國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【光電所碩士班】　　題號：435001

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　

Question 1 (20%)

(1) Prove following vector calculations. (10%)

$$|\mathbf{a}\times\mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a},\mathbf{b})^2$$

(2) Calculate the distance between two lines, $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z}{4}$ and $x=y=z$. (10%)

Question 2 (20%)

Calculate the following integration using the Laplace transform. Note that even if you obtain an answer, you do not receive any score unless you use the Laplace transform.

$$\int_0^\infty \frac{\sin ax}{x}dx$$

Question 3 (20%)

The Fourier transform is often used to relate the time domain parameter t and the frequency domain parameter ω. Suppose that a function x(t) has Fourier transform X(jω). Now, consider another function g(t) whose shape is the same as the shape of X(jω), that is

$$g(t) = X(jt).$$

Show that the Fourier transform G(jω) of g(t) has the same shape of 2πx(-t), that is, show that

$$G(j\omega) = 2\pi x(-\omega).$$

Question 4 (20%)

Solve following differential equations to obtain general solutions.

(1) $\cos x \sin y \frac{dy}{dx} = \sin x \cos y$ (6%)

(2) $(x+y+1)\frac{dy}{dx} = x+y-1$ (6%)

(3) $y+\frac{dy}{dx} = xy^3$ (8%)

Question 5 (20%)

Assuming $z = re^{i\theta}$ and $\varsigma = \rho e^{i\varphi}$, calculate following values.

(1) $\mathrm{Re}\frac{z+\varsigma}{z-\varsigma}$ (10%)

(2) $\mathrm{Im}\frac{z+\varsigma}{z-\varsigma}$ (10%)

(END OF QUESTIONS)

國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：電磁學【光電所碩士班】　　題號：435002

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1. (Total:15%) The circular arc of radius a shown in Fig.1 lies in the xy plane and has a constant linear charge density λ and center of curvature at the origin. (a)(10%) Find **E** at an arbitrary point on the z axis. (b)(5%) Show that when the curve is a complete circle your answer becomes

$$\mathbf{E} = \frac{\lambda a \vec{z}}{2\epsilon_0 (a^2 + z^2)^{3/2}}$$

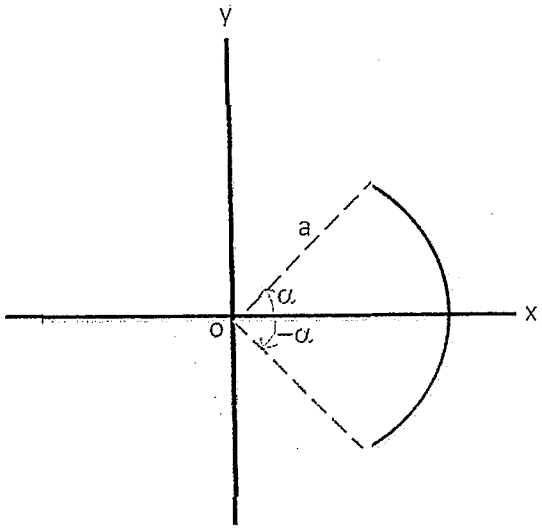

Fig. 1.

2. (Total:15%) (a)(10%) Please find out the potential $\Phi(z)$ in Fig.2 for points on the z axis for positive z. Assume the bound surface charge density on the uniformly polarized sphere is $\sigma_b(\theta') = P\cos\theta'$. (b)(5%) Also please find out the electric field on the z axis outside the sphere (|z|>a).

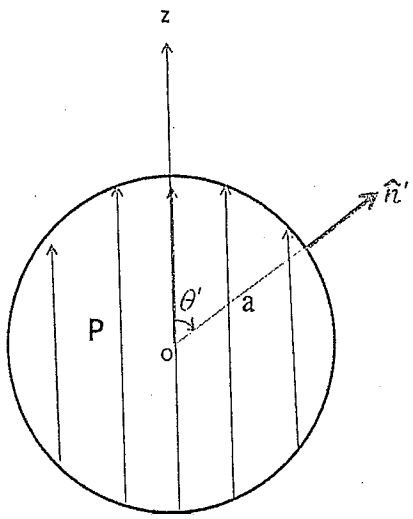

Fig. 2.

3. (10%) Please find the magnetic field a distance z above a long straight wire (finite) carrying a steady current I as shown in Fig. 3.

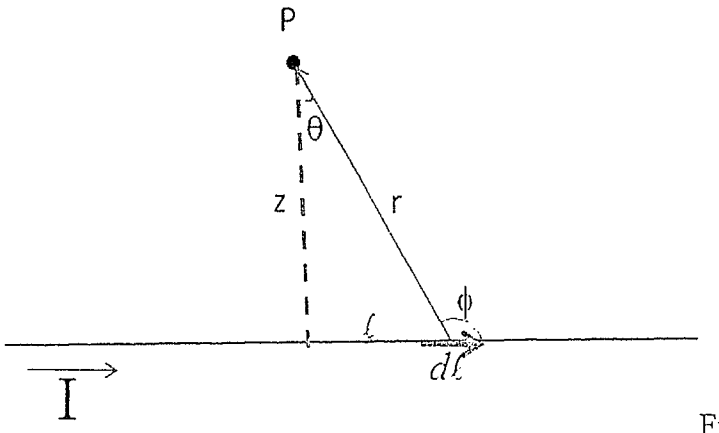

Fig. 3.

4. (15%) Consider the following boundary conditions shown in Fig. 4 and solve the potential $\phi(x,y,z)$ according to the boundary conditions

at x = 0	$\phi(0,y,z)=0$	plane 1
at x = L	$\phi(L,y,z)=0$	plane 2
at y = ∞	$\phi(x,\infty,z)=0$	
at y = 0	$\phi(x,0,z)=f(x)$	stripe 3

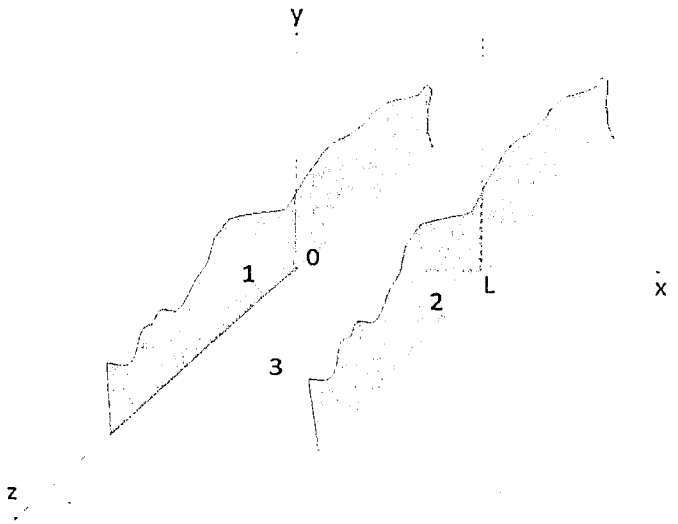

Fig. 4.

5. (Total: 25%) Please refer to Fig. 5. A plane wave traveling in a medium of impedance Z_1 is normally incident at z=0 on a second medium of impedance Z_2. The second medium has thickness L and behind it is another medium of impedance Z_3, which extends to the rest of space. (a)(10%) show that the ratio of the reflected and incident electric field amplitudes in the incident medium is given by

$$\frac{E_{0r}}{E_{0i}}=\frac{Z_2(Z_3-Z_1)\cos k_2L-i(Z_2^2-Z_1Z_3)\sin k_2L}{Z_2(Z_3+Z_1)\cos k_2L-i(Z_2^2+Z_1Z_3)\sin k_2L}$$

(b)(5%) Show that if $Z_1 \neq Z_3$, the reflected wave will be zero when L equals an odd multiple of a quarter wavelength in medium 2 and $Z_2=(Z_1Z_3)^{1/2}$

(c)(5%) Find the corresponding conditions for zero reflected wave when $Z_1=Z_3\neq Z_2$

(d)(5%) If light of wavelength $5*10^{-7}$m is normally incident in a vacuum upon a large slab of glass of index refraction 1.5. If the glass is to be coated with a layer of material in order that the light not be reflected, find the required index of refraction and minimum thickness of the coatings.

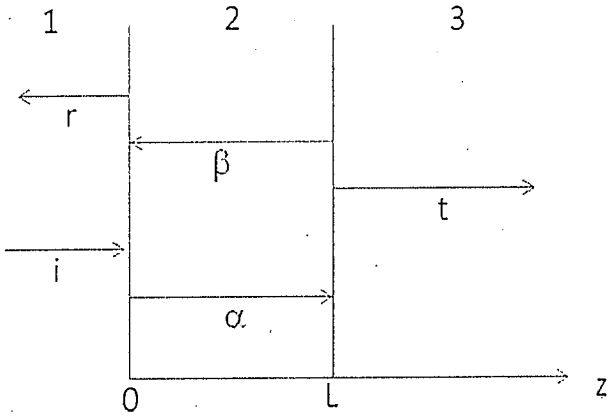

Fig. 5.

6. (20%) Consider the infinitely long coaxial cylindrical conductors shown in Fig. 6. The inner conductor carries a total current I in the $\hat{z}$ direction, while the outer conductor carries a current I in the $-\hat{z}$ direction. Assume the currents to be uniformly distributed over their respective cross sections. Find **B** everywhere and plot your results as a function of ρ(the radial variable).

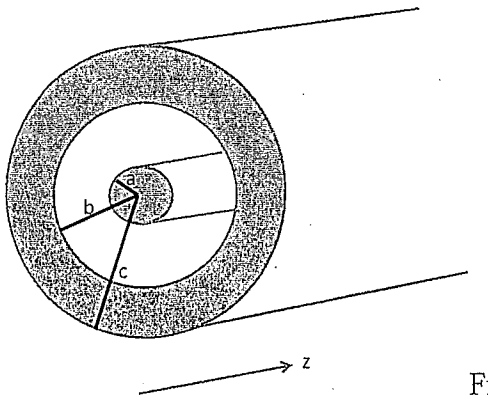

Fig. 6

國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：近代物理【光電所碩士班選考】　　題號：435003

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）　　

一、　簡答題:

1. The pion has an average lifetime of 26.0 ns when at rest. For it to travel 10.0 m, how fast must it move? _____ (5 分)

2. A proton has a kinetic energy of 1MeV. If its momentum is measured with an uncertainty of 5.0%, what is the minimum uncertainty in its position? _____ (5 分)

3. The density of gold is 19 g/cm^3, and its molar weight is 197 g/mol. What is the Fermi energy of gold at 0 K? _____ (5 分)

4. (a) What is the associated operator of energy [E]? _____ (3 分)
   (b) What is the associated operator of momentum [P]? _____
   (3 分)

二、　討論題:

5. For the simple harmonic oscillator, the potential energy function is $U(x)=1/2\ mw^2x^2$,
(a) Write down the time-dependent Schrodinger equation. (5 分)
(b) Derive state energy and wavefunction from Schrodinger equation. (15 分)
(c) Use uncertainty principle to calculate the ground-state energy of a harmonic oscillator. (10 分)
(d) How do the quantum probabilities agree well with classical probabilities? (5 分)

6. The central force on an atomic electron is one directed toward a fixed point, the nucleus.
(a) Derive the radial wave equation with orbital quantum number ℓ and magnetic quantum number m. (10 分)
(b) For hydrogen-like atom, the force is the coulomb force, with its associated potential energy $U=kZe^2/r$. Calculate the radial wave functions $R_{n\ell}(r)$ for n=1, 2 (15 分)
(c) What are the physical meanings of these quantum mumbers (n, ℓ, m) and how do they relate to each other? (9 分)
(d) Explain the Zeeman effect and how do Stern-Gerlach experiment detect space quantization. (10 分)

Electron mass: $9.11*10^{-31}$ Kg
Proton mass: $1.67*10^{-27}$ Kg.
Planck constant: $6.625*10^{-34}$ Js
Boltzmann constant: $1.38*10^{-23}$ J/K

國立中山大學102學年度碩士暨碩士專班招生考試試題

科目名稱：電子學【光電所碩士班選考】　　題號：435004

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(1)(20%)

(a) (5%)Please give an equation to express current flow in a semiconductor material. You should explain all the terms you give.

(b) (5%)Please write down the equation for current-voltage (I-V) relation of a pn diode and also draw a schematic of I-V relation. You should define the reversed and forward bias regimes.

(c) (10%)Please explain the basic operation principles of MOS and BJT, also write down and explain their small signal circuit models.

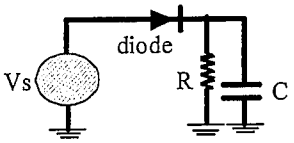

Figure 1

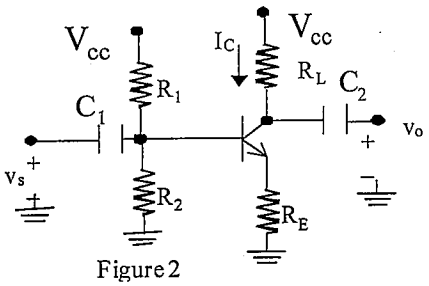

Figure 2

(2) (20%)As shown in figure 1, a diode serially connected with a resistor **R** and a capacitor **C** is served as a rectifier circuit. The voltage source **Vs** is a time-varying voltage source and the output voltage is placed across **R**.

(a) (5%)Please state the operational principle of this circuit and also plot the large signal equivalent circuit model for this circuit.

(b) (10%)If $V_s=V_o \cdot \sin(2\pi \cdot f \cdot t)$, please draw the schematic diagram of the output waveform within a period of time. V_o, f, and t are source amplitude, frequency, and time. If removing the capacitance, what is the difference in output waveform? Please explain all the plots in details.

(c) (5%)If the diode has turn-on voltage of $\mathbf{V_r}$ and series resistance of $\mathbf{R_f}$, use the large signal model to estimate the values of **R** and **C** to get output rectified DC voltage within 10% variation.

(3) (25%)In figure 2, a BJT common-emitter circuit is used for amplifier. BJT has common-base forward short-circuit gain $\alpha_F=0.98$. R_1, R_2, R_L, and R_E are 100, 10, 2, and 1KΩ respectively. $V_{CC}=12V$.

(a) (10%)Determine the operation point of BJT (i.e. I_C and V_{CE}) if ideal BJT is used. And also find the AC gain.

(b) (5%)The linearity of amplifier is quite important to amplifier operation. If the BJT is not ideal, what are the major factors leading to the nonlinearity as large input signal is coupled?

(c) (5%)What are the purposes of C_1 and C_2 in the circuit?

(d) (5%)Draw the equivalent circuit model of circuit at low-frequency and high-frequency regimes.

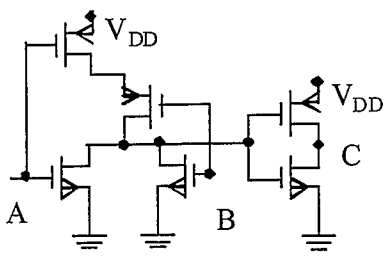

Figure 3

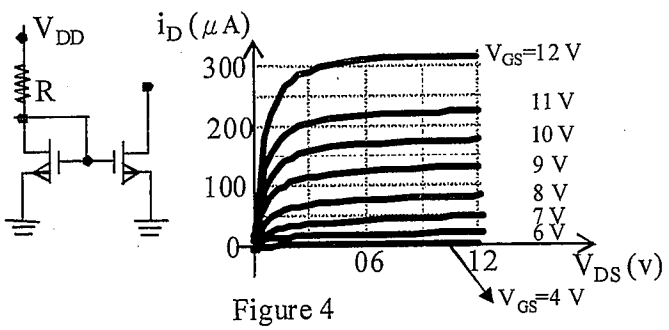

Figure 4

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科目名稱：電子學【光電所碩士班選考】　　題號：435004

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(4) (15%)

(a) (5%)Please draw the cross section of CMOS circuit in IC fabrication.

(b) (10%)Figure 3 plots a CMOS logic circuit with two inputs ports, **A** and **B**, and one output port, **C**. What is the logical function of this circuit? Please verify it by truth table.

(5)(20%)As shown in figure 4, a current mirror is formed by two identical MOS transistors, where the drain current with drain-source voltage (i_D-V_{DS}) at different V_{GS} levels is also shown in figure. **R**=40kΩ and $\mathbf{V_{DD}}$=12V.

(a)(5%) Please explain how the current mirror functions.

(b)(10%) Use i_D-V_{DS} relation to find the i_D of the output transistor. Give all the details in your calculation.

(c)(5%) If the mirrored current should be scaled up to a ratio of **r** (right i_D transistor / left i_D transistor = **r**), how to scale up the geometrical structure of such two MOS transistors? Explain that.