## 科目:工程數學 (光電所碩士班)

共/頁第/3

1. Short questions (60 points) (10 points for each question)

(a) 
$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$$
, then  $\frac{3\Gamma(\frac{3}{4})}{7\Gamma(\frac{11}{4})} = ?$ 

(b) Evaluate the real and imaginary parts of  $\cot\left(\frac{\pi}{3} - i \ln 3\right)$ .

(c) 
$$\int_0^\infty \frac{\cos mx}{x^2 + 1} dx = ?$$
, for  $m > 0$ .

- (d) Explain Gauss's theorem.
- (e) Explain Stoke's theorem.
- (f) Express  $\nabla^2 V = 0$  in spherical coordinates.

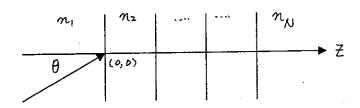
2. 
$$\frac{d^2y(x)}{d^2x} - 2\frac{d^2y(x)}{d^2x} + 2y(x) = e^{-x}, y(0) = y\left(\frac{\pi}{2}\right) = 0, \text{ find } y(x). (20 \text{ points})$$

3. Find the function f(x,y) satisfying the Laplace equation

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y} = 0, \text{ for } x^2 + y^2 = a, a > 0;$$

and the boundary condition  $f(x,y) = x^3$  for  $x^2 + y^2 = a$ . (20 points)

- 1. Write down the time-dependent Maxwell equations in both differential and integral forms. Also derive the continuity equation from the Maxwell equations. (20%)
- 2. A) Derive the one-dimensional vector wave equation for a plane wave in free space from two Maxwell equations and write the general solution of the one-dimensional wave equation (just the scalar case). If the plane wave is traveling in the z-direction, B) state clearly the field components for two set of independent solutions (20%)
- 3. Derive the magnetic field due to a current loop of radius a with a current I. Assume that the current loop is centered at the origin and lied entirely in the x-y plane. You may assume the observation points are far away from the origin. (Hint: using vector potential or Biot-Savart law) (20%)
- 4. Optical fibers are used to guide the EM waves in the visible and infrared frequency range. A) Describe the guiding principle of an optical fiber and explain why microwave waveguides or coaxial cables or parallel wires are not suitable to optical waves. Recently, UTP (unshielded twisted pair) cables and optical fibers are both used in gigabit local area network. B) Compare the differences, similarities and advantages/disadvantages of these two waveguides. (20%)
- 5. Multi-layered dielectric structures are often used in opto-electronic devices such as the DWDM filters, slab waveguides and optical coatings on glasses. Given that you have a N-layered dielectric structures (including the left and right two half spaces) shown below, an incident plane wave (assume time- harmonic case) is impeding on this structure, A) write down the functional forms of the total fields in each layer medium, and B) write down all interface conditions needed to solve this problem and C) Solve for the transmission coefficient for the case N=3. (20%)



科目: 椰子學 (光電所)(选考)

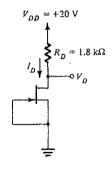
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#### I. 問答題: (40 points)

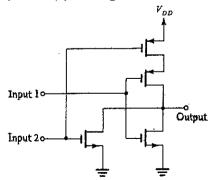
- 1. Explain what are the differences between Avalanche and Zener diodes? (20 points)
- 2. Consider an OP amplifier having 106-dB gain at DC and a single pole frequency response at 2 MHz. Estimate the magnitude of gain at 10 kHz? (20 points)

#### II. 選擇題: (60 points) 15 points for each question

1. What is the  $I_D$  of the circuit below ? Note: The  $V_p$  and  $I_{DSS}$  of the JFET are -4V and 10 mA, respectively. (a) 9.4 (b) 7.1 (c) 5.4 (d) 3.1 (e) 1.4 (f) 0.2 mA



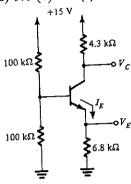
2. The CMOS logic circuit below can be served as an (a) AND (b) OR (c) NAND (d) NOR (e) XOR gate.



科目: 尼子學 (光電所)(选考)

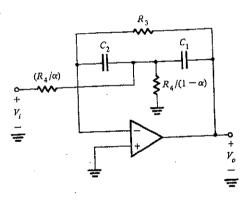
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3. For the circuit shown below, assume  $\beta$  to be very high. What is  $I_E$ ? (a) 2.5 (b) 2.0 (c) 1.5 (d) 1.0 (e) 0.5 (f) 0.2 mA



4. What can the circuit below used for? (a) low-pass filter (b) high-pass filter (c) band-pass filter (d) notch filter (e) voltage follower (f) integrator

Note:  $0 < \alpha \le 1$ 



近代物理 (光霉的)(送号) 共1頁第1頁

#### 問答題: (40 points)

- 1. Explain the origin of continuum X-ray? (20 points)
- 2. What is spin-orbit interaction? (20 points)

#### II. 選擇題: (60 points) 15 points for each question

- 1. Consider a very small dust of radius r=1 µm, density  $\rho=10$  g/cm<sup>3</sup>, moving with a velocity v=1 cm/sec. What is its de Broglie wavelength? (a) 9.2 (b) 8.7 (c) 6.5 (d) 4.3 (e) 1.6 (f) 1.1 x10<sup>-18</sup> m
- 2. A boy on top of a ladder of height H is dropping marbles of mass mto the floor and is trying to hit a crack in the floor. To aim, he is using equipment of the highest possible precision. Show that, despite his great care, the marbles will miss the crack by an average distance of the order of (a)  $(m/\hbar)^{1/2}(H/g)^{1/4}$  (b)  $(\hbar/m)^{1/2}(H/g)^{1/4}$  (c)  $(\hbar/m)^{1/2}(g/H)^{1/4}$  (d)  $(m/\hbar)^{1/2}(g/H)^{1/4}$  (e)  $(\hbar/m)^{1/2}(g/H)^{1/2}$  (f)  $(m/\hbar)^{1/2}(g/H)^{1/2}$  where g is the acceleration due to gravity.
- 3. Consider a particle with mass m under the influence of an 1-D infinite square potential well as described below,  $V(x) = \begin{cases} 0 & , -d < x < d \\ \infty & , x < -d, x > d \end{cases}$ . What is its ground state energy? (a) 1/8 (b) 1/4 (c) 1/2 (d) 1 (e) 2 (f) 4  $\frac{\pi^2 \hbar^2}{md^2}$ .
- 4. The probability of finding a particle at energy E is  $\frac{1}{(E-E_0)^2+(h\Delta\upsilon/2)^2}, \text{ where } E_0=100 \text{ eV}, h\Delta\upsilon=0.1 \text{ eV}. \text{ Then,}$ what is its mean lifetime? (a)  $2.2 \times 10^{-15}$  (b)  $6.6 \times 10^{-15}$  (c)  $2.2 \times 10^{-15}$  $10^{-13}$  (d) 6.6 x  $10^{-13}$  (e) 2.2 x  $10^{-11}$  (f) 6.6 x  $10^{-11}$  second.