國立中山大學100學年度碩士班招生考試試題

科目：工程數學【光電所碩士班】

(1. 務必在試題紙上作答，且務必按試題順序作答。請盡力發揮。)
(2. 作答務必簡潔，列出重要演算步驟，最後答案不需是數字但必需是最簡化的結果。)

1. 解下列方程式: $y' + y\tan x = \sin 2x, y(0) = 1.$ *(10%)*

2. 解下列方程式: $y'' + y = 2t,\ y(\frac{1}{4}\pi) = \frac{1}{2}\pi,\ y'\left(\frac{1}{4}\pi\right) = 2 - \sqrt{2}.$ *(10%)*

3. 求 Maclaurin series of $f(z) = \frac{1}{(1-z^3)}$, at $z_0 = 0.$ *(10%)*

4. 一個 $|z| = 2$ 的圓形，經 $w = z + \frac{1}{z}$ 映射(mapping) 後，會是何樣圓形?

   (提示:z=x+iy, w=u+iv). *(10%)*

5. 求 (a) $\nabla f(r)$, (b) $\nabla \cdot (\vec{r} f(r))$, and (c) $\nabla x(\vec{r} f(r))$, (提示: $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$). *(9%)*

6. 解下列方程式: $X' = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} X + \begin{bmatrix} 3e^t \\ e^t \end{bmatrix}.$ *(9%)*

7. 求 $\int_0^\infty \frac{dx}{1+x^4} = ?$ *(9%)*

8. 證明 $\cos(\theta) + \cos(\theta + \alpha) + \cdots + \cos(\theta + n\alpha) = \frac{\sin\frac{1}{2}(n+1)\alpha}{\sin\frac{1}{2}\alpha} \cos\left(\theta + \frac{1}{2}n\alpha\right)$。 *(7%)*

9. 敘述你(妳)可想到關於此方程式, $z = 2 + 2i$, 之任何數學觀念和應用。 *(8%)*

10. 簡述下列名詞及其重要性所在: *(18%)*
    - **(a)** Bessel's Functions
    - **(b)** Laplace Method
    - **(c)** Green's Theorem
    - **(d)** Taylor's Series
    - **(e)** Cauchy-Riemann Equation
    - **(f)** Spline interpolation

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科目：電磁學【光電所碩士班】

1. (10%) Please derive the two divergence equations of Maxwell's equations from the two curl equations of Maxwell's equations by making use of the equation of continuity.

2. (12%) A square loop shown in Fig. 1 with side $a = 0.2$ (m) in the xy-plane carries a current $I = 2$ (A) in a uniform magnetic field $\overline{B} = \hat{a}_x 5 + \hat{a}_y 4 + \hat{a}_z 3$ (T). Find the torque on the loop.

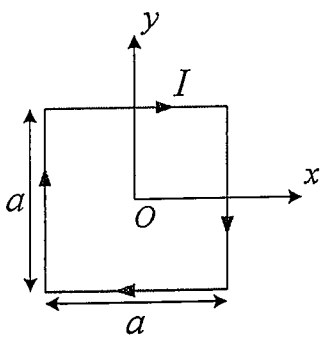

Fig. 1

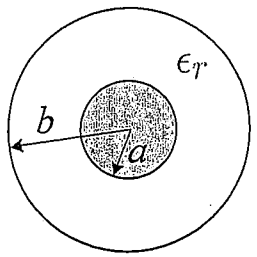

Fig. 2

3. (12%) A very long cylindrical capacitor shown in Fig. 2 consists of coaxial metallic surfaces of radii a and b. The dielectric material between the surfaces has a relative permittivity $\epsilon_r = 1 + (4/r)$. Please determine the capacitance per unit length of the capacitor.

4. (20%) Consider a 100 (Ω) distortionless transmission line. The line has an inductance of 0.25 (μH/m) and the attenuation on the line is 0.04 (dB/m).

   (a) Find the resistance, capacitance, and conductance per meter of the line. (12%)

   (b) Find the distance at which the amplitude of the voltage traveling wave decreases to 1% of its initial value. (8%)

5. (20%) A magnetic dipole formed by a small circular loop of radius a carries a current I and centers at the origin as shown in Fig. 3.

   (a) Find out the vector magnetic potential $\overline{A}$ at a point whose distance, R, from the center of the loop satisfies $R \gg a$. (10%)

   (b) Find out the magnetic flux density $\overline{B}$ from (a). (10%)

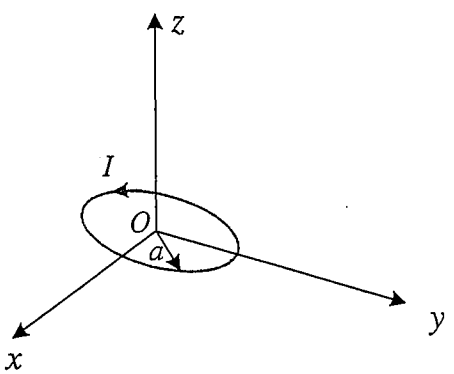

Fig. 3

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科目：電磁學【光電所碩士班】

6. (26%) Consider the problem of the normal incidence at multiple dielectric interfaces for a uniform plane wave in medium 1.

   (a) For a situation shown in Fig. 4(a), there is a dielectric discontinuity at $z = 0$, which can be characterized by an infinite medium with an intrinsic impedance $Z(0)$. Please find out $Z(0)$ and the effective reflection coefficient at $z = 0$. (10%)

   (b) If $\eta_1 \neq \eta_3$ and d is equal to a quarter wavelength in medium 2, please determine η_2 to have no reflection at $z = 0$. (6%)

   (c) Sometimes we cannot find a material having a suitable intrinsic impedance to satisfy the condition of no reflection. We can place one more layer with a thickness b in front of medium 2 to reduce the reflection as shown in Fig. 4(b). If b and d are quarter wavelengths in medium 4 and medium 2, respectively, please express the effective reflection coefficient at $z = -b$. (10%)

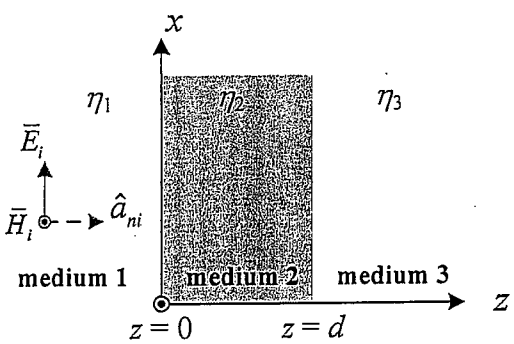

Fig. 4(a)

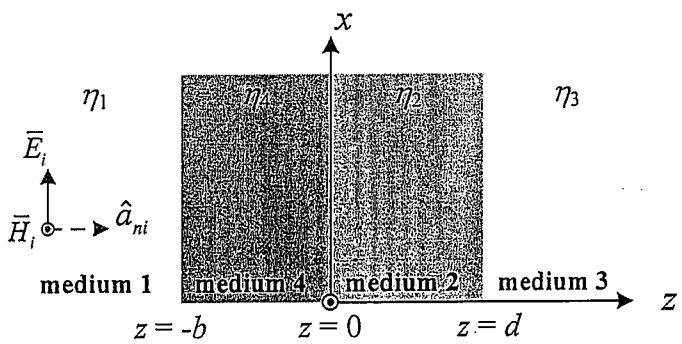

Fig. 4(b)

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科目：電子學【光電所碩士班選考】

(第一部分) 單選題 35%

Figure 1 shows a circuit based on a MOSFET which works as an amplifier. Assume the channel-length modulation cannot be neglected, thus induces an output impedance r_o; please choose the correct answer of the following questions.

1. (5%) What is the input impedance (R_{in}) of the circuit?

(a) R_1 (b) $R_1 + R_S$ (c) $R_1 \| R_S$ (d) $R_1 \| R_D$ (e) ∞

2. (5%) What is the output impedance (R_{out}) of the circuit?

(a) R_D (b) r_O (c) $R_D + r_O$ (d) $R_D \| r_O$ (e) $R_D \| r_O \| R_S$

3. (5%) What is the voltage gain of the circuit?

(a) $g_m R_D$ (b) $-g_m R_D$ (c) $g_m(R_D \| r_O)$ (d) $-g_m(R_D \| r_O)$ (e) $-g_m(R_D \| r_O \| R_S)$

4. (5%) Which one is **unchanged** if C_S is removed?

(a) input impedance (b) output impedance (c) voltage gain (d) all of the above

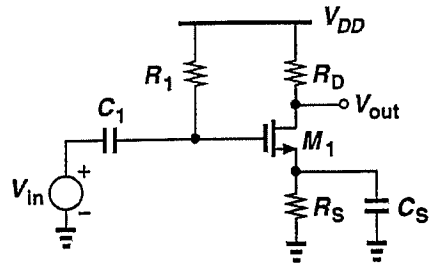

Figure 1

Figure 2 shows a circuit based on two BJT. Assume Early effect can be neglected, and the current gain of the two transistors Q_1 and Q_2 are β_1 and β_2, respectively.

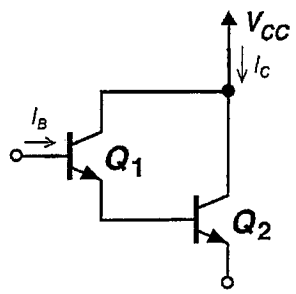

Figure 2

g_{m1} : transconductance of Q1

g_{m2} : transconductance of Q2

$r_{\pi 1}$: small-signal impedance of Q1 seen at the base

$r_{\pi 2}$: small-signal impedance of Q2 seen at the base

5. (5%) If the base of Q_1 is grounded, what is the impedance seen at the emitter of Q_2?

(a) $r_{\pi 1} + r_{\pi 2}$ (b) $r_{\pi 1} \| r_{\pi 2}$ (c) $(r_{\pi 1} \| \frac{1}{g_{m1}}) + r_{\pi 2}$ (d) $\frac{(r_{\pi 1} \| \frac{1}{g_{m1}}) + r_{\pi 2}}{\beta_2 + 1}$ (e) $\frac{(r_{\pi 1} \| \frac{1}{g_{m1}})}{\beta_2 + 1} + r_{\pi 2}$

6. (5%) If the emitter of Q_2 is grounded, what is the impedance seen at the base of Q_1?

(a) $r_{\pi 1} + r_{\pi 2}$ (b) $r_{\pi 1} \| r_{\pi 2}$ (c) $r_{\pi 1} + (\beta_1 + 1) r_{\pi 2}$ (d) $r_{\pi 1} + (\beta_2 + 1) r_{\pi 2}$ (e) $r_{\pi 1} \| (\beta_1 + 1) r_{\pi 2}$

7. (5%) what is the current gain I_C/I_B?

(a) $\beta_1 + \beta_2(1+\beta_1)$ (b) $\beta_1(1+\beta_2)$ (c) $\beta_1 + \beta_2$ (d) $\beta_1\beta_2$ (e) β_2/β_1

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科目：電子學【光電所碩士班選考】

(第二部分) 簡答題 **45%**, 題號請標註清楚

1. (5%) What are Zener breakdown and avalanche breakdown in pn junction?
2. (5%) Please define the drift current and diffusion current of semiconductors.
3. (5%) Draw the large-signal and small-signal model of a BJT operating in active region at low frequency.
4. (5%) A MOSFET can be viewed as a voltage-dependent resistor in triode region. Please draw the I_D-V_G characteristics and state the operational principles.
5. (5%) A MOSFET can be viewed as a voltage-controlled current source in saturation region. Please draw the I_D-V_G characteristics and state the operational principles.
6. (5%) Express the small signal transconductance g_m of a MOSFET operating in saturation region in terms of (a) I_D, V_{GS} and V_T and (b) I_D and W/L.
7. (5%) Please draw the physical structure of a PMOS and a NMOS including substrate, isolation island and contacts.
8. (5%) Please draw the circuit of a simple CMOS inverter and describe the operation principles.
9. (5%) What are the basic requirements on input impedance and output impedance for a "good" amplifier? Why?

(第三部分) 計算題 **20%**, 請寫出計算或推導過程

Figure 3 shows an op amp circuit.

1. (5%) Assume the op amp is ideal. Please derive the voltage gain V_{out}/V_{in} of the circuit and draw the equivalent circuit model.
2. (5%) If the op amp is not ideal and exhibits a finite open-loop gain A, please derive the voltage gain of the circuit.
3. (5%) In Figure 4, consider a capacitor C_1 is in parallel with R_2, please derive the expression for the transfer function $V_{out}(s)/V_{in}(s)$. Assume the op amp is ideal.
4. (5%) What is the dc gain and -3dB bandwidth of the circuit in Figure 4?

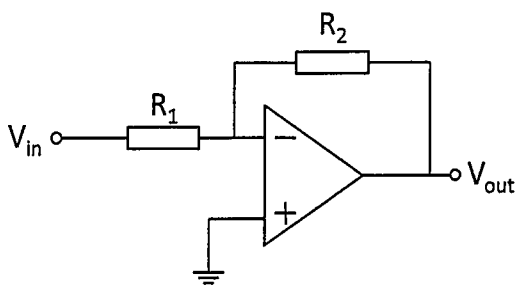

Figure 3

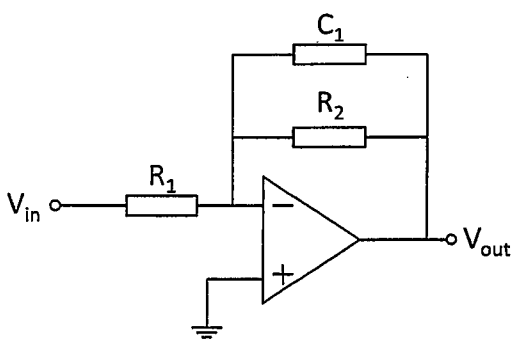

Figure 4

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科目：近代物理【光電所碩士班選考】

1. (20 points) A three-dimensional isotropic harmonic oscillator has the energy eigenvalues $\hbar\omega\,(n + 3/2)$ where $n = 0, 1, 2\ldots$ What is the degree of degeneracy of the quantum state n ?

2. (20 points) Use the variation principle to estimate the ground state energy of a particle in the potential

$$V = \infty \quad \text{for } x < 0,$$
$$V = cx \quad \text{for } x > 0.$$

Take xe^{-ax} as the trial function.

3. (30 points) Write expressions relating the wavelength to their energy for following particles. What are the energies for these particles of wavelength = 1 nm, respectively?
   (a) photons, (10 points)
   (b) electrons, (10 points)
   (c) neutrons. (10 points)

4. (30 points) For particle statistics, there are three distribution functions: (1) Maxwell-Boltzmann distribution, (2) Bose-Einstein distribution, and (3) Fermi-Dirac distribution.
   (a) Write down the three distribution functions. (10 points)
   (b) What are the particle properties of each distribution? (10 points)
   (c) For each distribution, give an example of the particle. (10 points)

Physical constants:

Reduced Planck constant $\hbar = 1.05458 \times 10^{-34}$ J-s

Electron volt 1 eV = 1.60218×10^{-19} J

Boltzmann constant $k_B = 1.38066 \times 10^{-23}$ J/K