國立中山大學115學年度
碩士班考試入學招生考試試題

科目名稱：工程數學【光電系碩士班】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

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科目名稱：工程數學【光電系碩士班】 題號：435001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題） 共1頁第1頁

Part 1: True or False (20 points, 2 points each question). Select A for True; select B for False. Scoring: +2 for a correct answer, −1 for an incorrect answer, and 0 for no answer.
(本大題請在答案卡上作答)

1. If matrix A is invertible, then the system $Ax = b$ must have a unique solution for every vector b.
2. For any square matrices A and B of the same size, $(A+B)^2 = A^2 + 2AB + B^2$.
3. If det(A) is not zero, then A must be invertible.
4. If $AB = 0$ for square matrices A, B, then $A = 0$ or $B = 0$.
5. If $v_1 \ldots v_k$ span R^n, then they must be linearly dependent.
6. Every symmetric matrix is diagonalizable by an orthogonal matrix.
7. If λ is an eigenvalue of A, then λ is also an eigenvalue of the adjoint of A.
8. If a n-by-n matrix has n distinct eigenvalues, then it is not diagonalizable.
9. If rank(A) = r, then the nullity of A equals r, too.
10. If U and V are subspace of R^n, then $U \cup V$ is always a subspace.

Part 2: Calculation and Proof (80 points)

11. Consider a vectorial field $\mathbf{F}(x,y,z) = (2xy + z^2, x^2 + 2yz, y^2 + 2xz)$. Let the closed loop C be the cross-section line of the sphere $x^2 + y^2 + z^2 = 2^2$ and the plane $x + y + z = 1$. Determine the value of the integral $\int_C \mathbf{F} \cdot d\mathbf{l}$ (10%)

12. Find the following inverse Laplace transforms (10%，5% for each question)

$$\text{(a) } L^{-1}\left\{\frac{1}{\sqrt{s^2+1}}\right\}, \quad \text{(b) } L^{-1}\left\{\frac{e^{-\sqrt{s}}}{\sqrt{s}}\right\}$$

13. Three functions $g_0(x) = a$, $g_1(x) = b_1 + b_2 x$, $g_2(x) = c_1 + c_2 x + c_3 x^2$ form an orthonormal set within an interval $-1 \le x \le 1$. Determine the values of a, b_1, b_2, c_1, c_2, and c_3. (10%)

14. Solve the following 2nd order differential equations (10%，5% for each question)

$$\text{(a) } xy'' - y' = x^2 e^{3x}, \quad \text{(b) } y'' + (4t - t^{-1})y' + 8t^2 y = 0,\ t > 0 \ \left(\text{Hint: Try } x = e^{t^2}\right)$$

15. Solve PDE $\nabla^2 u(r,\theta) = 0$ within an area $0.5 \le r \le 1$, $0 \le \theta \le 2\pi$. B.C. $u(1,0) = 0$, $u(0.5,\theta) = \cos^2\theta$ (10%)

16. It is known that $\nabla \times \mathbf{A} = \mathbf{B} + i\mathbf{A}$ and $\nabla \times \mathbf{B} = -\mathbf{A} + i\mathbf{B}$, where $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$. Show that (10%)

$$\oint\!\!\oint_S (\mathbf{A} \times \mathbf{B}) \cdot ds \ge \frac{1}{2}\int_V |\mathbf{A} + \mathbf{B}|^2\, dV$$

17. Find the following integrands using complex analysis techniques. (10%，5% for each question)

$$\text{(a) } \int_0^{\pi} \frac{d\theta}{(a + b\cos\theta)^2} = ?, \quad \text{(b) } \int_0^{\infty} \frac{x^{1/p}}{x^2+1}\, dx = ?,\ p \in N$$

18. Assuming that $f(x) = e^{i\omega_1 x} + e^{i\omega_2 x}$ and $g(x) = \dfrac{1}{x^2 + a^2}$, evaluate their convolution integral $h(x) = f(x) * g(x)$, where $*$ denotes convolution product.(10%)