Institute of Electro-Optical Engineering
1998 Entrance Examination of Engineering Mathematics

Part I: Fill in the blanks (40%)

In this session, each blank is worth 5%. *You are not required to fill in any detail calculation. Just give the final result in the corresponding blank spaces in your answer sheet. Your answers must be labeled clearly and written sequentially.*

A Write down an mathematical identity that relate the following four mathematical constants: π, i, -1, and e (natural base). Ans: ___(1)___.

B Find the two independent homogeneous solution of $y(t)$ that satisfies the differential equation

$$y'' + ay' + by = 0,$$

where a and b are constants. If $a = 3$, $b = 2$ $y(t) =$ ___(2)___, and if $a = 2$, $b = 1$ $y(t) =$ ___(3)___.

C Find a unit vector perpendicular to the plane defined by $x + 2y + 3z = 6$. Ans: ___(4)___.

D Given matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find both eigenvalues and corresponding eigenvectors of A = ___(5)___. Find matrix function e^{At} = ___(6)___.

E Find the vector gradient of the scalar function $f(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$ where $\vec{r}$ is the coordinate vector (x, y, z) and $\vec{k} = (k_x, k_y, k_z)$ is any constant vector in space. Ans: ___(7)___. (Note: you get full credit if you simplify your answer so that it contains only terms like f and $\vec{k}$ but no component terms)

F If $\vec{k} = (1, 2, 4)$ in problem E, find $\nabla \times (f(\vec{r})\hat{v})$ where $\hat{v}$ is a unit vector and $\hat{v} \cdot \vec{k} = 0$. Ans: ___(8)___. (see note of problem E)

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Part II: Short questions on Mathematical concept and theorems. (20%)

For the following two problem in this session, you are required to write down a concise paragraph not to exceed 75 words in Chinese or in English in order to get full credit.

1 In complex analysis of multi-value functions, explain the idea of branch point and branch cut. Please use $\sqrt{z}$ function as an example. (10%)

2 Explain Gibb's phenomenon in the theory of Fourier Analysis. (10%)

Part III: Proof and Calculation (40%)

1 Prove that the transpose of a matrix product is the product of the transposed matrices in reverse order. You may assume both A, B matrices are N by N matrices. (10%)

$$(AB)' = B'A'$$

2 Using the factor that $\int_0^\infty e^{-x^2} dx = \sqrt{\pi}/2$, find the Fourier transform of the Gaussian function: (15%)

$$F(\omega) = \int_{-\infty}^{\infty} e^{-t^2} e^{i\omega t}\, dt$$

3 Evaluate the following complex line integral (15%)

$$\int_C \frac{1}{1+z^3}\, dz$$

where C is the path along the positive imaginary axis from 0 to ∞.

Institute of Electro-Optical Engineering
1998 Entrance Exam on Electromagnetism

Part I: Fill in the blanks (50%)

In this session, each blank is worth 5%. Please do your calculation on the scratch paper and copy the final result in the corresponding blank space in your answer sheet. For those problems that require numerical values, try to give your best estimates. You shall get full credit if your answer is within 80% accuracy. Partial credit is also awarded if your answer is not too far off. For problems e, h and j, you get full credit only if you write your answer in less than 25 words in English or Chinese.

A The electric field intensity $\vec{\mathbf{E}}$, and the magnetic flux density $\vec{\mathbf{B}}$ are related to the scalar potential V and vector potential $\vec{\mathbf{A}}$ as ____(1)____. (in the time domain)

B The Lorentz condition is used to simplify the coupled equations of scalar potential V and vector potential $\vec{\mathbf{A}}$. Write down this condition: ____(2)____.

C The Lorentz force acting on a point test charge q traveling at a speed $\vec{v}$ in the presence of $\vec{\mathbf{E}}$, $\vec{\mathbf{B}}$ field is ____(3)____.

D If $\mathbf{A}$ is a vector function and f a scalar function then $\nabla\cdot(f\mathbf{A}) =$ ____(4)____.

E Write down the two interface conditions for $\vec{\mathbf{B}}$ and $\vec{\mathbf{D}}$ fields ____(5)____.

F Write down the formula for both instantaneous and time-harmonic Poynting vectors ____(6)____.

G Give the operating frequency ranges of the following systems 1) AM stations in Taiwan, 2) FM and broadcasting TV stations in Taiwan. A: ____(7)____. (Units are required for your answer)

H Determine the two lowest cut-off frequencies for the rectangular waveguide of width a = 3cm and height b = 1cm. Please also indicate these waveguide modes. A: ____(8)____. (Units are required for your answer)

I State the main guiding principle of the optical fiber. A: ____(9)____.

J Give the speed of light and its impedance in free space A: ____(10)____. (in M.K.S. units)

Part II: 50%

1 A. Explain the astronomical phenomenon "red shift" based on EM wave theory.
B. It is said that our universe is expanding; therefore, the stars are moving away from each others. If so will we be observing any "blue shift" on the spectrum of the stars in the sky? (10%)

2 A dielectric slab ($\epsilon_r > 1$) is inserted into the empty region of a fully charged parallel plates (like a charged capacitor). The plates are kept at a constant distance at all time. If you are holding the dielectric slab, would you experience an attractive force or a repulsive force acting on the slab while the slab is being inserted into the capacitor?. Please explain why? (10%)

3 Derive the one-dimensional wave equation from Maxwell's equations for the y-component $\vec{H}$ field propagating in the z direction. (10%)

4 Derive the reflection and transmission coefficients of an obliquely incident plane wave propagating across an interface of two media whose indexes of reflection are n_1 and n_2. You may assume the incident wave to a TE mode (i.e. $\vec{E}$ field in perpendicular to the plane of incidence say the $x-y$ plane). Please use θ_i (incident), θ_r (reflected), θ_t, (transmitted) as the relevant angles. You must clearly show each step of your derivation. (20%)

Note that to answer the following problems, you are required to provide the necessary details or key points in order to get the full credit, and you should answer the problems in number sequentially in your answer sheet.

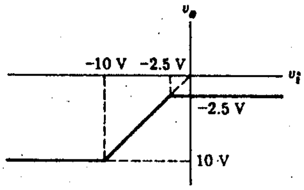
1. The transistor-transistor logic (TTL) and emitter-coupled logic (ECL) gates. (10%)
What's the typical output voltage of the logic levels [V(1) and V(0)] at the output port of (a) a TTL and (b) an ECL gates?

2. The emitter follower. (10%)
(a) Describe its typical characteristics of voltage gain and input/output impedance, and (b) what kinds of application of this circuit are commonly used.

3. The stability problem of the feedback-circuit systems. (10%)
(a) What is the Nyquist criterion? (b) Please draw the loci on the Nyquist plane for both stable and unstable feedback-circuit systems.

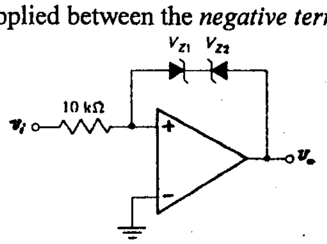
4. The class B push-pull amplifier. (10%)
(a)Draw the circuit of a class B push-pull power amplifier. (b) State two advantages and one disadvantage of class B over class A amplifier.

5. The analog diode circuit. (10%)
Construct a circuit whose transfer characteristic (v_o versus v_i) has the form shown in the figure. Use ideal diodes and give the numerical values of all elements in your circuit.

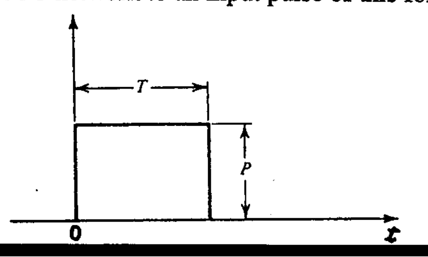


6. The charge-coupled device (CCD). (15%)
(a) Explain how a potential energy well is formed under an electrode of a CCD? (b) If the substrate is *p*-type, are electrons or holes captured in the well? (c) What determines the minimum and maximum frequencies of operation of a CCD?

7. The comparator circuit. (15%)
(a) For the shown comparator circuit, plot and explain the transfer characteristic if the OP AMP gain is infinite and $V_{Z1} = V_{Z2} = 10$ V. (b) Repeat part (a) if the large-signal gain is 50,000. (c) Repeat part (a) if a voltage of 4V is applied between the *negative terminal and ground.*



8. The pulse response of single-time-constant (STC) networks. (20%)
For a pulse signal whose height is P and whose width is T. (a) Draw the responses of a low-pass STC network to an input pulse of this form shown in the figure, with a time constant τ, for $\tau \approx T$ and $\tau >> T$, respectively. (b) Draw the responses of a high-pass STC network to an input pulse of this form, with a time constant τ, for $\tau \approx T$ and $\tau << T$, respectively.



1. Use the Wilson-Sommerfeld quantization rule to calculate the allowed energy levels of a ball which is bouncing elastically in a vertical direction. (15%)

2. A particle of mass m moves one-directionally to the right of a hard wall at $x = 0$ in a potential $V(x)$, where

   $V(x) = -B,\ 0 < x < b,\ B > 0; \quad V(x) = 0\ ,\ x > b.$

   For fixed b, there is a minimum value of B below which there are no bound states. Find this minimum, B_{min}. (20%)

3. For particle statistics, there are three distribution functions: Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac distributions. (20%)
   (a) What are the particle properties of each distribution? For each distribution, give an example of the particles
   (b) Write down the three distribution functions.

4. Two small electrodes facing each other are separated by 50 μm in vacuum. One is made of gold, the other of silver, both have a disk shape and a 1 mm diameter. They are electrically connected. The gold electrode vibrates sinusoidally with an amplitude equal to 5 μm and a frequency 100 Hz (displacement parallel to the normal to the two disks). Calculate the amplitude of the a.c. current (in units of Amps) induced by this vibration. Assume a constant contact potential difference of 1 Volt between the two metal disks. (20%)

5. For x-rays and electrons are both of 4 keV energy, find their wavelengths, respectively. (10%)

6. Give mathematical expressions, values and units (if any) of the following. (15%)
   (a) Bohr magneton
   (b) Compton wavelength
   (c) fine structure constant

useful constant: Plank's constant $h = 6.6262 \times 10^{-34}$ J•s
$1\ eV = 1.60218 \times 10^{-19}$ J
Electron mass $= 9.1095 \times 10^{-31}$ kg
Permittivity in vacuum $\varepsilon_0 = 8.85418 \times 10^{-14}$ F/cm