# 國立中山大學九十學年度碩博士班招生考試試題

科目：工程數學【光電所碩士班】



May 6, 2001

1. Explain the mathematical meanings and their consequences of the following subjects. (Please limit your answer to 75 words or less for each item) (20%)
   a. Sturn Liouville theorm.
   b. Riemann surface.
   c. Gibb's phenomenon.
   d. Stokes theorem.

In problems 2-5, you must show your work to get full credit.

2. Find the Fourier transform of the following function: (20%)

$$f(t) = e^{-i\pi t^2}, \quad t \in (-\infty, \infty),$$

where $i$ is the imaginary number.

3. a. Write down the differential equation that Bessel functions satisfy (Use $z$ as the independent variable and $\nu$ as the order variable.) (5%)
   b. Write down the mathematical symbols for Bessel functions of the first and the second kinds. Explain the differences in mathematical and physical meanings among these functions (10%)

4. Given the following system of differential equations:

$$\frac{dy_1}{dt} = -ib_1 y_1 - icy_2, \quad \frac{dy_2}{dt} = -ib_2 y_2 - icy_1,$$

where $i$ is the imaginary number, $b_1$, $b_2$ and $c$ are constants

   b. Find the two eigenfunctions and their associated eigenvalues. (15%)
   c. Solve for $y_1(t)$ and $y_2(t)$ if the initial conditions are $y_1(0) = y_{10}$, $y_2(0) = 0$. (10%)

5. a. Evaluate the expression: $i^i$ (5%)

   b. Solve for the Green's function satisfying given boundary conditions (15%) (Hint: Expand the Green's function in series of eigenfunctions satisfying the same boundary conditions)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right) G(x,y) = -\delta(x)\delta(y),$$

$$x \in (-1,1), \quad y \in (-1,1)$$

$$\frac{\partial G(x,y)}{\partial x} = \frac{\partial G(x,y)}{\partial y} = 0, \quad \text{for } x = \pm 1, \text{ or } y = \pm 1$$

**Institute of Electro-optical Engineering**
**2001 Entrance Examination of Electro-magnetics**

1. Please use $\nabla \times E = -\partial B/\partial t$ and $\nabla \times H = \partial D/\partial t$ to show that the tangential components of $E$ and $H$ fields are continuous across the boundary. Detailed descriptions including your drawings are required (10%).

2. A conducting sphere of radius R is surrounded by free space. Initially, a charge density of $\rho_o$ is distributed uniformly throughout the sphere. Please find the current density $J$ of the sphere at t = 0 and $t \to \infty$. The dielectric constant and conductivity of the sphere are $\varepsilon$ and $\sigma$, respectively (6%).

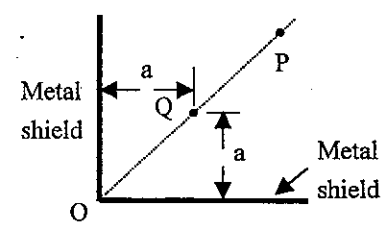
3. In free space, charges of magnitude Q are installed in a parallel-plate capacitor. The dimension of the parallel plate is a×b, and the spacing between the plates is d. Please find the energy stored in the capacitor (6%).

4. Consider an amount Q of positive charge is uniformly distributed on the surface of a conducting sphere of radius $r_1$. Given the Laplacian in spherical coordinates $\nabla^2\phi = \frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\phi}{dr})$, please determine the potential at every point in the free space surrounding the sphere (6%).

5. Consider a system with a point charge over a large conducting plane. Please draw the equipotential surfaces and $E$ lines of the system (6%).

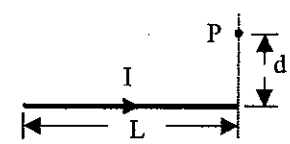
6. A long wire with a radius $r_0$ is carrying a constant current $I$ along the z direction. Please draw the $H$ field distribution versus radial distance in any plane perpendicular to the z axis (6%).

7. A charge q is placed at the point Q. Please determine the potential at the point P shown in the figure (10%). Note that $\overline{OQ} = \overline{PQ}$.




8. A circular polarized wave is incident from medium 1 into medium 2 at the Brewster angle. Please describe the polarization states of both the reflected wave in medium 1 and the transmitted wave in medium 2 (10%).

9. A straight-line segment of length L is carrying a current I. A point P is located at a vertical distance d to the line. Please find the magnetic field $B$ at the point P (10%).



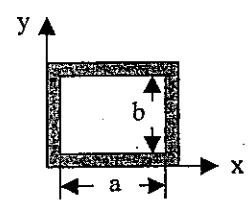

10. Consider an electron moving with a constant velocity $v$ along the z direction. At

the same time a long-wavelength EM wave is also propagating at the same direction. Please describe in detail the trajectory of the electron (8%).

11. What are the group velocity and phase velocity of an EM wave? Are they the same? Please explain your answer (6%).

12. The propagation constant of a transmission line is $\gamma = [(R + j\omega L)(G + j\omega C)]^{0.5}$, where $R$, $L$, $G$, and $C$ are the resistance, inductance, conductance and capacitance per unit length of the line. Please show that when $\frac{L}{R} = \frac{C}{G}$, the line is called *distortionless* line (6%).

13. A rectangular waveguide is shown in the following figure. The operation frequency of the waveguide is 5 GHz for $TE_{10}$ mode. For safety reasons the operation frequency should be 20% larger than the cutoff frequency ($f_c = \frac{c}{2a}$, where $c$ is the speed of light). Please fined the lower and upper bounds of $a$ (10%).



I. Short questions: 30 points (10 points each)

(i) What is the Early effect in bipolar junction transistor ?
(ii) What is Miller effect ?
(iii) What is common mode rejection ratio ?

II. Long questions: 70 points

(i) Write down the truth table of a digital full adder and design it with basic logic gates. (25 points)

(ii) Use an OP amplifier to design a N-channel analog signal adder. Write down its transfer function. (25 points)

(iii) Give reasonable values of $R_1$, $R_2$, $R_E$, and $R_c$ to complete the DC bias of the figure below. The power supply is 12 Volts and the emitter current, $I_E$, is 1 mA. What is the effect of $C_E$ ? (20 points)

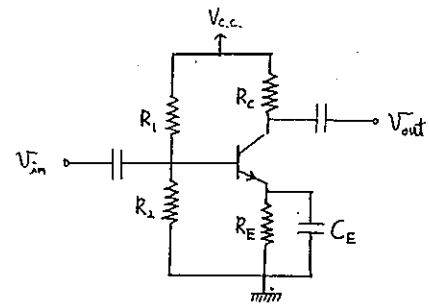



Figure 1

I. Short questions: 40 points (10 points each)

(i) A particle with wavefunction, $\Psi$, travels in the 3-D space. What is the quantum mechanical expectation value of its dynamical quantity, g ?

(ii) What is the 3-D quantum-mechanical Hamiltonian of one particle ?

(iii) What is Zeeman effect ?

(iv) What is Fermi probability distribution ?

II. Long questions: 60 points

(i) What is the maximum speed for which the classical expression $\frac{1}{2}mv^2$ will yield an error in the kinetic energy no greater than 1 % ? (18 points)

(ii) Consider a X-ray beam with $\lambda$=0.2 nm and also a $\gamma$–ray beam with $\lambda$=2 pm. If the radiation scattered from free electrons is viewed at 90 degrees to the incident beam,

1. What is the Compton wavelength shift in each case? (8 points)
2. What kinetic energy is given to a recoiling electron in each case? (8 points)
3. What percentage of the incident photon energy is lost in the collision in each case? (8 points)

(iii) Write down the Schroedinger's equation for a simple harmonic oscillator. And explain the procedures to solve it. (18 points)