國立中山大學104學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【光電所碩士班】　　題號：435001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. Let $\mathbf{A}=\begin{bmatrix}1 & 2 & 1\\ 1 & 2 & 0\\ 2 & 1 & -1\end{bmatrix}$. Find the rank and the determinant of $\mathbf{A}^5=\mathbf{AAAAA}$. Show the details of your work. (10%)

2. Transform the quadratic form: $7x_1^2+5x_2^2+6x_3^2-4x_1x_3+4x_2x_3=210$ to principle axes (to a canonical form; i.e. the form of $\lambda_1y_1^2+\lambda_2y_2^2+\lambda_3y_3^2=210$). Express the new coordinate vector $\mathbf{y}=[y_1 \quad y_2 \quad y_3]^T$ in terms of the original coordinate vector $\mathbf{x}=[x_1 \quad x_2 \quad x_3]^T$. (15%)

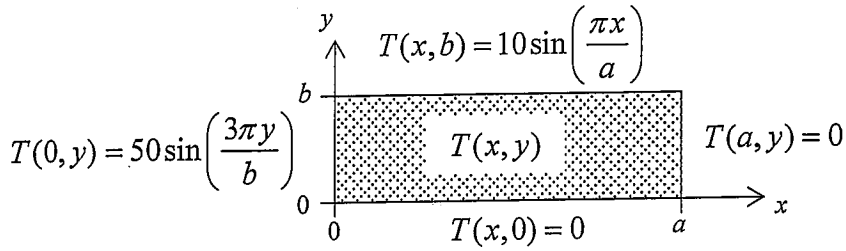
3. Find respective general solution $y(x)$ for the following ordinary differential equations. (Note: $y'=\frac{d}{dx}y(x)$, $y''=\frac{d^2}{dx^2}y(x)$, and $y'''=\frac{d^3}{dx^3}y(x)$)

   a. $y'+y-x\sqrt{y}=0$ (10%)

   b. $y'''-3y''+3y'-y=e^x+x^2$ (10%)

   c. $y''+y=\cos^2 x$ (10%)

4. Find the steady-state temperature $T(x,y)$ of the thin rectangle in the following figure, if the left side is kept at $50\sin(3\pi y/b)$, the upper side is kept at $10\sin(\pi x/a)$, and the other sides are kept at 0. (15%)



5. For a given Sturm-Liouville problem: (Note: $y'=\frac{d}{dx}y(x)$ and $y''=\frac{d^2}{dx^2}y(x)$)

$$x^2y''+xy'+\lambda y=0, \quad 1\le x\le 3$$

$$y'(1)=0 \text{ and } y(3)=0$$

   a. Find the eigenvalues λ and eigenfunctions $y(x)$. (10%)

   b. Verify orthogonality of the eigenfunctions. (Hint: rewrite the given equation in the form of $[p(x)y']'+[q(x)+\lambda\times w(x)]y=0$ to find the weight function $w(x)$) (10%)

6. Show the triangle inequality for complex numbers, $|z_1+z_2|\le|z_1|+|z_2|$ (10%)

國立中山大學104學年度碩士暨碩士專班招生考試試題

科目名稱：電磁學【光電所碩士班】　　題號：435002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共1頁第1頁

1. In the general orthogonal coordinates, prove that the divergence of a vector function $\vec{B}$ is given by

$$\nabla\cdot\vec{B}=\frac{1}{h_1h_2h_3}\left(\frac{\partial B_1h_2h_3}{\partial u_1}+\frac{\partial B_2h_1h_3}{\partial u_2}+\frac{\partial B_3h_1h_2}{\partial u_3}\right)$$

Where B_i, h_i, and u_i (i = 1,2,3) are the coefficients, metric coefficients, and coordinates in the general orthogonal coordinate system, respectively. (10%)

2. A point charge +q is at a distance d from the center of a grounded conducting sphere of a radius a (a<d) as indicated in Figure 1. Please determine (a) the potential distribution and electric field intensity inside and outside the sphere (12%) and (b) the charge distribution induced on the surface of the sphere (6%). (c) If the conducting sphere is not grounded, please recalculate the potential distribution inside and outside the sphere. (10%)

3. An infinitely long coaxial transmission line has a solid inner conductor of radius a and a very thin outer conductor of inner radius c, in which two magnetic materials μ_1 and μ_2 are filled as shown in Figure 2. Determine the inductance per unit length of the line. (15%)

4. Please derive the equation of continuity from Maxwell's equations. (10%)

5. Please describe Maxwell's corrections to Ampere's Law. (10%)

6. Determine the dielectric constant and thickness of a transparent coating layer deposited on a glass substrate (ε_r = 4, μ_r = 1) to eliminate the reflection of red light (660 nm). (8%)

7. For a step-index fiber shown in Figure 3, assume the core index is n_1 = 1.45 and the cladding index is n_2 = 1.44. Also assume the free space wavelength of the light is λ_0 = 1.5 μm.
(a) Determine the numerical aperture NA and acceptance angle. (8%)
(b) What is the range of the propagation velocity along the fiber axis according to geometrical optics? (6%)
(c) What is the maximum value of the core radius R allowed for this fiber to operate at single mode condition for the wavelength range of 1.2~1.6 μm? (5%)
(Hints: The V-number of an optical fiber is defined as $V = 2\pi R(NA)\lambda_0^{-1}$. For single-mode operation, V < 2.405)

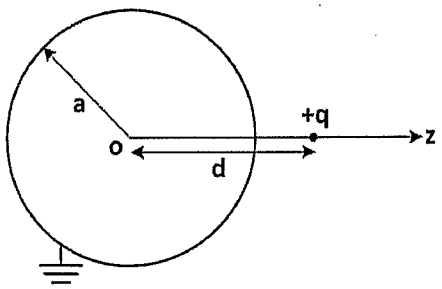


Figure 1

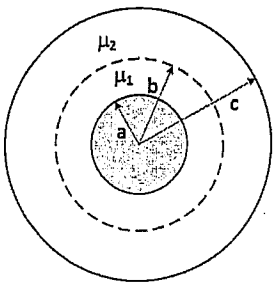


Figure 2

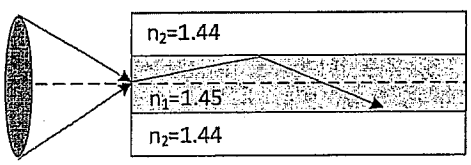


Figure 3

國立中山大學104學年度碩士暨碩士專班招生考試試題

科目名稱：近代物理【光電所碩士班選考】　　題號：435003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

請精簡扼要作答。

1. 列舉四項，你（妳）認為最重要的(32%)
   - 甲、 近代物理學名稱是(8%)
   - 乙、 上述最重要的基本觀念是(8%)
   - 丙、 上述最重要的數學公式是(8%)
   - 丁、 上述最重要的應用是(8%)

2. 一個具相對論性粒子(relativistic particle)的總能量(E)和動量(p)的關聯性，可由下列數學式來表示 $E^2=c^2*p^2+m^2*c^4$，其中 c 是光速，m 是靜止質量。若用 $E=h\omega/2\pi$和 $p=hk/2\pi$關係，其中 h 是普朗克常數，ω是角頻率，k 是波數。
   - 甲、 若 v_p 是相速，v_g 是群速，請計算 $v_p * v_g$ 是等於甚麼？(10%)
   - 乙、 舉一個相對論性粒子之例子。(5%)

3. 一維薛丁格方程式 $\left(-\frac{\hbar^2}{8\pi^2 m}\right)\frac{d^2\psi(x)}{dx^2}+U(x)\Psi(x)=E\Psi(x)$
   - 甲、 請解釋上列三項符號(←+↑=→)及Ψ(x)所代表物理含意。（8%）
   - 乙、 若一質量 m 的粒子，在一個長度為 a 的一維箱子中，其位能在 x=0 及 x=a 是無窮大，而在 0<x<a 是零。請解此一維薛丁格方程式，算出Ψn(x)及 En，n 是整數 (n>0)。(10%)
   - 丙、 簡單說明為何會有量子化現象。(5%)

4. 在單位體積中，含有 n 個質量為 m 的傳導(conduction)電子，
   - 甲、 請導出其費米(Fermi)能量為(10%)
   - 乙、 簡述在導出上述公式時，所用之重要物理概念為(5%)

   (提示: $n_{occupied}=n(E)*P(E)$，$n(E)=\frac{8\sqrt{2}\pi m^{\frac{3}{2}}}{h^3}E^{\frac{1}{2}}$，P 為機率函數)

5. 波長為 11.0pm 的 X 光，被一電子散射時，
   - 甲、 算出此 X 光，在 45 度被散射的波長(5%)
   - 乙、 算出此 X 光，在被散射中之最長波長(5%)
   - 丙、 算出此 X 光在被散射中，反彈(recoil)電子最大動能(5%)

   (提示:電子的 Compton 波長是 2.426pm)

國立中山大學104學年度碩士暨碩士專班招生考試試題

科目名稱：電子學【光電所碩士班選考】　　題號：435004

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　

1. A PN-junction operated reversed bias exhibit I-V characteristic as illustrated in Fig. 1 (a).
   (1) Please state and explain possible mechanisms leading to such character. (10%).
   (2) When operated at the breakdown region, a diode can be used as a rectifier shown in Fig. 1(b). Please plot its equivalent circuit (4%) and find the dc line regulation $\frac{\Delta V_o}{\Delta V^+}$(4%).
   (3) $V^+ = 10V$, $R = 10k\Omega$ and $C = 100nf$. When a small signal $v_s = 0.1sin(2\pi f_s t)$ is added upon V^+, estimate the maximum frequency f_s achievable when the diode still works in breakdown region (4%) and estimate the associated phase angle between signal and output (4%).

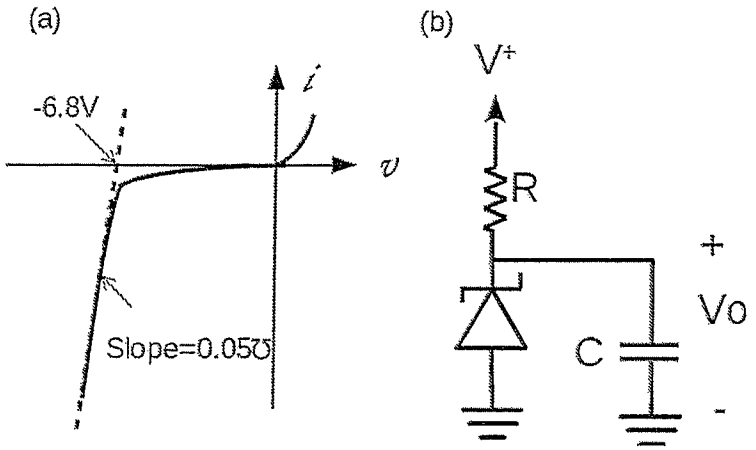


Fig. 1

2. An amplifier circuit is shown in Fig. 2 and its frequency response is given in Fig. 3. For $R_4 = 10k\Omega$, please find the values of R_1, R_2, R_3, C_1, C_2 and g_m(12%). If one insert a resistor $R_5 = 2.2k\Omega$ between node 1 and ground, what kind of feedback topology was utilized (3%)? What is the closed-loop gain $A_f \equiv \frac{V_o}{V_i}$ at zero frequency (3%)?

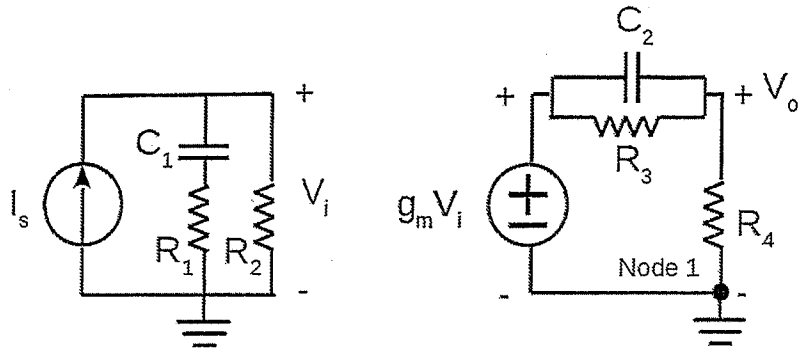


Fig. 2

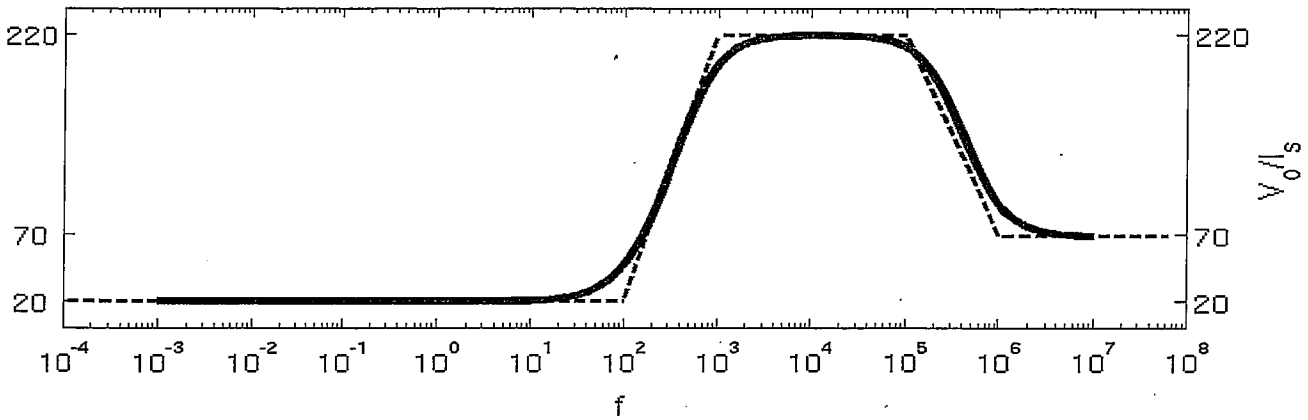


Fig. 3

3. A common-Emitter differential amplifier in Fig. 4(b) is biased by a current source built by circuit in Fig. 4(a).
   (1) Assume the current gains in Q_1 and Q_2 and are β, and in Q_3 it is β'. Obtain the expression for I_{bias}. (5%)
   (2) Assume the Early voltage of Q_3 is V_A, what is the output resistance of the current source in Fig. 4(a). (5%)
   (3) Transistors Q_1,Q_2, Q_4 and Q_5 have identical current gain β and Q_3 has its current gain β'. The Early voltage for Q_1, Q_2, Q_3 and Q_5 is V_A and Q_4 has its early voltage $(1+x)V_A$ where x is much smaller than 1. Find the expression of common mode rejection ratio (CMRR) of the CE differential amplifier in Fig. 4(b). (10%) ** credits without detailed calculation procedure.

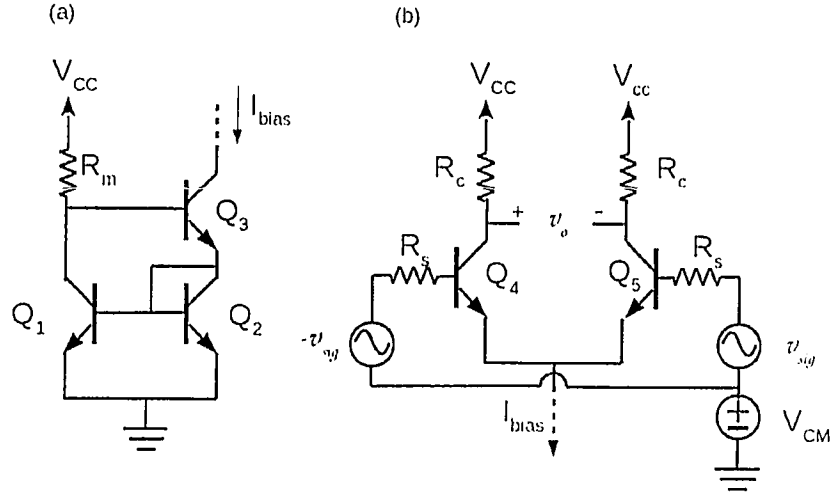


Fig. 4

4. MOSFET characteristics:
   (1) Plot the structure of n-type MOSFET. Indicate in your plot the depletion region at thermal equilibrium when drain and sources are grounded. (4%)
   (2) Explain the how depletion region is formed.(4%)
   (3) Explain the basic operations of n-type MOSFET (4%).
   (4) Explain body effect; how does it affect the device characteristics; plot the equivalent circuit model including the body effect (4%).
   (5) Explain channel length modulation and how it affect the MOSFET characteristics. (4%).
   (6) Two MOSFETs are fabricated on one substrate using the same technology. The dimension of the transistor are listed in table 1 and MOSFET are in operated in saturation region. Calculate the ratio of the following parameters: drain current (2%), channel conductance(2%), Early voltage (2%), output resistance (2%), gate-to-channel capacitance (2%) of the two transistor , i.e.

$$\frac{\text{parameteroftransistor1}}{\text{parameteroftransistor2}}$$

   (7) Explain the operation of CMOS inverters and elucidate the effect if the two transistors are not matched. (6%)

Transistor	Oxide thickness	Channel length	Channel width
#1	4 nm	0.4μm	2μm
#2	6 nm	0.8μm	8μm

Table 1