國立中山大學115學年度
碩士班考試入學招生考試試題

科目名稱：離散數學與演算法【資工系資安碩班碩士班】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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- 違規者依本校招生考試試場規則及違規處理辦法處理。

國立中山大學115學年度碩士班考試入學招生考試試題

科目名稱：離散數學與演算法【資工系資安碩班碩士班】　　題號：485002

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　

There are 9 problems in this test. Note that you should write down detailed steps for the solution to each problem; otherwise, no credits for that problem will be given.

1. [10%] Write a computer program/algorithm to convert a positive integer in base 64 to base 10.

2. If A = {2,3,4,6}, and B = {1,3,7,8}, give examples of
   (a) [5%] four empty relations from A to B;
   (b) [5%] four nonempty relations on A.

3. (a) [5%] How many two-factor unordered factorizations, where each factor is greater than 1, are there for 22,069,649?
   (b) [5%] In how many ways can 22,069,649 be factored into two or more factors, each greater than 1, with no regard to the order of the factors?

4. (10%) Find the generating function for the sequence 1, 2, 4, 7, 11, ...

5. Two boxes of chocolate bars, 36 bars of one brand and 36 bars of another, are distributed among six surveyors who are conducting taste tests. In how many ways can the 72 bars be distributed so that each surveyor gets
   (a) [10%] at least four chocolate bars of each brand?
   (b) [10%] at least three bars of one particular brand and at least one of the other?

6. Find the exponential generating function for the number of ways to arrange n letters, $n \geq 0$, selected from each of the following words,
   (a) [5%] BOOKKEEPER
   (b) [5%] ACCOMMODATIONIST

7. (Algorithm points) [10%] Please describe the algorithm of Binary Search Tree and analyze its time complexity in detail with mathematical derivation.

8. (Algorithm points) [10%] Please describe the algorithm of Heap Sort and analyze its time complexity in detail with mathematical derivation.

9. (Algorithm points) [10%] Please describe the algorithm of Tower of Hanoi and analyze its time complexity in detail with mathematical derivation.

國立中山大學115學年度
碩士班考試入學招生考試試題

科目名稱：作業系統【資工系資安碩班碩士班】

—作答注意事項—

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國立中山大學115學年度碩士班考試入學招生考試試題

科目名稱：作業系統【資工系資安碩班碩士班】　　題號：485001

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　

INSTRUCTIONS: If any question is unclear or you believe some assumptions need to be made, state your assumptions clearly at the beginning of your answer.

1. [Operating System: 80%]

(1). Please explain the redundant array of independent disks (RAID) and the levels of RAID from RAID 0 to RAID 6, i.e., RAID 0, RAID 1, RAID 2, RAID 3, RAID 4, RAID 5, and RAID 6. Note that both descriptions and figures are required to depict all these levels, such as name, basic idea, and how it works. (20%)

(2). Please explain which four conditions that happen simultaneously in a system may result in a deadlock. Please also explain the prevention, avoidance, detection, and recovery of deadlock. (20%)

(3). In a dynamic protection system, we may encounter issues when revoking access rights to objects shared by multiple users, so we must consider various scenarios. Please explain the following schemes that implement revocation for capabilities: reacquisition, back-pointers, indirection, and keys. (20%)

(4). Please explain the basic idea of cloud computing, edge computing, and fog computing, as well as draw a figure to show their relationships. (10%)

(5). Please explain the basic idea of logic bomb and distributed denial-of-service, as well as draw a figure to show their relationships. (10%)

2. [Security: 20%]

(1). Explain what is Indirect Prompt Injection Attack? (5%)

(2). Explain what is Backdoor Attacks in AI? (3%)

(3). Explain what is Quantum Migration? (3%)

(4). Explain what is Model Inversion Attacks? (3%)

(5). Please describe what is Advanced Persistent Threat (APT)? (3%)

(6). Please explain what AI Hallucination is. (3%)

國立中山大學115學年度
碩士班考試入學招生考試試題

科目名稱：計算機結構【資工系碩士班甲組、乙組】

—作答注意事項—

考試時間：100分鐘

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國立中山大學115學年度碩士班考試入學招生考試試題

科目名稱：計算機結構【資工系碩士班甲組、乙組】
※本科目依簡章規定「不可以」使用計算機(問答申論題)

題號：434001

Note: If any question is ambiguous or insufficiently specified, you are allowed to make reasonable assumptions. Please state and justify these assumptions clearly in your answer.

1. Short Conceptual Questions (20 points)

Q1.1 (7 points)
For several decades, processor performance improvements were largely achieved by increasing clock frequency. However, this approach eventually stopped being the primary driver of performance gains. Modern processors now rely heavily on techniques such as instruction-level parallelism, pipelining, speculative execution, and hierarchical memory systems.

Please explain why increasing clock frequency alone has become an ineffective and unsustainable method for improving processor performance. In your explanation:

1. *(4 points) Identify two fundamental physical or architectural constraints that limit the benefits of higher clock frequencies.*
2. *(3 points) Explain how each constraint directly motivates the architectural shift toward parallelism, prediction, and memory hierarchy.*

(Your answer should be based on core computer architecture principles rather than manufacturing or marketing considerations.)

Q1.2 (7 points)
A program executes on a processor with the following characteristics:

- Total instruction count: 2×10^9 instructions
- Instruction mix:
  - 50% arithmetic instructions, CPI = 1
  - 30% memory instructions, CPI = 5
  - 20% control instructions, CPI = 2
  - Processor clock frequency: 2 GHz

Please answer the following questions:

1. *(3 points) Compute the total CPU execution time of the program.*
2. *(4 points) The processor designer improves the memory system so that the CPI of memory instructions is reduced from 5 to 2. Compute the new execution time and the overall speedup.*

Q1.3 (6 points)
Many architectural optimizations are designed to "make the common case fast." *Please explain what this principle means in practice and why violating it often leads to worse overall performance, even if rare cases become faster.*

2. Performance Analysis (20 points)

Q2.1 (10 points)
A program executes on a processor with the following characteristics:

- 35% of instructions are arithmetic instructions (CPI = 1)
- 45% are memory access instructions (CPI = 3)
- 20% are control instructions (CPI = 2)
- Clock cycle time = 0.4 ns

The program executes 1 billion instructions.
Please calculate:

1. (5 points) *Average CPI*
2. (5 points) *Total CPU execution time*

Q2.2 (10 points)
An architect accelerates a floating-point unit so that floating-point instructions run five times faster. Originally, floating-point instructions account for 30% of total execution time. *Please use the fundamental performance laws to compute the maximum possible speedup of the entire program.*

3. Instruction Set Architecture (15 points)
Q3.1 (8 points)
A processor designer is considering adding complex instructions that directly operate on memory operands (thus potentially doing more work per instruction), which would reduce the number of instructions in programs but increase the complexity and latency of each instruction. *Considering architectural trade-off principles, please explain why modern instruction set architectures favor simple, RISC-style instructions even if this means executing more instructions to accomplish the same task.*

Q3.2 (7 points)
In modern 64-bit processors with fixed-length instructions and deep pipelines, conditional branch instructions almost always use PC-relative addressing, where the branch target is encoded as an offset from the current program counter, instead of using an absolute target address.
Consider the following design and system context:

- Programs may be loaded at different memory addresses due to dynamic linking, shared libraries, or address space layout randomization.
- Branch instructions are frequent and often target nearby instructions such as loop back-edges or conditional blocks.
- Deep pipelines require early branch target calculation to reduce control hazards.
- Instruction cache performance is sensitive to instruction size and code density.

Based on the above context, please answer the following two questions:

(a) (4 points) *State two reasons at the instruction-set / system level why PC-relative addressing is preferred over absolute addressing.*
(b) (3 points) *State one reason at the microarchitectural level why PC-relative addressing is especially beneficial for pipelined processors.*

Your answers should be concise and focus on architectural principles.

4. Computer Arithmetic (15 points)

Q4.1 (8 points)
A processor represents signed integers using 8-bit two's complement format. The most significant bit is used as the sign bit, and arithmetic is performed using standard two's complement rules. Consider the following design and usage context:

- All arithmetic operations are performed modulo 2^8.
- Negation is implemented by bitwise inversion followed by adding 1.
- The same hardware is used for both signed and unsigned arithmetic.

Please answer the following three questions

(a) (3 points) determine the minimum and maximum integers that can be represented in this system.

(b) (3 points) Explain why the set of representable values is asymmetric, i.e., why there is one more negative value than positive value.

(c) (2 points) Identify which specific bit pattern causes this asymmetry and explain its role in two's complement arithmetic.

Your answers should be concise and based on representation properties rather than examples alone.

Q4.2 (7 points)

A processor implements floating-point arithmetic using a fixed-precision binary floating-point format that rounds the result of every arithmetic operation to the nearest representable value. Consider the following scenario:

- You are given a very large array of floating-point numbers whose magnitudes vary widely.
- Two programs compute the sum of all elements in the array:
  - Program A sums the elements from left to right.
  - Program B sorts the elements by magnitude and sums from smallest to largest.
- Both programs use the same floating-point format and run on the same hardware.

Despite using identical arithmetic instructions, the two programs produce different results.

Please answer the following questions:

(a) (4 points) State two fundamental properties of floating-point arithmetic that explain why the two programs may produce different sums.

(b) (3 points) Explain which summation order is expected to produce a more accurate result and why.

Your answers should refer to representation and arithmetic properties of floating-point numbers, not implementation bugs or software errors.

5. Pipelining & Hazards (15 points)

Q5.1 (8 points)

Consider a 5-stage pipeline (IF, ID, EX, MEM, WB). A load instruction is immediately followed by an instruction that uses the loaded value (for example, LW R1, 0(R2) followed by ADD R3, R1, R4). *Please explain in detail: (1) (3 points) Why a data hazard occurs in this scenario, (2) (3 points) Why forwarding alone is insufficient to resolve it, and (3) (2 points) The minimum pipeline stall required (if any) to handle the hazard.*

Q5.2 (7 points)

Modern high-performance processors often use deep pipelines to increase clock frequency. However, deeper pipelines suffer higher penalties when branch predictions are incorrect. Assume the following simplified model:

- A branch is resolved at the end of the pipeline
- A mis-predicted branch causes all younger instructions in the pipeline to be flushed
- Branch prediction accuracy is fixed at 90%
- Each pipeline stage takes 1 cycle

Please answer the following two parts.

(a) Numerical Analysis (3 points)

For the following pipeline depths, please compute the branch misprediction penalty and the average misprediction cost per branch. Please draw a new table on your answer sheet for your answers.

Pipeline Depth (stages)	*Misprediction Penalty (cycles)*	*Average Cost per Branch (cycles)*
5	*?*	*?*
10	*?*	*?*
20	*?*	*?*

(b) Based on your numerical results and graph, explain why deeper pipelines rely more heavily on accurate branch prediction, even when prediction accuracy does not change. (4 points)

6. Cache & Memory Hierarchy (15 points)

Q6.1 (8 points)
A system has a cache and memory with the following characteristics:

- Cache hit time = 1 cycle
- Cache miss rate = 2% (0.02)
- Miss penalty (time to fetch from lower-level memory on a miss) = 100 cycles

Please calculate the average memory access time (AMAT) for this system.

Q6.2 (7 points)
Increasing cache associativity reduces conflict misses but may degrade performance. *Please explain why increasing associativity eventually hurts performance, using architectural cost reasoning.*

國立中山大學115學年度
碩士班考試入學招生考試試題

科目名稱：作業系統與資料結構【資工系碩士班甲組】

—作答注意事項—

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國立中山大學115學年度碩士班考試入學招生考試試題

科目名稱：作業系統與資料結構【資工系碩士班甲組】　　題號：434003

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　

1. (20%) Fill-in questions (2 points each)
   (a) The number of processes currently in memory is known as the degree of ___.
   (b) The modify bit (also called ___ bit) is set when page content is changed.
   (c) The ___ is a module in the CPU-scheduling function, which provides the control of a CPU core to the process selected by the CPU scheduler.
   (d) Any solution to the ___ problem must satisfy three requirements of mutual exclusion, progress, and bounded waiting.
   (e) The main mass-storage system is secondary storage, which is usually provided by hard disk drive devices and ___ memory devices.
   (f) To encapsulate details and oddities of different I/O devices, the kernel of an operating system is structured to employ device-___ modules.
   (g) Among all necessary deadlock conditions, the condition of ___ implies the condition of hold and wait.
   (h) File is a ___ collection of related information that is recorded on secondary storage.
   (i) The ___ principle says that at any time, a process can access only resources that it currently requires to complete its task.
   (j) Breach of ___ means unauthorized reading of data.

2. (a) (6%) What are three cases that cause a running process to give up the CPU core involuntarily? Please also mention the queue it will be placed in.
   (b) (4%) Explain warm cache from the perspective of processor affinity.

3. (a) (6%) Please give three common ways to handle a deadlock problem in operating systems.
   (b) (4%) Explain copy-on-write from the perspective of memory management.

4. (10%) What are the five steps involved in a typical sector-sparing transaction?

5. What is printed by each of the following C programs?
   (a) (5%)

```
int x = 12;
int y = 25;
printf("%d\n", x ^ y & ~x | y << 2);
// ^: bitwise XOR; &: bitwise AND; ~: bitwise NOT; |: bitwise OR; <<: left shift
```

   (b) (5%)

```
int mystery(int *p, int n) {
   static int count = 0;
   count++;
   if (n <= 0) return count;
   if (*p > 10)
      return *p + mystery (p + 1, n - 1);
   else
      return mystery(p + 1, n - 1);
}
int main() {
   int arr[] = {15, 5, 20, 8, 30};
   printf("%d\n", mystery(arr, 3));
   return 0;
}
```

6. (10%) Given the inorder traversal ABCDEFGHIJ and the postorder traversal ABDCGIHJFE of a binary tree, determine the preorder traversal.

7. (10%) Consider a hash table of size $M = 11$, where each bucket can hold only one key. The table uses double hashing. The primary hash function is $h_1(k) = (k \bmod 11)$, and the secondary hash function is $h_2(k) = 7 - (k \bmod 7)$. Insert the keys 11, 33, 66, 7, 77 (in this order) into the empty hash table. Show the final state of the hash table after all insertions.

8. Consider the adjacency matrix of an undirected graph as follows:

	A	B	C	D	E	F
A	0	2	4	0	5	0
B	2	0	1	9	0	7
C	4	1	0	0	6	10
D	0	9	0	0	0	8
E	5	0	6	0	0	3
F	0	7	10	8	3	0

   (a) (8%) Starting from vertex A, show the order in which the edges are added into the minimum spanning tree using Prim's algorithm. Use the weight to represent edges in your answer.
   (b) (2%) Let V and E denote the number of vertices and edges in a graph, respectively. What is the time complexity of Prim's algorithm when it is implemented using an adjacency list and a binary heap?

9. For each of the following sorting algorithms, state its worst-case time complexity using Big-O notation (e.g., $O(n)$).
   (a) (2%) Bubble sort
   (b) (2%) Insertion sort
   (c) (2%) Quick sort
   (d) (2%) Merge sort
   (e) (2%) Heap sort

國立中山大學115學年度
碩士班考試入學招生考試試題

科目名稱：離散數學【資工系碩士班甲組】

—作答注意事項—

考試時間：100分鐘

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- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
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- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學115學年度碩士班考試入學招生考試試題

科目名稱：離散數學【資工系碩士班甲組】　　題號：434004

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　

Simple-Answer Questions

Write down simple answers in answer sheet, e.g. (A1) ***Answer***, with one line for each Question-Answer pair. **7** points per question for a subtotal of **70** points.

(A1) How many arrangements of the letters in ENGINEERING have no adjacent E's?

(A2) How many triplets (x, y, z) satisfy $x + y + z = 10$ with $x, y, z \in \{1, 2, 3, 4, 5\}$?

(A3) How many 3-digit numbers (i.e. from 100 to 999) are divisible by 3 or 5 or 7?

(A4) What is the greatest common divisor of $1,066,601$ and $343,033$?

(A5) Suppose a stone is allowed to move from a square to an adjacent square in the direction of Right, Up, or Right-and-Up. For example, from $(1, 1)$, it can move to $(1, 2)$, $(2, 1)$, or $(2, 2)$. Consider a board of 5×5 squares. What is the number of possible paths that the stone can move from lower-left corner $(1, 1)$ to upper-right corner $(5, 5)$?

(A6) Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the total number of 3-member subsets of S without consecutive integers, e.g. $\{1, 3, 5\}$ but not $\{1, 2, 8\}$?

(A7) What is the smallest positive integer x such that $2^x \equiv 1 \pmod{13}$?

(A8) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many functions $f : A \to A$ satisfy $f^{-1}(\{1, 3\}) = \emptyset, f^{-1}(\{4, 6\}) = \{1, 3, 7\}$, and $f^{-1}(\{7, 9\}) = \{8, 9\}$ simultaneously?

(A9) Let $\Sigma = \{a, b, x, y\}$ and $L = \bigcup_{k=1}^{5} \Sigma^k$. How many strings in L have substring bay?

(A10) Adam and Eve gamble with fair games. The winner takes 1 dollar from the loser in each game. Initially, each has 5 dollars. What is the expected number of games until the first time that one of them has no money left?

Computation Questions

Write down the answer and the inference/deduction process, e.g. (B1) ***Answer. Inference/deduction process***, for each question attempted. **10** points per question for a subtotal of **30** points.

(B1) A fair die has 6 faces $1, \ldots, 6$. Find the probability that the sum of 5 throws of this die is 15.

(B2) Let p, q, r be primitive statements. Construct the truth table for the statement

$$p \leftrightarrow [(q \wedge r) \to \neg(p \vee r)]$$

(Explanation is not required for this question.)

(B3) (Proof question) In PleasantVille, every pair of people *either* know each other *or* are strangers. Adam, Bill, Chad, Dan, Elon, and Fred live in PleasantVille. In this group of 6 people, prove that there is a subgroup of 3 people such that *either* they totally know each other *or* they are totally strangers.

國立中山大學115學年度
碩士班考試入學招生考試試題

科目名稱：工程數學【資工系碩士班乙組】

—作答注意事項—

考試時間：100分鐘

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國立中山大學115學年度碩士班考試入學招生考試試題

科目名稱：工程數學【資工系碩士班乙組】　　題號：434002

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　

A. Ordinary differential equations

1. (20%) Find $z(x)$ to satisfy the following equation and condition

$$z' = z^2 x + x, \text{ and } z(0) = 0$$

2. (20%) Find $v_1(t)$ and $v_2(t)$ to satisfy the following equations and conditions

$$\begin{cases} v_1' = v_2, \\ v_2' = 2v_1 + v_2, \end{cases} \quad \text{and } v_1(0) = 1,\ v_2(0) = 2$$

B. Linear algebra

3. (15%) Find matrices B and C of rank 1 to satisfy $\mathrm{M} = \mathrm{B} + \mathrm{C}$ where

$$\mathrm{M} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

4. (15%) Find matrix A with eigenvalues $\lambda_1 = -1, \lambda_2 = 0, \lambda_3 = 1$ with respective eigenvectors

$$\mathrm{s}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},\ \mathrm{s}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ \mathrm{s}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

C. Fourier series and Fourier integral

A periodic function $f(t)$ of period 2π can be represented by a Fourier series at differentiable points. Specifically

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nt) + b_n \sin(nt))$$

where

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt,\ a_{n\geq 1} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(nt)\, dt,\ b_{n\geq 1} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(nt)\, dt$$

A piecewise continuous function $g(t)$ can be represented by a Fourier integral at differentiable points. Specifically

$$g(t) = \int_0^{\infty} A(\omega) \cos(\omega t)\, d\omega + \int_0^{\infty} B(\omega) \sin(\omega t)\, d\omega$$

where

$$A(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} g(t) \cos(\omega t)\, dt,\ B(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} g(t) \sin(\omega t)\, dt$$

5. (15%) Define

$$\alpha \triangleq 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{2k-1} = \sum_{k=1}^{\infty} \frac{\sin\left((2k-1)\frac{\pi}{2}\right)}{2k-1}$$

Find α with the Fourier series representation of a periodic rectangular wave function $f(t)$ with ± 1 as amplitudes and $(-\pi, \pi)$ as a period, or otherwise.

6. (15%) Define

$$\beta \triangleq \int_0^{\infty} \frac{\sin \omega}{\omega}\, d\omega$$

Find β with the Fourier integral representation of a unit pulse function $g(t)$ over support $(-1, 1)$, or otherwise.