光電所工程數學

May 4 2003

1. Solve the following differential equation: (15%)

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0, \quad \text{boundary conditions:} \quad y(0)=1, y(1)=0.$$

2. Show that eigenvales (λ) of a real symmetric matrix **A** are real and that the eigenvectors ($\vec{x}$) are orthogonal to each other. (15%)

$$\mathrm{A}\vec{x}_i=\lambda_i\vec{x}_i, \quad \vec{x}_i^T\vec{x}_j=0, \quad i\neq j.$$

3. Using the fact that: (15 %)

$\int_0^\infty e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$, perform the definite integral $\int_{-\infty}^{\infty} e^{-ax^2}\cos bx\, dx$.

4. Compute the complex refracted angle θ_2 under total reflection. This occurs when light enters from water (with high index n_1) to air (with low index $n_2=1$) with an incident angle θ_1 greater than the critical angle $\theta_c=\sin^{-1}(\frac{n_1}{n_2})$. You must express this angle both the real and imaginary parts in a closed form with hyperbolic functions (15%) (Hint: Apply the Snell law $n_1\sin\theta_1=n_2\sin\theta_2$)

5. a. State the Divergence Theorem and Stokes Theorem. (10%)
   b. Apply both theorems above show that: (10%)

$$\nabla\cdot(\nabla\times\vec{F})\equiv 0$$

Here $\vec{F}$ is a vector function of 3 spatial variables.

6. Find the first two eigen-functions for the 2-D Helmholtz equation satisfying Neumann boundary conditions. (Hint: apply the technique of separation of variables.) Please denote Bessel function as $J_n(z)$ and use $\chi_{n,i}$ as the i^{th} root of the derivative of the Bessel function: i.e. $J_n'(\chi_{n,i})=0$ (20%)

$$\left(\nabla^2+k^2\right)u(r,\theta)=0, \ r\in(0,1), \quad \theta\in(0,\frac{\pi}{2}),$$

$$\frac{\partial u(r,\theta)}{\partial n}=0 \quad \text{on the boundary, } \bar{n} \text{ is the normal vector.}$$

National Sun Yat-sen University Institute of Electro-Optical Engineering
2003 Entrance Examination for Engineering Electromagnetics

[*30points*]

1. (a)Write down the Maxwell equations and name all the vector and scalar quantities in the equation.
   (b)In the vacuum and source-free medium (no charge and current), derive the vector wave equations.
   (c)In the problem (b), prove that the solutions (in scalar) for all the plane-waves in x-direction can be written as forms of **f(x - v · t)**, **f(x + v · t)**, or their linear combination.
   (d)Continue the problem (b) and (c), find the **v** in terms of some scalar quantities given in problem (a) and explain what are the meanings and their difference for **f(x - v · t)** and **f(x + v · t)**.
   (e)Write down the definitions of time-harmonic solution and phasor. Based on the time-harmonic assumption, repeat the problem(b) and write down the general solutions for one-dimension vector wave equation in terms of wave number (**k**) and angular frequency(ω).

[*20points*]

2. Consider two large parallel conducting plates of area A are separated by a distance d and a voltage drops V_o is applied across these two plates. As shown in the figure1, two dielectric materials of thickness d_1 and d_2, permittivities ε_1 and ε_2 are fully filled between the plates. Answer the following questions,
   (a)If both dielectrics are lossless, write down the boundary conditions and determine the electric fields in both dielectrics.
   (b)If both dielectrics are lossy and their conductivity are σ_1 and σ_2 respectively, write down the boundary conditions and determine the electric fields and current density in both dielectrics. And state what is the difference between lossy and lossless cases? And explain that.
   (c)Write down the equivalent circuit for lossy dielectrics.
   (d)if $\sigma_1 \gg \sigma_2$, what do the equivalent circuit approach ? And explain that.

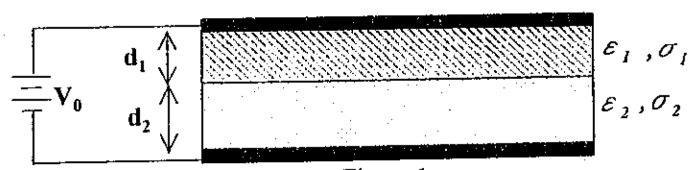


Figure 1

[*10points*]

3. Derive the electric-fields and plot the schematic equal-potential and electric-field lines for an electric dipole $\mathbf{Q} \cdot \mathbf{d}\hat{z}$, where **d** the distance between two charges +**Q** and -**Q**. You must state each step clearly (You may use the potential V to calculate the electric field).

[*25points*]

4. For a coaxial cable, as shown in figure2, the radius for inner and outer conductors is **"a"** and **"b"**. The space between the conductors is filled with air.
   (a) Explain briefly what are the reasons the coaxial cable can transmit electrical signal.
   (b) If the cable is lossless and we assume the current is uniformly distributed inside the inner conductor. Derive briefly the microwave wave propagation constant and characteristic impedance. You may be able to use the transmission line theory.
   (c) Since the signal is carrier by time-varying electrical and magnetic fields, could we say that the current is uniformly distributed inside the inner conductor, why?
   (d) Once the conductor is not perfect (finite conductivity), what causes the microwave propagation loss? And if the frequency of the modulated electrical signal increases, will the propagation-loss decrease or increase? Explain the answers you give.

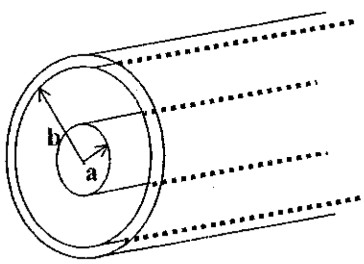


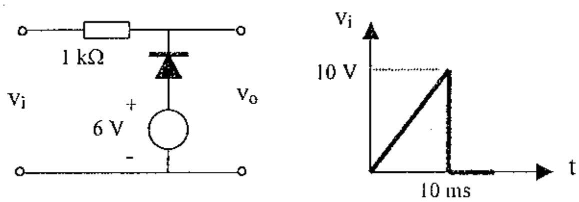
Figure 2

[*15points*]

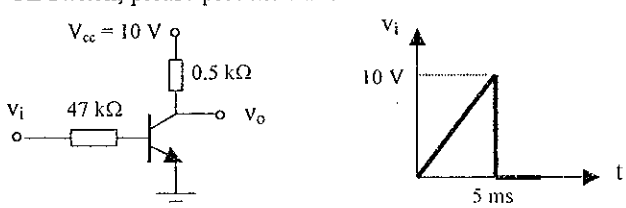
5. Briefly answer the following questions with reasons :
   (a) What is the Poynting's vector and its physical meaning ?
   (b) Do the magnetic flux lines always form closed loops? Why?
   (c) What is the group velocity and the phase velocity? At what kind of situation they are the same?

Answer the following questions:

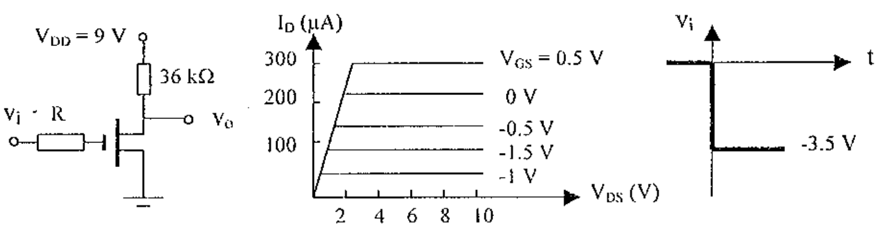
1. How many components are included in a VLSI chip? (6%)
2. What are the differences with regard to the operation principle between a BJT and a FET? (6%)
3. An n-type Si layer with a donor concentration of 5×10^{14} cm^{-3} is used as a conducting line on an IC. What is the conductivity of this sample? The intrinsic carrier density and mobility of the layer are 1.45×10^{10} cm^{-3} and 1500 $cm^2/V\cdot sec$. (8%)
4. Please draw the charge density, electric field intensity, and electrostatic potential for electrons in the depletion region of a pn junction. (8%)
5. For the given circuit, please find the output v_o as a function of input v_i. What is the breakpoint of this circuit? The cut-in voltage and forward resistance of the diode is 0.6 V and 10 Ω. (8%)



6. Please draw the structures of an IC npn transistor and an n-channel MOSFET including substrate, isolation island and contacts. (6%)
7. For the CE switch, please plot the transfer characteristics of the circuit. (10%)

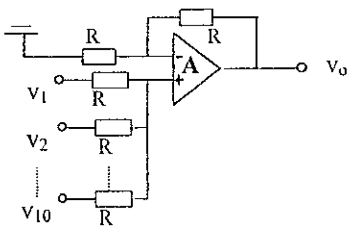


8. The characteristics of a MOSFET are given in the following figure. Please draw the output waveform of the circuit. (10%)



9. Please draw a logic diagram of a 5-bit shift register using SR Flip-Flops. (8%)

10. Please find the output v_o as a function of multiple input v_n (n = 1-10). (10%)

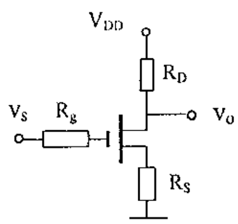


11. The return ratio of a two-pole amplifier is

$$T(s) = \frac{100}{(1+s/10^6)(1+s/10^7)}$$

Is this amplifier stable? Why? (10%)

12. Please draw the equivalent circuit without feedback of the following CS amplifier. What is the return ratio of this circuit? (10%)



I. 選擇題: (45 points) 15 points for each question

1. Consider an electron of speed 600 m/sec with an uncertainty of 0.01%. With what fundamental accuracy could we have located the position, if the position is measured simultaneously with the speed in the same experiment? (a) 0.1 (b) 1 (c) 10 (d) 100 (e) 1000 (f) 10000 cm

2. Consider an electron with a wavefunction of the following expression,

$$\psi = \begin{cases} \cos\left[\frac{\pi(x-a)}{a}\right]\exp\left(\frac{-iEt}{\hbar}\right) & , \frac{a}{2} < x < \frac{3a}{2} \\ 0 \;, & \text{elsewhere} \end{cases}$$

   The mean position ($\bar{x}$) of the electron is
   (a) 0 (b) $a/2$ (c) a (d) $a\pi/2$(e) $a\pi/2$ (f) $3a\pi/2$.

3. In hydrogen atom, a quantum state of l, s, j are 2, 1/2, and 5/2, respectively. What is its $\vec{S}\cdot\vec{L}$? (a) 1/8 (b) 1/4 (c) 3/8 (d) 1/2 (e) 5/8(f) 1 $\hbar^2$

II. 問答題: (55 points)

1. Consider a photon and an electron of the same energy, E, express the wavelength of each particle in terms of E and other fundamental quantities. (20 points)
2. In a three particle system, assume ψ_i^j is the i^{th} state of the j^{th} particle, where i, j= 1, 2, 3. Determine the form of the normalized anti-symmetric total eigenfunction, in which the interactions between the particles can be ignored. (20 points)
3. Illustrate the three transition processes of a two-energy-state atom in the presence of an electromagnetic field. (15 points)