

1. $A = \begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$ (a) Is the matrix A Hermitian, skew-Hermitian or unitary?

(b) Please find its eigenvalues & eigenvectors. (10%)

2. Please find the inverse of the given linear transformation: (10%)

$$x^* = 19x + 2y - 9z$$

$$y^* = -4x - y + 2z$$

$$z^* = -2x + z$$

3. Please find the inverse Laplace transform of the function $\frac{3s+4}{s^2+4s+5}$ (10%)

4. Please evaluate the real integral: $\int_{-\infty}^{\infty} \frac{\cos x}{x^4+1} dx$ (10%)

5. Find the Fourier transform of the function $f(x)$. (15%)

$$f(x) = e^{-ax^2}, \text{ where } a > 0$$

6. Please solve the initial value problem:

$$x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0, y(1) = 2, y'(1) = 1, y''(1) = -4 \quad (y^{(n)} \equiv \frac{d^n y}{dx^n}) \quad (15\%)$$

7. Please evaluate $\oint_C \frac{2z^3 + z^2 + 4}{z^4 + 4z^2} dz$, C the circle $|z-2|=4$, clockwise. (15%)

8. Please solve $x \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = xt$, $u(x,0) = 0$ if $x \geq 0$, $u(0,t) = 0$ if $t \geq 0$. (15%)

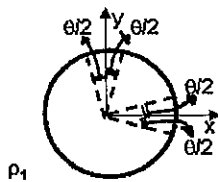
國立中山大學光電工程研究所

2005 年研究所入學考試

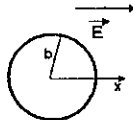
電磁學

一 填充題(每格四分，共十格)40%

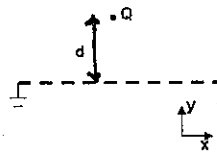
1. 在真空中有數段均勻帶電的導線，其線電荷密度為 ρ_l 。導線圍成圖型如圖一(沿半徑 a 之圓)。圓心處 O 處電場強度為 (E_x, E_y) ，則 E_x, E_y 分別為何? 1, 2
2. 兩個半徑為 $r=a$ 及 $r=b(b>a)$ 之無限長之同軸圓柱面上，分別帶有面電荷密度 ρ_{sa} 及 ρ_{sb} 。請問為了使在 $r>b$ 之區域中的 E 為零，則 a 與 b 之關係為何? 3
3. 見圖二，簡單的古典電子模型包含了帶 Ne 正電的原子核，該原子核被帶相同電荷的球型電子雲所環繞。外加電場 E_0 ，可使的電子雲內中心的電子核產生位移 r_0 ，也就是該原子被極化了。假設該原子雲內的電荷分布均勻且其半徑為 b ，試求 r_0 。 4
4. 見圖三，距一大之接地導電板上 d 處有一點電荷 Q 。求導電板上的面電荷密度 ρ_s 。 5
5. 一長 a 寬為 b 之長方形導電迴路(如圖四)，承重物質量為 m ，陰影處為一均勻磁場 B ，迴路處於該均勻且方向垂直於電流 I 方向之磁場 B ， B 之方向為指向朝內(垂直於重力常數 g 之方向)，多大之電流可平衡以支撐住重物。 6
6. 見圖五，假設導線中電流密度正比於所在位置與軸心距離($J=kr$; J 為電流密度， r 為距離， k 為比例常數)，試求體電流 I 。 7
7. 見圖六，赫姆霍茲線圈:有兩相同同軸線圈，半徑為 a ，間距為 b ，同時在每條線圈上有 M 匝繞線，如圖，每條線圈中電流方向相同。請問位於兩線圈之間格距離中點的磁通密度 $B=?$ 8
8. 有一具均勻磁性物質之圓球，其磁化向量大小 M 方向平行 Z 軸(如圖七)，請問圓球內磁通密度 $B=?$ 9
9. 見圖八，一長空心螺管每單位長度上有 n 匝線圈，螺線管橫截面積為 S ，電流為 I 。試求單位長度的電感。 10



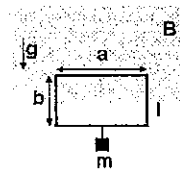
圖一



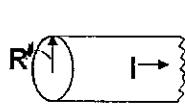
圖二



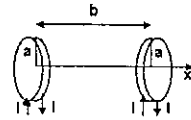
圖三



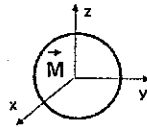
圖四



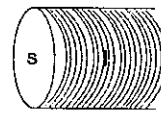
圖五



圖六



圖七



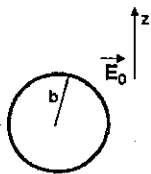
圖八

二 簡答與說明題(20%)

- (a)何謂肌膚深度?(4%) ;金、銀、銅等三金屬其 σ (S/m)分別為 4.1×10^7 , 6.17×10^7 , 5.8×10^7 (b)請問其對同一頻率電磁波之肌膚深度大小觀係為何? (4%) ; (c)試說明何以鍍金、鍍銀與鍍銅鏡何以顏色看來不同?(4%)
- (a)試說明微波爐加熱原理(4%),為何金屬放入微波爐會爆炸?(4%)

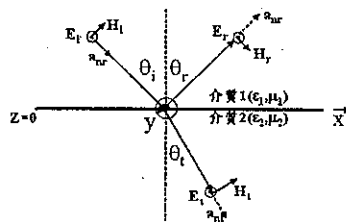
三 計算題(40%)

- 見圖九,於一起初均勻的電場(大小為 E_0 , 方向為正 Z 方向)中,有一長且接地的導電圓柱體,其半徑為 b ,位於 Z 軸上。試求圓柱體外的電位分布 $V(r, \phi)$ 20%

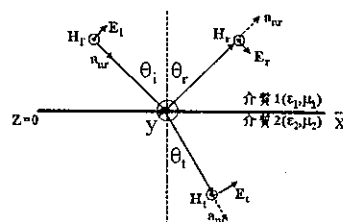


圖九

- 如圖十,試分別推導 TE 及 TM waves 所對應之 Fresnel's equation(描述電場振幅之反射與折射率之關係式),並說明只有何種偏極才有零反射之布魯斯特角(Brewster angle)(20%)



TE waves



TM waves

圖十

Modern Physics 2005

1. Write down the name(s), in English, associated with the following major events in modern physics and the year (full credit if within 5 years) of occurrence. (5% each) (25%)
 - (a). Special theory of relativity.
 - (b). First stable atomic structure model.
 - (c). Wave mechanics (quantum mechanics) for the electrons in matters.
 - (d). Exclusive principle for electrons in an atom.
 - (e). Big-bang theory for the origin of the universe.

2.
 - (a). Describe the Max Planck's theory of black body radiation and its importance in revolution of modern physics. (10%)
 - (b). Derive the equation for the black body radiation distribution. You must clearly state all the assumptions needed for this equation. (15%)

3.
 - (a). Compute the photon flux density inside the core of a single mode step-indexed optical fiber carrying 1 milli-watt of power operating at $1.3 \mu\text{m}$. For this calculation you need to estimate, from EM waveguide theory, the core diameter. Please state clearly all the necessary assumptions. (10%)
 - (b). At extremely low power level, continuous EM theory is insufficient and the particle-like photon characteristics become apparent. Estimate the maximum theoretical bandwidth capacity (Giga-bit/second) of this milli-watt optical fiber assuming that ten photons are needed to carry one single bit information. (15%)

4.
 - (a). Write down the time harmonic Schrodinger equation in spherical coordinate for the single-electron hydrogen atom. (10%)
 - (b). Outline the mathematical procedure for solving this equation and list the special functions used in the solutions and discuss their properties. (15%)

Planck's constant $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$

Electron's mass $m_e = 9.11 \times 10^{-31} \text{ kg}$

Speed of light $c = 3.0 \times 10^8 \text{ M/s}$

電子學(選考) 光電工程研究所碩士班

(20points)

(1) A pn junction with a bias voltage V is shown in figure 1. (20 points)

(a) Please draw the schematic current-voltage (I - V) relation in forward- and reverse-biased PN junction. You should indicate voltage polarization, the turn voltage, the breakdown voltage, dark current level in the plot.

(b) Using the plot in (a), draw and explain the equivalent circuit models for forward- and reverse- biases.

(c) In figure 1, please draw schematic diagram for charge density, electric field density, and electrostatic potential with position x .

(d) Define depletion capacitance and diffusion capacitance of a PN junction. As in reverse bias, is there any diffusion capacitance effect in the junction? why?

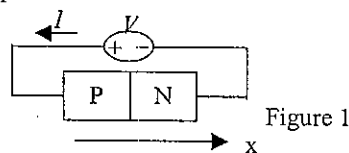


Figure 1

(15points)

(2) (a) Explain and compare the functions of BJT and FET.

(b) State the Early effect.

(c) Explain the function of charge-couple-device (CCD).

(15points)

(3) (a) Plot the circuit diagram and cross sectional view of a CMOS inverter and state its operation principle.

(b) Explain that the CMOS inverters have advantages over NMOS and PMOS inverters.

(30points)

(4) A discrete BJT common-emitter (CE) amplifier stage is shown in figure 2.

(a) Explain the effects for all the capacitances.

(b) Draw the large-signal D.C. equivalent circuits.

(c) If $V_c=30$ V, $R_c=7$ k Ω , $R_1=90$ k Ω , $R_2=10$ k Ω , $R_E=1$ k Ω , $\beta=100$, then calculate the operation point, i.e. the I_c and V_{ce} of BJT. (You could assume $I_{c0}\sim 0$)

(d) At the low frequency, draw the small signal A.C. model using hybrid- π model and derive the expression for voltage gain (V_o/V_s). You must define all the parameters in BJT.

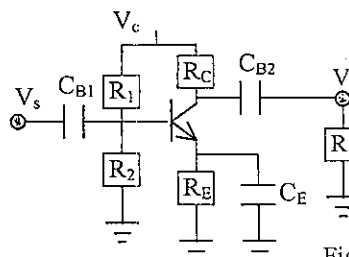


Figure 2

(20 points)

(5) OP-Amp is shown in figure 3.

(a) Define an ideal operational amplifier (Op-Amp).

(b) For an ideal noninverting OP-Amp, what is the voltage gain ($A=v_o/v_i$) in terms of R_1 , R_2 and R_3 ? (You should state your reasons).

(c) If the OP-Amp is not an ideal one, please find the output voltage gain in terms of R_1 , R_2 , R_3 , A_v (the voltage gain of Op-Amp), R_i (input resistance of Op-Amp) and R_o (output resistance of Op-Amp). And explain why the results will approach to the results of (b) in an ideal case.

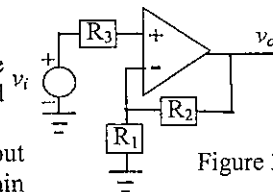


Figure 3