國立中山大學八十八學年度碩博士班招生考試試題

科 目：工程數學 (光電所)

1. 利用拉氏 (Laplace) 轉換，試解下列積微分方程

(i) $y(t) = t^2 + \int_0^t \sin(u)\, y(t-u)\, du$ (15%)

(ii) $y''(t) + 2y'(t) + y(t) = -u(t-1)$

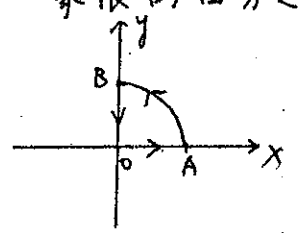
其中 $y(0) = 0$, $y'(0) = 1$, $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$ (15%)

2. 假設 $f(x) = \begin{cases} 0, & -1 \leq x < 0 \\ x, & 0 \leq x \leq 1 \end{cases}$

試求 $f(x)$ 於 $[-1, 1]$ 之 Fourier 級數。(20%)

3. 假設 $\nabla \times \vec{A} = -\hat{z}(x+1)$，試求 $\oint_C \vec{A} \cdot d\vec{\ell} = ?$ (20%)

其中 C 為半徑 =1，且在第一象限的四分之一圓的周圍 OABO，如圖。



4. 求 $\int_0^\infty \frac{\cos(mx)}{x^2+1}\, dx = ?$ $m > 0$, (10%)

5. 求矩陣 $A = \begin{bmatrix} -3 & 2 \\ 6 & 1 \end{bmatrix}$ 之 (1) 反矩陣 A^{-1}, (5%) (2) 特徵值及特徵向量 (5%)

6. 若期望值 $E(X) = 2$，且變異數 (variance) $Var(X) = 1$

求 $Var(3X+2) = ?$ (10%)

I. Problems (40%) :

1. As shown in Fig. 1, the inner conductor of the coaxial cable is now carrying a DC current of magnitude I flowing in the direction out of the plane of the paper, and the thin outer conductor is carrying an current of equal magnitude but in opposite direction. If the current flows uniformly in the conductor, please find the inductance per unit length of the cable. (20%)

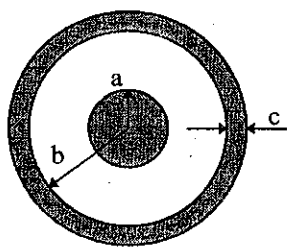


Fig. 1

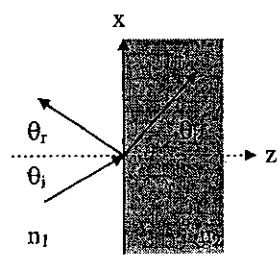
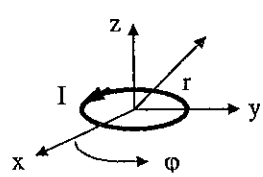


Fig. 2

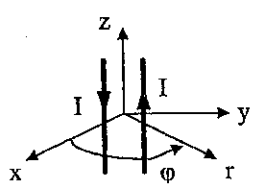
2. An EM wave is propagating from medium 1 into medium 2 as shown in Fig. 2. The refractive index of the medium 1 (n_1) is larger than that of the medium 2 (n_2). Please use the poynting theorem to verify that there is no energy propagating along the z direction when $\theta_i > \theta_c$ (the total internal reflection angle). (20%)

II. Questions (60%):

1. A conducting sphere of radius R is surrounded by free space. At t = 0, a charge density of ρ_o is distributed uniformly throughout the sphere. Please find the $\boldsymbol{E}$ field as a function of time for r < R. The dielectric constant and conductivity of the sphere are ε and σ, respectively (8%).
2. Please draw the $\boldsymbol{E}$ field between the conducting plates of a capacitor with a finite dimension. Charges of magnitude Q are installed in the capacitor (5%).
3. Consider an electric dipole separated by a distance d. Please draw the equipotential surfaces and $\boldsymbol{E}$ lines of the dipole (6%).
4. A point charge q is completely enclosed inside a metal sphere shell of radius r_o. The thickness of the metal shell is d ($< r_o$). Please find the potential as a function of r outside the sphere (7%).
5. If $\boldsymbol{A}$ is a vector function and f is a scalar function then $\nabla \times (f\boldsymbol{A}) = ?$ (5%).
6. A visible light is launched directly onto the following metals : gold, silver, chromium, zinc and copper. Please choose the one from above list with the highest reflection of the incident light (5%).
7. A circular polarized light is incident from medium 1 into medium 2 at the Brewster angle. Please describe the polarization states of both the reflected light in medium 1 and the transmitted light in medium 2 (8%).
8. Please find the guided wavelength of the TE_{10} mode of a rectangular waveguide with a width of 3 cm and a height of 1 cm (5%).
9. Please draw the directions of the vector potential $\boldsymbol{A}$ of the magnetic dipoles shown in the following. (6%)?



Magnetic dipole I



Magnetic dipole II

10. Please write down the two interface conditions for both $\boldsymbol{H}$ and $\boldsymbol{E}$ fields (5%).

Note that to answer the following problems, you are required to provide the necessary details or key points in order to get the full credit, and you should answer the problems in number sequentially in your answer sheet.

1. The field-effect transistor (FET) source follower. (10%)

(a) Describe its typical characteristics of voltage gain, and input/output impedance, and (b) what kind of application of this circuit is commonly used.

2. The operational amplifier (op amp). (10%)

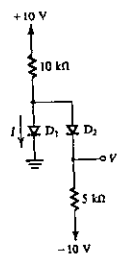
Describe (a) the definition of the common-mode rejection ratio (CMRR) of an op amp, and (b) the gain, input resistance, and output resistance characteristics of an ideal op amp.

3. The negative feedback amplifier circuit. (10%)

(a) What means negative feedback? (b) Describe the general properties (advantages) of a negative feedback amplifier.

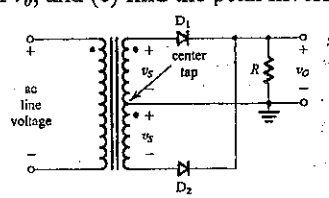
4. The ideal diode. (10%)

For the following circuit using ideal diodes, find the values of the (a) voltage, V, and (b) current, I, indicated.



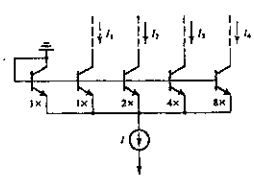
5. The full-wave rectifier. (20%)

For the following full-wave rectifier circuit, sketch (a) the transfer function, v_o versus v_s, (b) the input and output waveforms, i.e., both v_s and v_o, and (c) find the peak inverse voltage (PIV) of the diode.



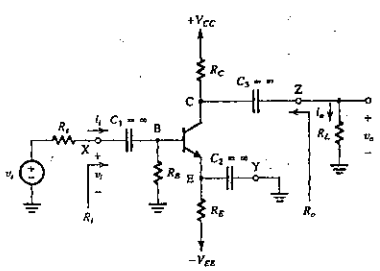
6. The digital-to-analog (D/A) converter circuit. (20%)

The bipolar junction transistors (BJTs) in the following D/A converter circuit have their base-emitter junction areas scaled in the ratios indicated. Find I_1 to I_4 in terms of I.



7. The operation of a single-stage bipolar junction transistor (BJT) amplifier. (20%)

For the following circuit let $R_B = 100$ kΩ, $R_E = 10$ kΩ, $R_C = 10$ kΩ, $R_s = R_L = 10$ kΩ, $V_{CC} = V_{EE} = 10$ V, and let the BJT have $\beta = 100$ and $V_A = 100$ V. (a) Find the dc value of V_B, V_E, I_C, and V_C. (b) Also find the values of R_i, R_o, A_v (= v_o/v_s), and A_i (= i_o/i_i) .



1. Write down the year (full credit if within 5 years) when the following events took place in the history of modern physics. (15%)
   a. Max Planck theory on black body radiation.
   b. Einstein published paper on photoelectric effect.
   c. Neil Bohr's theory on atomic structure model.
   d. Erwin's Schrodinger introduced theory on wave mechanics.
   e. Wofgans Pauli suggested exclusive principle for electrons.

2. a) Explain the meaning of "time-dilation" and "length contraction" under the special theory of relativity. b) If you are traveling at a speed of V=0.6c on a spaceship, compute the time (your time) it needs for you to arrive at a distance planet which is 12 light years (measured from earth's coordinate) from earth (the star is assumed to be fixed relative to earth). c) As soon as you arrive at the planet, you send a radio signal back to the earth. For the people on the earth, when will they receive your signal after you take off the earth? (20%)

3. A 0.5um CW laser produces a collimated optical beam (2 cm diameter). Assume the total output power is 1 watt, compute a) the photon flux density and b) the optical pressure exerted on a mirror of 100% reflectivity. State clearly your assumption and provide both numerical values and the units of your answers. (15%)

4. a) Write down the time independent one dimensional Schrodinger equation and explain the physical meaning of the wave function. (8%)
   b) Solve for lowest three energy levels and associated wave functions for an electron in a infinitely deep well of width L=20nm. Please include both derivation and numerical value of your answers (12%)

5. State the significant features of Rutherford's theory of atomic scattering. (10%)

6. State the particle properties of Bosons and Fermions (also give example of the particle, their quantum statistical distribution, etc.) (10%)

7. What is the Mossabauer effect? How does the physicist measure small change of energies of the photons due to the motion of the photon source. (10%)

Planck's constant $h = 6.63 \times 10^{-34}$ J•s
Electron's mass $m_e = 9.11 \times 10^{-31}$ kg