國立中山大學八十九學年度碩博士班招生考試試題

科　目：工程數學　光電工程研究所碩士班　　

總分為100分

1. 試求 $x\frac{dy}{dx}-y=\frac{x^2}{y}$ 之通解. (15%)

2. 判斷下列級數之斂散性:

   (a) $\sum_{n=1}^{\infty}\frac{n!}{n^n}$ (5%), (b) $\sum_{n=1}^{\infty}\frac{3^n}{n^3}$ (5%)

3. 試解擴散方程 (20%)

$$\frac{\partial u}{\partial t}=9\frac{\partial^2 u}{\partial x^2}, x\in(-\infty,\infty), u(x,0)=\delta(x)$$

4. (a)試求 $f(x)=\begin{cases}a, |x|<a\\ 0, |x|>a\end{cases}$ 之 Fourier 轉換,

   a 爲正值之常數.(10%)

   (b) 試求 $\int_{-\infty}^{\infty}\left(\frac{\sin\ \omega a}{\omega a}\right)^2 d\omega\ =\ ?$ (10%)

5. 試求 $f(x)=|x|, -\frac{\pi}{2}<x<\frac{\pi}{2}$ 之 Fourier 級數. (20%)

6. 試求 $\oint_C \frac{z}{1+4z^2}dz$ 之值, C 爲單位圓. (15%)

Institute of Electro-Optical Engineering
2000 Entrance Examination of *Engineering Electro-magnetics*

Part I: Questions and Short Answers (60%).

In this session, each blank is worth 3%. You are not required to fill in any detail calculation. Just give the final result, or key point and brief description in the corresponding blank spaces sequentially in your answer sheet.

A. For a plane electromagnetic (EM) wave in medium 1 (with dielectric constant of ε_{r1}) being incident on a medium 2 (ε_{r2}), define the *Brewster angle:* $\theta_B =$ __(1)__, and when does it exist at an interface of two nonmagnetic media with $\mu_1 = \mu_2$? __(2)__. Determine *the Brewster angle:* $\theta_B =$ __(3)__, and the corresponding *angle of transmission:* $\theta_t =$ __(4)__ for an EM wave impinges from air on the surface of water, which has a dielectric constant of 80.

B. For a plane EM wave in medium 1 (with dielectric constant of ε_{r1}) being incident on a medium 2 (ε_{r2}), define the *critical angle:* $\theta_c =$ __(5)__, and when does *the total reflection* exist at an interface of two nonmagnetic media with $\mu_1 = \mu_2$? __(6)__.

C. If a uniform plane EM wave has a wavelength λ of 1.5 μm in free space, give the phase velocity $u_p =$ __(7)__, and its frequency $\nu =$ __(8)__ (in M.K.S. units) when this plane wave traveling in a medium with dielectric constant ε_r of 2.25.

D. Given two point charges: $Q_1 = 10$ (μC) at (2, 0, -4) and $Q_2 = -60$ (μC) at (0, -1, -2), determine the electric field intensity, $\mathbf{E} =$ __(9)__, at Q_1 due to Q_2, and the magnitude of the force experienced by Q_1, $F =$ __(10)__. All dimensions are in meters.

E. If a static electric field **E** in a charged region, then which one is true __(11)__ of the following statements: (I) $\nabla \bullet \mathbf{E} = 0$ and $\nabla \times \mathbf{E} = 0$, (II) $\nabla \bullet \mathbf{E} = 0$ and $\nabla \times \mathbf{E} \neq 0$, (III) $\nabla \bullet \mathbf{E} \neq 0$ and $\nabla \times \mathbf{E} = 0$, (IV) $\nabla \bullet \mathbf{E} \neq 0$ and $\nabla \times \mathbf{E} \neq 0$.

F. Determine the work done, $W =$ __(12)__, by the electric field $\mathbf{E} = \mathbf{a}_x x - \mathbf{a}_y 2y$ (V/m) in moving a unit positive charge from position $P_1(-2, 0, 0)$ to position $P_2(5, -1, 3)$. The distances are in meters.

G. Give the boundary conditions: __(13)__ and __(14)__ for the electrostatic fields of $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{D}_1$, and $\mathbf{D}_2$ across the interface between two media.

H. Write down the Lorentz's force equation: __(15)__.

I. Write down the ***SI*** units (*International System of Units*) of the following fundamental electromagnetic field quantities and universal constants: **E**: __(16)__, **D**: __(17)__, **B**: __(18)__, **H**: __(19)__, μ_o and ε_o: __(20)__.

Part II: Problems and Calculations/Derivation (40%)

In this session, you are required to provide the necessary calculation or derivation details in order to get the full credit, and you should answer the problems in number sequentially in your answer sheet.

1. Consider two spherical conductors with radii b_1 and b_2 ($b_2 > b_1$) that are connected by a conducting wire. The distance of separation between the conductors is assumed to be very large in comparison to b_2 so that the charges on the sphere conductors may be considered as uniformly distributed. A total charge Q is deposited on the spheres. Find (a) the charges on the two spheres (Note: $Q = Q_1 + Q_2$), and (b) the electric field intensities at the sphere surfaces. (10%)

2. A parallel-plate capacitor of width w, length L, and separation d has a solid dielectric slab of permittivity ε in the space between the plates. The capacitor is charged to a voltage V_o by a battery, as indicated in Fig. 1. Assuming that the dielectric slab is withdrawn to the position shown and the switch is opened, determine the force acting on the slab. (10%)

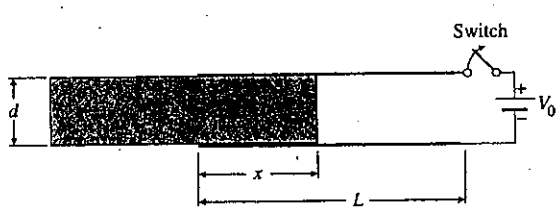

Fig. 1

3. A 10 (cm) by 10 (cm) square conducting loop having a resistance $R = 0.5$ (Ω) rotates in a constant magnetic field $\mathbf{B} = a_y\, 0.04$ (T) with an angular frequency $\omega = 100\pi$ (rad/s) about one of its legs, as shown in Fig. 2. Assuming the loop initially lies in the xz-plane, find the induced current i if the self-inductance of the loop is neglected. (10%)

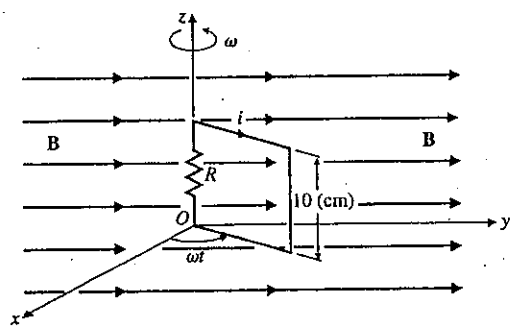

Fig. 2

4. Show that the instantaneous Poynting vector of a circularly polarized plane wave propagating in a lossless medium is a constant that is independent of time and distance. (10%)

Part I Questions: 60%

1. Please explain the operation principles of a BJT and a MOS transistor. (6 %)
2. Please specify the maximum and the minimum thickness of the base region of a BJT. (5%)
3. Feedback amplifier has four different structures including current-voltage converter, voltage-current converter, current amplifier and voltage amplifier. Please specify the input and output impedance of above four different amplifier structures? (5 %)
4. Please draw the DC and dynamic load lines of the amplifier stage given in fig. 4a and 4b. The $V_{cc} = 5V$, $R_1 = R_2 = 2\Omega$, $R_c = 4\ \Omega$, $R_E = 5\ \Omega$ and $R_L = 10\ \Omega$? (8 %)

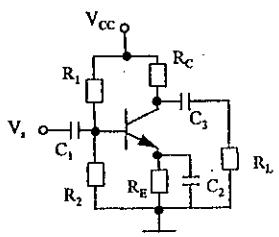

Fig. 4a

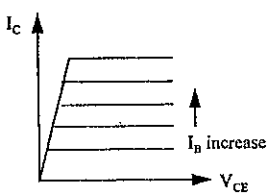

Fig. 4b

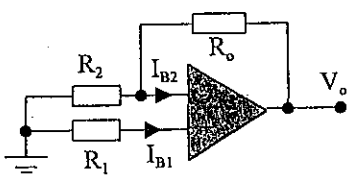

Fig. 5

5. Consider a non-inverting Op-Amp shown in the fig. 5. What is the value of R_1 (in terms of I_{B1}, I_{B2}, R_2 and R_o) so that $V_o = 0$? (8 %)
6. Please draw the block diagrams of a two-stage 32-output demutiplexer tree and a two-stage 32-to-1 mutiplexer tree. (5 %)
7. Please draw the cross sectional view of a CMOS inverter. A p-MOS FET, a n-MOS FET and the connection between the two FETs' must be included in your drawing. (6%)
8. Consider the following dielectric materials: SiO_2, Si_3N_4, TiO_2, Al_2O_3 and AlN. Which one is the commonly used material for fabricating the dielectric gate of a MOSFET device? Which dielectric materials are used in a DRAM cell for fabricating the thin film capacitor? (6%)
9. Please explain why a large emitter resistance is required in a typical differential amplifier? (5%)
10. Please describe the physical mechanisms which produce the Zener breakdown and the avalanche breakdown. (6%)

Part II Problems: 40%

1. Consider a diode circuit operated at 20 °C shown in Fig. 1a. The ideality factor η and the cut-in voltage of the diode is 2 and 0.5 V, respectively. The load resistor R_L = 50 Ω. Please draw the output voltage V_L as a function of time t for the given bias shown in Fig.1b. (20 %)

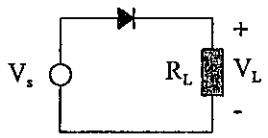

Fig. 1a

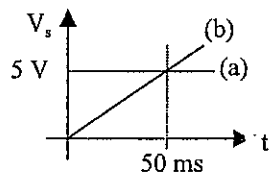

Fig.1b

2. The circuit shown in Fig. 2 is a common-source amplifier with source resistance. The transconductance of the amplifier is g_m, and the drain and the source resistance of the FET are negligible. (a) Please draw the equivalent circuit of the amplifier without feedback. Note that you only need to consider the internal capacitor C_{gs} between gate (G) and source (S). (b) What is the return ratio T of the amplifier? (c) Please draw the normalized Bode plot (including the relative magnitude and phase to the relative frequency) of the transfer function of the circuit. What is the 3-dB frequency of the circuit? (20 %)

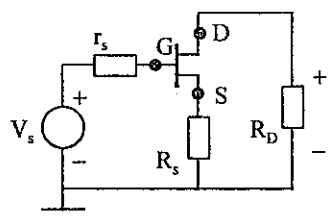

Fig.2

1. Explain what is black body radiation. What is the relation between radiation center wavelength and temperature? Why? (25 points)

2. What is the coherence of light ? How does it related to stimulated emission ? (25 points)

3. To understand the structure of atom, what did Bohr postulate? Estimate the Bohr radius. (25 points)

4. What is spin-orbit interaction ? Estimate the associated orientational potential energy. (25 points)