國立中山大學資訊工程學系碩士班考試
計算機結構試題

NOTE: The problems have been carefully checked before they are given to you. However, if there are questions which seem unclear or not well-defined to you, don't worry. You can make your own assumptions in such cases. Please state clearly in the answer sheet what's the problems and the assumptions you made. The grading is flexible.

1. Logic Skill (25%)

請判斷以下之句子是否有一個以上之涵義。若有，請列舉之。

1.1 (5%) 張三說瑪麗喜歡自己。

1.2 (5%) 五個正方形和圓形。

已知以下各題之前提爲真，請判斷根據該前提所導出之結論是否正確。

1.3 (5%)

前提 1：假如騎車超速，則被警察罰款。
前提 2：沒被警察罰款
結論： 沒騎車超速

1.4 (5%)

前提 1：假如路平，則車不顛。
前提 2：路不平
結論： 車顛

1.5 (5%)

前提 1：他若感冒，則會發燒。
前提 2：他發燒
結論： 他感冒

2. Digital Logic Circuits (25%)

2.1 (10%) Simplify the following Boolean function in sum-of-products form by means of a three-variable map. Note that in describing the boolean function, the variable x is the most significant bit, while z being the least significant bit.

$$F(x, y, z) = \Sigma(3, 5, 6, 7) \quad \text{(EQ 1)}$$

2.2 (5%) Implement the simplified boolean function in Question 2.1 with AND-OR gates. Assume that both the positive and negative versions of the input signals are available, for example, x and x' ($\bar{x}$). Here the gates can have any number of inputs.

2.3 (10%) Implement the simplified boolean function in Question 2.1 with **only** two-input NAND gates. Assume that both the positive and negative versions of the input signals are available, for example, x and x' ($\bar{x}$).

3. Pipeline (25%)

3.1 (5%) Draw a space-time diagram for a four-segment (four-stage) pipeline showing the time (number of cycles) it takes to completely process four tasks. Assume that each segment takes one cycle to complete its execution.

3.2 (10%) Consider the four instructons in the following program. The instructions execute on a four-segment pipeline. The four segments are: instruction fetch (IF), instruction decode and calculate effective address (ID), fetch operands (FO), and instruction execution (EX). Assume that each segment takes one cycle to complete its execution. Suppose that the first instruction starts from cycle 1. Specify what instructions are performed in the four segments, respectively, during cycle 6.

```
LOAD  R1 ← m[312]
ADD   R2 ← R2 + m[313]
INC   R3 ← R3 + 1
STORE m[314]← R3
```

3.3 (10%) Is there any problem with the above execution during cycle 6? If yes, how can you fix the problem?

4. Performance Analysis (25%)

4.1 (10%) A design team of the MIPS 40000 processor has found that the critical path that sets the clock cycle length of the datapath is data memory access for loads and stores. This has caused their newest implementation of the MIPS 40000 to run at a clock rate of 500 MHz

rather than the target clock rate of 750 MHz. However, a designer has a solution. Originally, the operation of the data memory access takes one clock cycle. If the operation of the data memory access is broken into two clock cycles, then the machine can run at its target clock rate. Use the given instruction type frequency information and CPI information to determine that how much faster the machine with the two-cycle data memory access is compared with the 500 MHz machine with single-cycle datamemory access. The instruction type frequencies are: load (21%), store (12%), arithmatic/logic (46%) and jump/branch (21%). The CPIs for the 500 MHz machine are: load (5), store (4), arithmatic/logic (4) and jump/branch (3).

4.2 (10%) In the above problem, if the data memory access is broken into three clock cycles, instead of two cycles, then the new machine can run at 780 MHz. Please recalculate that how much faster the new machine is compared to the original 500 MHz machine?

4.3 (5%) Which of the possible solutions is faster, the 750 MHz machine or the 780 MHz machine?

註: CPI = cycle per instruction

1. Explain each of the following terms. (12%)
   (a) biconnected component of a graph
   (b) bipartite graph
   (c) indexed sequential file
   (d) AVL tree

2. A *tridiagonal matrix* is a kind of sparse matrix that arises often in numerical analysis. In a tridiagonal matrix A of size n-by-n, $A[i,j] = 0$ if the absolute value of $i - j$ is greater than 1.
   (a) What is the maximum number of nonzero elements in A? (4%)
   (b) Suppose that a linear array B, in the *row-major* style, is used to store $A[i,j]$ if the absolute value of $i - j$ is 1 or less. Note that the first element of A is $A[1,1]$, which is stored in the first location of B, that is $B[1]$. Give a simple method to calculate the location of B storing $A[i,j]$. (8%)

3. A sequence $(a_1, a_2, \cdots, a_n)$ is *bitonic* if there exists i, $1 \le i \le n$, such that $a_1 \le a_2 \le \cdots \le a_i$ and $a_i \ge a_{i+1} \ge \cdots \ge a_n$. For example, (1,2,3,4,10,19) is bitonic since $i = 6$ and (1,7,12,13,14,10,9,5) is also bitonic since $i = 5$. There is an important theorem as follows.
   Given a bitonic sequence $A = (a_1, a_2, \cdots, a_{2n})$, *let* $b_i = min\{a_i, a_{n+i}\}$ *and* $c_i = max\{a_i, a_{n+i}\}$. *Then the two sequences* $B = (b_1, b_2, \cdots, b_n)$ *and* $C = (c_1, c_2, \cdots, c_n)$ *are both bitonic and* $b_i \le c_j$ *for all* i *and* j.
   Suppose that the minimum and maximum of a pair of numbers can be found with a comparison. In other words, b_i and c_i can be obtained with a comparison performed on a_i and a_{n+i}.
   (a) Given a bitonic sequence A of length $n = 2^k$, k is a positive integer, how do you sort A into nondecreasing order by applying the above theorem? Design your algorithm, and use the example (1,7,12,13,14,10,9,5) to illustrate your algorithm. (10%)
   (b) What is the time complexity of your above algorithm? (4%)

4. A binary tree is said to be *strictly binary* if every non-leaf node of the binary tree has exactly two sons. Write a recursive C or Pascal function to determine whether a binary tree is strictly binary or not. You can use the following declaration in your C program. (12%)

```
sturct nodetype {
   int info;
   struct nodetype *left;
   struct nodetype *right;
}
int strict(struct nodetype *tree)   /* return true or false */
```

Or, you can use the following declaration in your Pascal program.

```
type nodeptr= ↑ nodetype;
     nodetype=record;
        info: integer;
        left: nodeptr;
        right: nodeptr;
       end;
function strict(nodeptr tree):boolean;   /* return true or false */
```

You have to state explicitly which language you are using to write the program.

5. **15% [Job and Processor Scheduling]**

The following are common scheduling criteria.

(a) I/O-boundedness of a process
(b) CPU-boundedness of a process
(c) is the process batch or interactive?
(d) urgency of a fast response
(e) process priority
(f) page fault frequency
(g) frequency of a process being preempted by higher priority processes
(h) priorities of processes waiting for resources held by other processes
(i) accumulated waiting time
(j) accumulated execution time
(k) estimated run-time to completion

For each of the following, indicate which of the preceding scheduling criteria is most appropriate.

(1) In a real-time spacecraft monitoring system, the computer must respond immediately to signals received from the spacecraft.
(2) Even though a process has been receiving occasional service, it is making only nominal progress.
(3) How often does the process voluntarily give up the CPU for I/O before its quantum expires?
(4) Is the user present and expecting fast interactive response times, or is the user absent?
(5) One goal of processor scheduling is to minimize average waiting times.

6. **20% [Distributed Computing: The Parallel Computation View]**

Consider the following program segment taken from a Pascal program

```
for i := 1 to 20 do
  if x = 0 then
    y(i) := 3.5
  else y(i) := 5
```

(a) Recode the program segment so that it will run much more efficiently on a uniprocessor.
(b) Assume a four-processor symmetrical multiprocessor is available. Produce the most efficient concurrent program you can to accomplish exactly what the given loop does.
(c) Assume the availability of a symmetric massive parallel processor. Produce the most efficient concurrent program you can to accomplish exactly what

the given loop does.

(d) Compare the execution speed of the given loop with the execution speeds of the programs you produced in answer to (a), (b), and (c). Assume that all the processors have identical capabilities.

7. 15% [Virtual Storage Organization]

In any computer system, regardless of whether it is a real storage system or a virtual storage system, the computer will rarely refer to all the instructions or data brought into real storage. Let us call this *chunk* fragmentation because it is the result of handling storage items in blocks or chunks rather than individually. Chunk fragmentation might actually account for more waste of real storage than all other types of fragmentation combined.

(a) Why, then, has chunk fragmentation not been given the same coverage in the literature as other forms of fragmentation?

(b) How do virtual storage systems with dynamic storage allocation greatly reduce the amount of chunk fragmentation over that experienced in real storage systems?

(c) What effect would smaller page sizes have on chunk fragmentation?

(d) What considerations, both practical and theoretical, prevent the complete elimination of chunk fragmentation?

(e) What can each of the following do to minimize chunk fragmentation?

(i) the programmer

(ii) the hardware designer

(iii) the operating system designer

國立中山大學 89 學年度碩博士班招生考試試題

科 目：離散數學 資訊工程學系碩士班 共 1 頁 第 1 頁

共八題，除第八題外，各題都要寫出解的過程，只有答案的不計分。

1. Let $m > 1$ and $n > 1$ be two positive integers. Let $r(m, n)$ denote the maximum number of rectangles defined by m horizontal lines and n vertical lines in a plane. Derive a formula for $r(m, n)$. Note that rectangles may overlap. For example, $r(2, 3) = 3$, not 2. (10%)

2. Let $S = \{1, 2, \ldots, n\}$. Define a relation $\mathbf{R}$ on S, such that $x \mathbf{R} y$ if and only if $x = 2^k y$ for some integer k.

   (a) Show that the relation R is an equivalence relation. (5%)

   (b) Show the equivalence classes for $n = 20$ and $n = 25$, respectively. (5%)

   (c) Let $\lfloor x \rfloor$ be the largest integer less than or equal to x. Show that if $\lfloor \frac{n+1}{2} \rfloor + 1$ numbers are chosen from the set S, then there must be two numbers a and b such that a is divisible by b. (5%)

3. Solve the following recurrence equation for $T(n)$, $n > 0$. (15%)

$$T(n) = \begin{cases} 0, & \text{if } n = 1 \\ T(\lfloor n/2 \rfloor) + \log n, & \text{if } n > 1. \end{cases}$$

4. Let the sequence of numbers $g_0, g_1, \ldots, g_n, \ldots$ be defined by $g_0 = 0$, $g_1 = 1$ and, for every $n > 1$, $g_n = g_{n-1} + g_{n-2}$. Solve g_n, in terms of n, by the method of generating functions. (15%)

5. Assume that for any two people x and y, x is a friend of y if and only if y is a friend of x. Show that, in any group of two or more people, there are always two people with exactly the same number of friends inside the group. (10%)

6. Suppose that 5 points are chosen in a square whose sides have length 2. Show that there must be at least two points p and q such that the distance between them is no more than $\sqrt{2}$. (Hint: Divide the square into 4 equal parts and then use pigeonhole principle.) (10%)

7. A *tree* is a connected graph without cycles. Show that if G is a tree of more than 1 vertex, then G has at least two vertices of degree 1. (Hint: Consider the longest path of G.) (15%)

8. Suppose the vertices of a binary tree are listed in preorder as A, B, D, F, E, C, and in postorder as F, D, E, B, C, A. Draw the tree. (10%)