國立中山大學112學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學【光電系碩士班】

－作答注意事項－

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張，亦不得使用應考證空白處作為計算紙使用)。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學112學年度碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學【光電系碩士班】 **題號：435001**

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）(問答申論題) 共1頁第1頁

1. If an $n \times n$ matrix **A** has a basis of eigenvectors, then $\mathbf{D} = \mathbf{X}^{-1}\mathbf{A}\mathbf{X}$ is diagonal, with the eigenvalues of **A** as the entries on the main diagonal. **X** is the matrix with the corresponding eigenvectors as column vectors. In this problem, let

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}.$$

   (a). (5%) Find **D**.
   (b). (9%) Find **X**.
   (c). (7%) Find $\mathbf{X}^{-1}$ by the Gauss-Jordan elimination.

2. Calculate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ counterclockwise as seen by a person standing at the origin, for the following **F** and C.

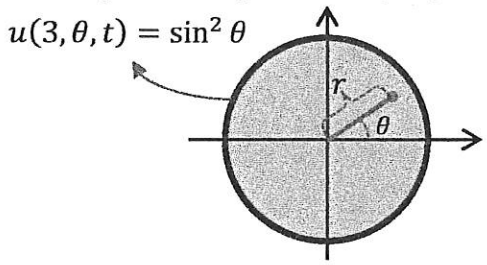
$$\mathbf{F} = [y^3, \quad x^3, \quad z^3 - x]$$

   around the triangle with vertices $(0, 0, 4)$, $(4, 0, 4)$, and $(4, 2, 4)$.
   (a). (9%) Calculate by direct integration.
   (b). (5%) Calculate by Stokes's theorem.

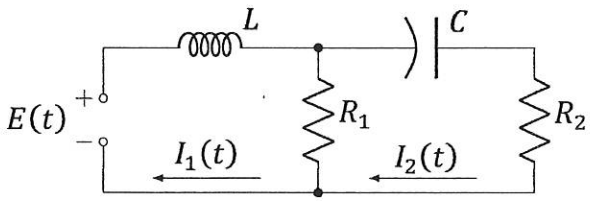
3. (15%) The temperature $u(r, \theta, t)$ of a thin circular disk is modeled by

$$\frac{\partial u}{\partial t} = c^2 \nabla^2 u = c^2 \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} \right).$$

   The equation is in the polar coordinates, and c is a constant. If the disk is perfectly insulated, and the edge of the disk is kept such that the boundary condition is $u(3, \theta, t) = \sin^2 \theta$, as shown in the following figure, find the steady-state temperature $u(r, \theta)$.



4. (18%) Find the steady-state current $I_2(t)$ in the circuit shown in the following figure, where $R_1 = 1\ \Omega$, $R_2 = 9\ \Omega$, $L = 1$ H, $C = 0.1$ F, and $E(t) = -\frac{2}{\pi} + \left|\cos\left(\frac{t}{2}\right)\right|$ V.



5.
   (a). (8%) Find a general solution $y(x)$:
   $y' + y\sin(2x) = x\exp(\cos^2 x)$
   (b). (8%) Find a general solution $y(x)$ with one known solution of $y(x) = x$:
   $x^3y''' - 3x^2y'' + (6 + 4x^2)xy' - (6 + 4x^2)y = 0$
   (c). (8%) Find a general solution $y(x)$:
   $x^2y'' + \frac{1}{2}xy' + xy = 0$
   (d). (8%) Find a general solution $y_1(x)$ and $y_2(x)$:
   $y_1' + 3y_1 + 4y_2 = 2x$ and $y_2' - 5y_1 - 6y_2 = 1$

試題請隨卷繳回，請留意背面是否有題