnetic wave is represented by

$$\overline{E}_{(z,t)} = \hat{y}0.02\cos(7.5\times10^{8}t - \beta z) \quad \text{V-m}^{-1}$$

Find the corresponding wave magnetic field $\overline{H}_{(z,t)}$ and the constant β

2. (20%) Two long coaxial cylindrical metal tubes (inner radius *a*, outer radius *b*) stand vertically in a tank of dielectric oil (susceptibility χ_e, mass density ρ) as shown in Fig. 1. The inner one is maintained at potential V, and the outer one is grounded. To what height (*h*) does the oil rise in the space between the tubes.

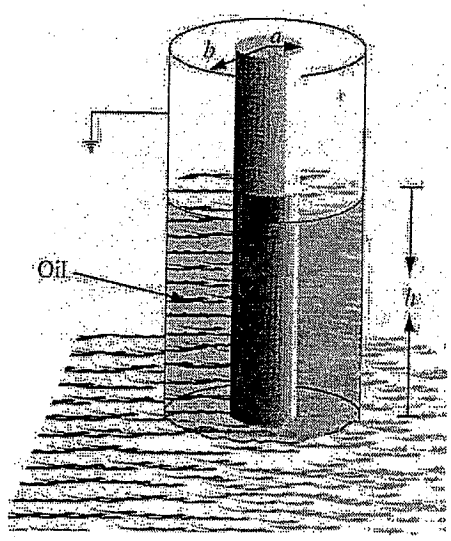

Figure 1.

3. (10%) An air-filled parallel-plate waveguide has a plate separation of 1.25cm. Find the cutoff frequencies of the TM_0 and TE_1 modes and the phase velocities of TM_0 and TE_1 modes at 15GHz.

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4. (20%) Determine the force between two coaxial circular coils of radii b_1 and b_2 separated by a distance d that is much larger than the radii ($d>>b_1, b_2$) as in Fig. 2. The coils consist of N_1 and N_2 closely wound turns and carry currents I_1 and I_2, respectively.

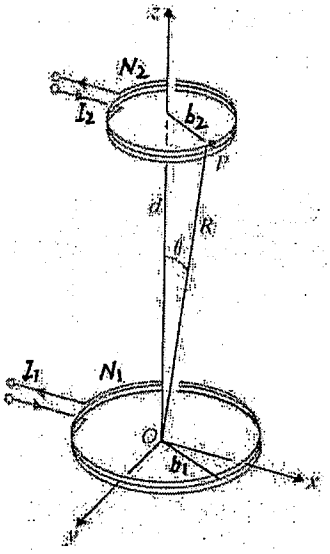

Figure 2.

5. (20%) (a)Derive the reflection coefficient of electric feld ($\Gamma = E_r / E_i$) for obilque incidence TE and TM waves. (Figure 3)
(b) Please explain the meaning of critical angle (the total internal reflection) and Brewster angle.

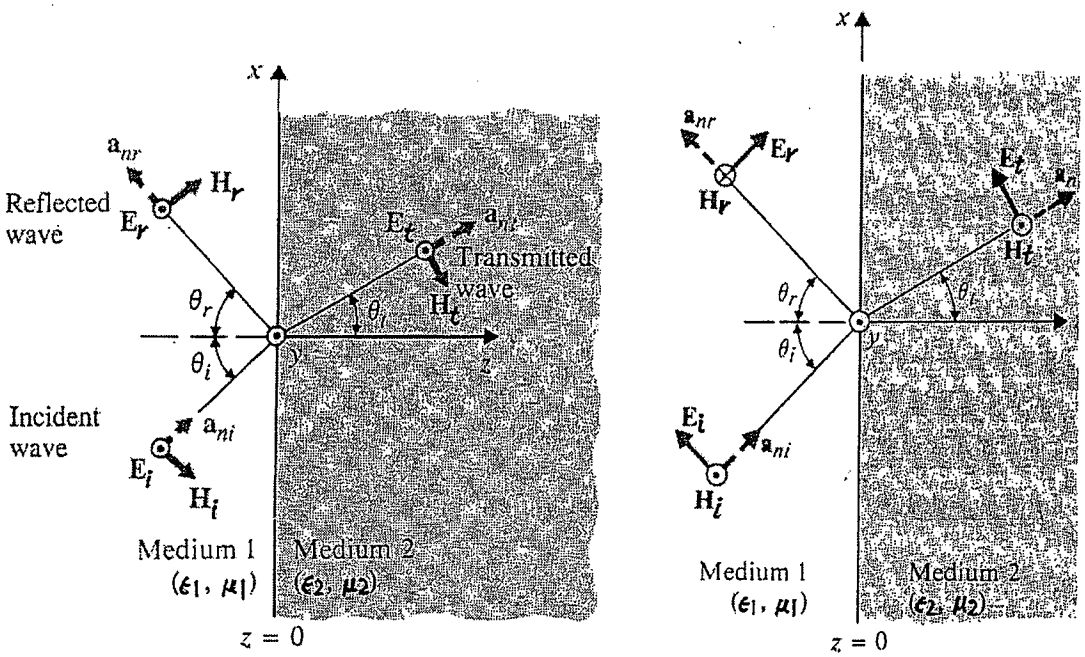

Figure 3.

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(1) (a)What is the difference between drift current and diffusion current in semiconductor? (6%)

(b)Why does an intrinsic semiconductor behave like an insulator at 0^0K? And at very high temperatures, why does it behave like a conductor? (6%)

(c)In a pn diode, please explain why a voltage drop is formed in the junction. And why does the maximum electric field occur in the junction? Why its current-voltage (I-V) relation is quite different in bias polarity? (6%)

(d)What is the operation principle of depletion mode and enhancement mode in n-MOSFETs? Please plot their cross sections in Si wafer. How does the channel length affect the drain current with applied drain-source voltage? (6%)

(e)Please explain the function of a BJT operating in active mode and write down Ebers-Moll equation with its circuit model for an npn BJT. (6%)

(2) Figure 1 shows a BJT inverter and the characteristic of BJT, where R_L=1kΩ, R_s=100kΩ. If the input waveform is Vs(t) ,as shown, please find and plot the output waveform Vo with time (from 0ms to 10ms). (15%)

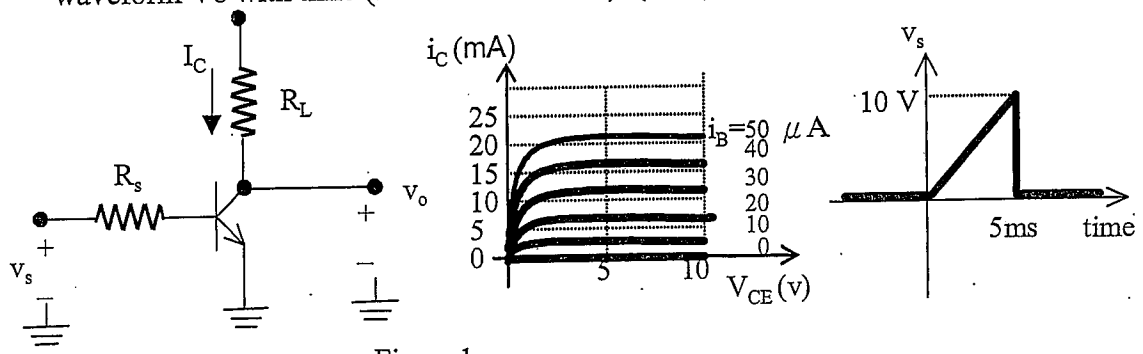

Figure 1

(3) As shown in figure 2, a logic gate defined by CMOS circuit. Please find and explain the logic function of output C with input A and B and write down its truth table. (10%)

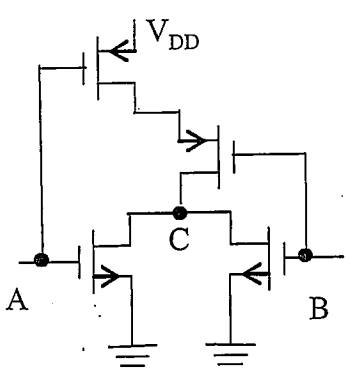

Figure 2

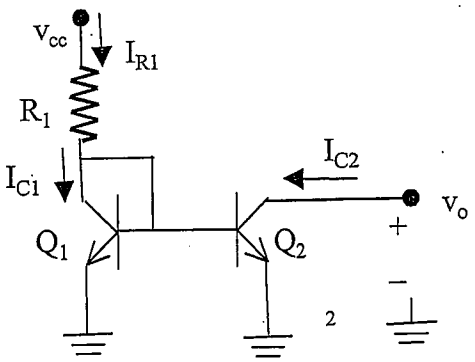

Figure 3

(4)A current source formed by two identical transistors is plotted in figure 3. (15%)

(a)please explain its operation principle by deriving the equation of I_{C2}, I_R in terms of V_{CC}, β, and R_1. (9%)

(b)In order to get a low level current source (I_{C2}), the general method is the so-called Wilder current source, i.e. by adding a resistor (R_2) following the emitter of Q_2. Please explain the function of R_2. (6%)

(5) A common-source amplifier with source resistance is shown in figure 4. (15%)

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(a)Please plot small-signal equivalent circuit model at low-frequency and high-frequency. (5%)

(b)Please derive the voltage gain at low-frequency. (You could assume s=jω=0.) (5%)

(c) Please derive the voltage gain at high-frequency if R_S=0. (5%)

(6)A zero-crossing detector is shown in figure 5. (15%)

(a)Please express its operation function. If an input waveform is Vi= $A \cdot \sin(2 \cdot \pi \cdot f \cdot t)$, where f=1/T. Please find and plot the output waveform of V_o(t), V_1(t), and V_L(t) by the parameters shown in figure.(9%)

(b)Continue (a), what is the pulse width of V_L? You could use full-width-half-maximum (FWHM) to define pulse width. (6%)

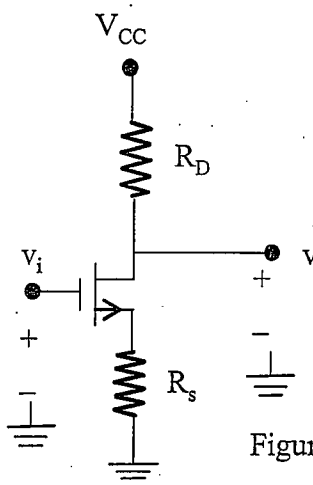

Figure 4

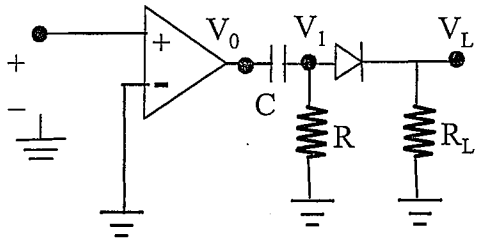

Figure 5

科目：近代物理【光電所碩士班選考】√

請在答案卷標明題號並務必照題目的順序作答，尚未回答的問題請標明題號並預留空白。

1. The following problems are on Max Planck's theory on black body radiation.
   a) List at least two critical assumptions made by Planck and discuss the significance of Max Planck theory on its role in modern physics. (10%)
   b) Outline steps in arriving for the black body radiation spectrum:

$$R(\lambda) = C\frac{h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1},$$

   where k is the Boltzmann's constant and h is the Planck's constant and T is the absolute temperature in Kelvin. (15%)

2. List the assumptions that Einstein made in his special theory on relativity and discuss the relation between the theory and the Lorentz transformation. (15%)

3. List important assumptions in Quantum mechanics and discuss the experimental proofs of some of your assumptions. (15%)

4. Quantum optics puts a theoretical limit on the maximum achievable bandwidth of an optical fiber. It requires that at least ten photons are needed to resolve a bit (weather it is a zero or a one). Given that, please estimate the maximum communication bandwidth that a single optical fiber (at $\lambda = 1.3\mu m$) can provide assuming that 1 mili-watt of power arriving at the receiving end of the fiber. (Hint: Planck's constant $h = 6.63\times10^{-34}$ Joule·sec). (15%)

5. Regarding the wave function of a hydrogen atom, please answer the following questions:
   a) Write down Schrödinger equation in the spherical coordinate system. (10%)
   b) The solution of (a) can be obtained via separation of variables. Give the names and associated properties of the three 1-D functions. (10%)
   c) State the physical significances of the principle quantum number and the angular quantum number appear in the combined solution of (a) and (b). (10%)