國立中山大學108學年度
碩士暨碩士專班招生考試試題

科目名稱：電磁學【光電系碩士班】

—作答注意事項—

考試時間：100分鐘

- **考試開始響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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國立中山大學108學年度碩士暨碩士專班招生考試試題

科目名稱：電磁學【光電系碩士班】　　題號：435002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. (20%) As shown in figure 1, two line charges of uniform density ρ and $-\rho$ (total charge Q and -Q) form two semicircles of radius **r** on the x-y plane of a space. Two semicircles are separated with an infinitely small gap. The permittivity of the space is ε.
   (a) (10%)Please find the electrical field at the center of semicircle.
   (b) (5%)Please find the equal potential area.
   (c) (5%)If such dipole oscillates by timely varying charge density with a time harmonic way, an EM wave propagation will be formed. What are the directions of the maximum and minimum EM field intensities and also their corresponding polarizations?

2. (20%) As shown in figure 2, two parallel conducting plates with area of **A** are placed by a distance **d** ($d=d_1+d_2$). As a voltage is applied to these two plates, a capacitance is formed. Two dielectric materials with thickness d_1 and d_2, permittivities ε_1 and ε_2, and conductivities σ_1 and σ_2 are fully filled between the plates.
   (a) (10%) If both dielectric materials are electrical isolators ($\sigma_1=\sigma_2=0$), please find the bound charge between two dielectrics and also the corresponding capacitance.
   (b) (10%) If both dielectric materials are conductive, find the boundary conditions and determine the electric fields and current densities in both dielectrics.

3. (20%) Consider a coaxial cylindrical conductor plotted in figure 3. The inner conductor is a solid conductor of radius "**r**" and the outer conductor is an infinitely thin and perfect conductor of radius "**R**".
   (a) (10%)If a current of "I" is injected into the inner conductor, as shown, and flows back to the outer conductor. Assuming "I" is uniformly distributed over the inner conductor, please find the inductance of coaxial cable per unit length.
   (b) (5%)As the "I" is timely varied, will the current intensity be still uniformly distributed over the inner conductor? Please state the reasons for your answer.
   (c) (5%) Please explain briefly the working function of transmitting electrical signal through such coaxial cable.

4. (15%) EM wave propagation
   (a) (10%)Using the Maxwell's equations, derive and describe the wave propagation form of electric field **E** and magnetic field **H** in a free space. This is so called EM wave.
   (b) (5%)Will the EM wave be a longitudinal wave? Please state the reasons for your answer.

5. (15%) A flat metal with infinite conductivity ($\sigma = \infty$) is placed under a light excitation. As shown in figure 4, when a time harmonic light with angular frequency of ω (rad/s) and TE polarization along the metal plane is incident with an angle of θ. Such light can be assumed to be a plane EM wave with field intensities of $\vec{E}$ and $\vec{H}$.
   (a) (10%) Using boundary condition on the plate, explain that only surface current density $\vec{J}$ and surface carrier density ρ exist on the surface of the plate, and no bulk current density can be found inside the metal. Find the $\vec{J}$ and ρ. Please also indicate the polarization of $\vec{J}$. (You should give the answer step by step.)
   (b) (5%)What is the reflection?

6. (10%) In figure 5, a three-layer dielectric material forms an optical waveguide structure. The refractive index of the center layer and outer cladding layers is n_1 and n_2, respectively. Please derive the acceptance angel, Ω, condition for coupling light into waveguide and also guiding light inside the n_1 medium, i.e., the light can be made with total reflection inside the center layer.

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※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

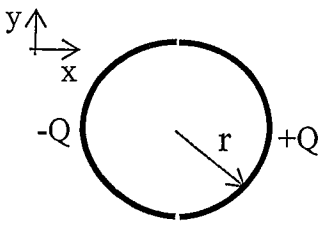

Figure 1

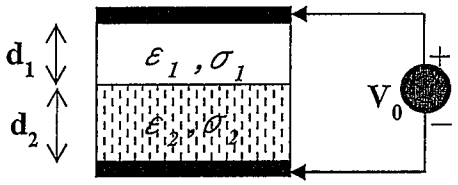

Figure 2

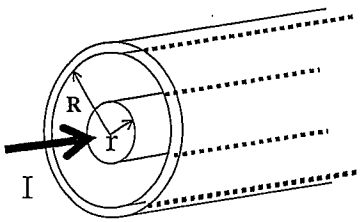

Figure 3

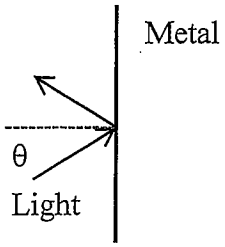

Figure 4

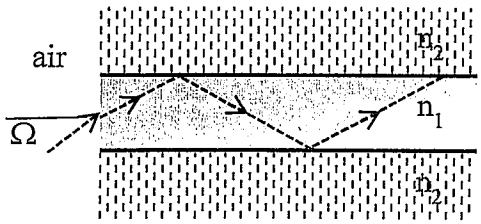

Figure 5

試題請隨卷繳回

國立中山大學108學年度
碩士暨碩士專班招生考試試題

科目名稱：工程數學【光電系碩士班】

—作答注意事項—

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國立中山大學108學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【光電系碩士班】　　題號：435001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共1頁第1頁

1. Find the general solutions of the following differential equation with given initial or boundary conditions. You must provide the details and conditions (for existing of solutions) to get full credit. (20%)

$$y'' = \lambda^2 y, \quad \lambda \text{ is a constant,}$$
$$\text{(1A)}, \quad y(0) = a, \quad y'(0) = b,$$
$$\text{(1B)}, \quad y(0) = 0, \quad y'(1) = 0.$$

2. Find the Fourier transform of the following Gaussian function (a is a positive real constant) by direct integration along the temporal axis. Details required. (20%)

$$f(x) = \exp\left(-a^2 t^2\right).$$

3. (3A) Please provide **coordinate independent** definitions of the gradient, divergence and curl operators. (12%)

   (3B) Write down the Laplacian operator over the scalar function $f(\rho, \phi, z)$ (using the gradient and divergence operators) in the cylindrical coordinate system. (8%)

4. (4A) Show that eigenvalues (λ_i) of a real symmetric matrix **A** are real and that the eigenvectors ($\vec{x}_i$) corresponding to distinct (unequal) eigenvalues are orthogonal to each other. (10%)

$$\mathbf{A}\vec{x}_i = \lambda_i \vec{x}_i, \quad \vec{x}_i^T \vec{x}_j = 0, \quad \left(i \neq j, \quad \lambda_i \neq \lambda_j\right).$$

   (4B) Please first write down the definition of the matrix exponential $e^{\mathbf{A}}$, then write down the procedures of using similarity transform (**to** diagonalize the **A** matrix) to compute the matrix exponential of a given matrix **A**. (10%)

5. What are branch points and branch cuts? Please specify the branch points and draw branch cuts of the following multi-value complex functions: (20%, 5% for each subproblem)

$$\text{(5A)}, \quad \sqrt{z},$$
$$\text{(5B)}, \quad \frac{1}{\sqrt{z^2 - a^2}},$$
$$\text{(5C)}, \quad \ln(z),$$
$$\text{(5D)}, \quad Y_0(z), \quad \text{(Bessel function of second kind)}$$

試題請隨卷繳回

國立中山大學108學年度
碩士暨碩士專班招生考試試題

科目名稱：普通物理【材光系碩士班丙組】

－作答注意事項－

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Problem 1. [Mechanics: 40 points]

Materials are made of atoms linked by the electrostatic force. Around the equilibrium positions, the electrostatic force can be modeled as a spring with elastic constant k.

(a) [5 points] If the potential between two atoms is:

$$V(r) = e^{-\frac{r}{a}} - \frac{C}{r},$$

where r is the distance between the two atoms, a and C are constants. If this potential function has a minimum at $r = r_0$, do Taylor's expansion of $V(r)$ around $r = r_0$:

$$V(r) \approx V_0 + \frac{1}{2}\, k\, (r - r_0)^2 + \cdots$$

to obtain V_0 and k (express your answers with r_0, a, and C).

(b) [5 points] Use the result of (a), what is the force between the two atoms?

(c) [10 points] Consider 2 non-identical atoms with mass m_1 and m_2, connected by a force modeled as a spring with elastic constant k. The center of mass is stationary (not moving). Assume the 2 atoms only move in x-direction, solve the angular frequency of the vibration motion of the 2 atoms (express your answer with m_1, m_2, and k. How many vibration modes?

(d) [20 points] Now consider 3 identical atoms with mass m, connected by a force modeled as a spring with elastic constant k. The center of mass is stationary (not moving). Assume the 3 atoms only move in x-direction, solve the angular frequencies of the vibration motion of the 3 atoms (express your answer with m and k. How many vibration modes?

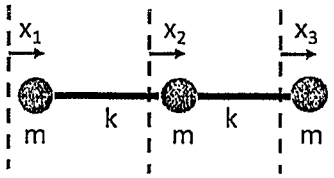

Figure 1

Problem 2. [Electromagnetism: 30 points]

Electromagnetic fields are the dominant forces in materials science. They are also directly related to optoelectronic applications. The electric field at a distance r from a point charge Q is:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\boldsymbol{r}},$$

where $\hat{\boldsymbol{r}}$ is the unit vector along the vector $\boldsymbol{r}$. Consider the following questions in the vacuum, and use the units in the above formula.

(a) [5 points] Find the electric field (magnitude and direction) a distance s away from the midpoint between two charges: q at $z = -d/2$ and q at $z = +d/2$.

(b) [5 points] Write down the electric potential V of the configuration in **(a)**. Calculate the electric field from the potential V.

(c) [5 points] Find the capacitance of two parallel infinite plates, with opposite uniform charge density σ and $-\sigma$. The distance between the two plates is d.

(d) [5 points] Calculate the electric field and potential difference between the two plates for the configuration of (c).

(e) [5 points] Calculate the energy per unit area stored in the configuration of (c).

(f) [5 points] Consider a particle with mass m and charge q moving with velocity v in a uniform magnetic field B. Since magnetic force is perpendicular to the velocity, its effect is to change the direction of the velocity without changing its magnitude, resulting in a circular motion. Solve the radius and angular frequency of the circular motion.

Problem 3. [Thermodynamics: 20 points]

(a) [10 points] Considering an ideal gas, the probability of the particles with velocity v to $v+dv$ follows the Maxwell-Boltzmann distribution:

$$f(v)d^3v = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} d^3v,$$

where m is the mass of the particle, k is the Boltzmann's constant, and T is the temperature. Calculate the square root of the mean square speed $v_{rms} \equiv \sqrt{\langle v^2 \rangle}$ (the symbol $\langle \cdots \rangle$ is the expectation value).
[You might need the integration: $\int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, $\int_0^\infty x^3 e^{-x^2} dx = \frac{1}{2}$, $\int_0^\infty x^4 e^{-x^2} dx = \frac{3}{8}\sqrt{\pi}$]

(b) [5 points] Based on the result in (a), calculate the average kinetic energy .

(c) [5 points] Based on the result in (b), calculate the specific heat per particle of the ideal gas.

Problem 4. [Waves/Optics/Modern physics: 10 points]

(a) [5 points] In a stretched string, the wave $e^{ik(x-\omega t)}$ can be reflected from the end of the string in two ways: $+e^{-i(kx+\omega t)}$ and $-e^{-i(kx+\omega t)}$. The reflected wave would interfere with the incoming wave. Calculate the standing waves obtained from such interference.

(b) [5 points] A wave has an angular frequency ω and a wavelength λ. Calculate the speed of the wave.