

# 國立中山大學 111 學年度 碩士班暨碩士在職專班招生考試試題

科目名稱：計算機結構【資工系碩士班甲組、乙組】

## — 作答注意事項 —

考試時間：100 分鐘

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# 國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：計算機結構【資工系碩士班甲組、乙組】

題號：434001

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 2 頁第 1 頁

## Problem 1: (10%)

1. (2%) Please explain why we need virtual memory and explain its pros and cons.
2. (2%) TLB is usually much smaller than the page table. Hence, fully associative is a good choice to design the TLB. Is it still a good choice to use fully associative if we enlarge the TLB size? Please explain your reasons.
3. (2%) The multi-level cache is usually applied in modern systems. Please determine what kind of strategy is proper for cache consistency in writing data.
4. (2%) Please explain the difference between the multi-word cache organization and set-associative organization and explain their pros and cons.
5. (2%) Please explain why the nonblocking cache can improve the cache bandwidth.

## Problem 2: (30%)

You have to run a program, which requires 4GB of memory space. Unfortunately, your computer system only allocates 2GB DRAM to this program. Hence, applying the virtual memory technique is necessary, and we assume that the total page size is 4KB. Besides, you also implement the write-back consistency policy and the LRU replacement policy in this virtual memory technique.

1. (10%) How large is the involved page table at least?
2. (10%) We assume that the CPU in your system accesses the data in this program randomly (i.e., no data locality). At the quasi-stationary, please show the approximated page fault rate.
3. (10%) If we have a TLB in this system and the TLB miss rate is 75% at the quasi-stationary, how large is the involved TLB at least?

## Problem 3: (25%)

Now, you have a 32-bit processor, including a cache with 16KB data. We assume that the write-back consistency policy and the LRU replacement policy are both implemented in this cache as well. Please analyze the different architecture designs below.

1. (5%) How many total bits are required if the direct-mapping strategy is involved, and the one-word block is considered?
2. (5%) How many total bits are required if the direct-mapping strategy is involved, and the four-word block is considered?
3. (5%) How many total bits are required if the two-way set-associative strategy is involved, and the four-word block is considered?
4. (5%) TLB is usually adopted in modern systems. If we use the 32-bit address to access the TLB with 10 entries and each page size is 4KB, how large is the involved TLB?
5. (5%) By following the previous question, how large is the involved page table?

## Problem 4: (20%)

The clock cycle time of a given processor is 1ns. We assume only one level cache is installed in this system and every data can be found in the main memory. Besides, we know the miss penalty is 20 clock cycles; the average miss rate is 5%; the cache access time is one clock cycle. Today, we have a program, and 40% of codes can be executed parallelly.

1. (10%) Please find the average memory access time of this system.
2. (10%) How many cores do you have to use to double the performance of this system? Please explain your answer.

# 國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：計算機結構【資工系碩士班甲組、乙組】

題號：434001

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 2 頁第 2 頁

**Problem 5: (15%)**

Figure 1 shows a 5-stage pipelined MIPS CPU to support a simple code in Figure 2. We need to spend one clock cycle to complete all tasks in each pipeline stage.

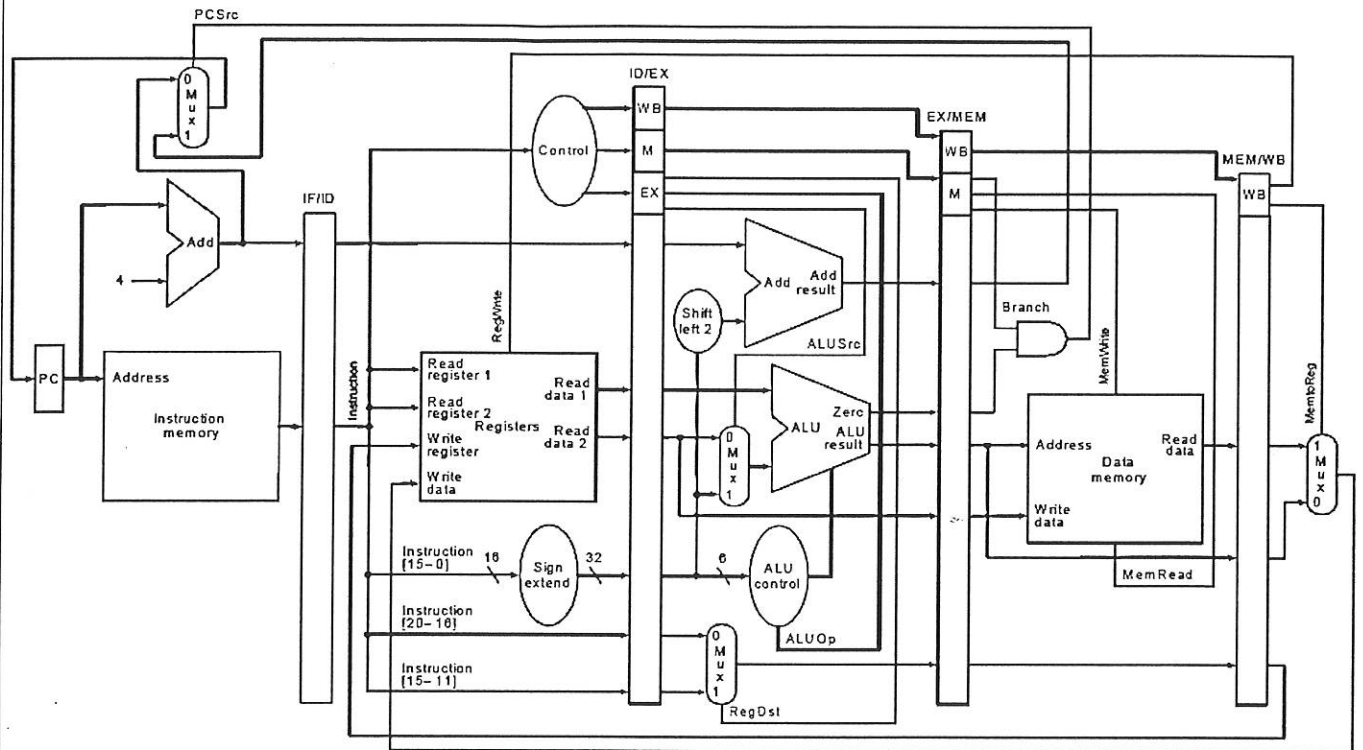


Figure 1

```

sub $2, $1, $3
and $12, $2, $5
or $13, $6, $2
add $14, $2, $2
sw $15, 100($2)
```

Figure 2

1. (5%) Do we need to stall this architecture in Figure 1 to run the code in Figure 2? If yes, please determine how many cycles we should stall and explain your reasons. If not, please describe your reason.
2. (5%) To support the code in Figure 2, please design a minimum viable product (MVP) of the ALU in Figure 1. You need to show the ALU architecture.
3. (5%) Is it possible to move the Adder from the third pipeline stage to the second pipeline stage? If yes, please explain your re-design strategy. If not, please describe your reason.

# 國立中山大學 111 學年度 碩士班暨碩士在職專班招生考試試題

科目名稱：作業系統與資料結構【資工系碩士班甲組】

## —作答注意事項—

考試時間：100 分鐘

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# 國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：作業系統與資料結構【資工系碩士班甲組】

題號：434003

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 2 頁第 1 頁

INSTRUCTIONS: If any question is unclear or you believe some assumptions need to be made, state your assumptions clearly at the beginning of your answer.

1. What are printed by each of the following C program?

(a) (5%)

```
void f(int v[ ], int *w, int *x, int y[ ], int z[ ])
{
    w[2] += 4;
    printf("%d %d %d %d %d \n", v[0], w[2], x[5], *(y+3), z[2]);
}
int main( )
{
    int e[ ]={10,11,12,13,14,15,16,17,18,19,20};
    f(&e[2]+4, &e[2+4], e, e+4, &e[2]);
}
```

(b) (10%)

```
void g(char a[ ], int k, int m)
{
    if (k == m) {
        for (int i = 0; i <= m; i++) cout << a[i] << " ";
        cout << endl;
    }
    else
        for (int i = k; i <= m; i++) {
            swap(a[k], a[i]); // exchange
            g(a, k+1, m);
            swap(a[k], a[i]);
        }
}
int main( )
{
    char a[ ]="abcde";
    g(a, 0, 2);
}
```

2. (10%) Suppose four symbols A, B, C, D are input into a stack sequentially. Four PUSH operations and four POP operations are used for the stack. There are some sequences are impossible output from the all possible combinations of PUSH and POP. Please give the number of each impossible case with AXXX, BXXX, CXXX, DXXX separately.
3. (10%) The following numbers are input into an empty binary search tree sequentially: 3, 6, 4, 5, 2, 7, 1. And, then 3 is deleted from the tree. When a nonleaf node is deleted, its inorder successor will replace its position. Please draw the tree after the deletion of 3.
4. (a) (10%) If we use the heap sort method to sort 14, 4, 19, 2, 18, 7, 17, 9, 16, 13 into increasing order, the first phase is to build a maximum heap. Please draw the heap tree after the first phase finishes.
- (b) (5%) The second phase of the heap sort is to output the solution. Please draw the heap tree after the first maximum is output and the heap is restored.
5. Given an  $i$ -node with ten direct blocks and three levels of indirect blocks and assuming that the sizes of a pointer and a block are, respectively, 8 bytes and 8 Kbytes, answer the following questions.

試題請隨卷繳回，請留意背面是否有題

# 國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：作業系統與資料結構【資工系碩士班甲組】

題號：434003

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 2 頁第 2 頁

- (a) (5%) How many blocks are needed for the smallest file?
- (b) (5%) How many blocks are needed for the largest file?
6. (10%) A computer whose processes have 4096 pages in their address spaces keeps its page tables in memory. The overhead required for reading a word from the page table is 500 nsec. To reduce this overhead, the computer has a TLB, which holds 128 (page, frame) pairs and can do a lookup in 50 nsec. What hit rate is needed to reduce the mean overhead to 100 nsec or less, *assuming that the page table is consulted only if there is a miss*?
7. (10%) A disk has 10000 cylinders, each with 10 tracks of 512 blocks. A seek takes 1 msec per cylinder moved. If no attempt is made to put the blocks of a file close to each other, two blocks that are logically consecutive (i.e., follow one another in the file) will require an average seek, which takes 5 msec. If, however, the operating system makes an attempt to cluster related blocks, the mean interblock distance can be reduced to 2 cylinders and the average seek time reduced to 500 microsec. How long does it take to read a 100 block file in both cases, if the rotational latency is 10 msec and the transfer time is 20 microsec per block?
8. (10%) A computer has 8 GB of RAM allocated in units of 4 KB. How many KB are needed if a bit map is used to keep track of free memory?
9. (10%) A machine has 48-bit virtual addresses and 32-bit physical addresses. Pages are 4 KB. How many entries are needed for the page table?

# 國立中山大學 111 學年度

## 碩士班暨碩士在職專班招生考試試題

科目名稱：離散數學【資工系碩士班甲組】

### — 作答注意事項 —

考試時間：100 分鐘

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# 國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：離散數學【資工系碩士班甲組】

題號：434004

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 1 頁第 1 頁

*There are 8 problems in this test. Note that you should write down detailed steps for the solution to each problem; otherwise, no credits for that problem will be given.*

- [10%] Let  $S$  be a set of seven positive integers the maximum of which is at most 21. Prove that the sums of the elements in all the nonempty subsets of  $S$  cannot all be distinct.
- [10%] Construct a state diagram for a finite state machine with  $I = O = \{0,1\}$  that recognizes all strings in the language  $\{0,1\}^*\{11\} \cup \{0,1\}^*\{01\}$ .
- [10%] Let  $n \in \mathbb{Z}^+$  with  $n > 1$ , and let  $A$  be the set of positive integer divisors of  $n$ . Define the relation  $R$  on  $A$  by  $xRy$  if  $x$  (exactly) divides  $y$ . Determine how many ordered pairs are in the relation  $R$  when  $n$  is 6680.
- [10%] Let  $n$  be a (fixed) positive integer, with  $n \geq 5$ . If  $1 \leq t \leq \lfloor \frac{n}{2} \rfloor$ , how many palindromes of  $n$  start with  $t$ ?
- If a 26-digital ternary (0,1,2) sequence is randomly generated, what is the probability that:
  - [10%] It has an even number of 0's and an even number of 2's?
  - [10%] It has an odd number of 0's, an odd number of 1's, and an even number of 2's.
- [10%] For  $n \geq 1$ , let  $a_n$  count the number of binary strings of length  $n$ , where there are no consecutive 0's. Find and solve a recurrence relation for  $a_n$ .
  - [10%] For  $n \geq 1$ , let  $b_n$  count the number of binary strings of length  $n$ , where there are no consecutive 1's and the last bit of the string is 0. Find and solve a recurrence relation for  $b_n$ .
- [10%] If a first case of COVID-19 is recorded in a country, let  $p_n$  denote the probability that a least one case is reported during the  $n$ th week after the first recorded case in case that the percentage of the population that has received at least two doses of the COVID-19 vaccine achieves 70%. The country records provide evidence that  $p_n = p_{n-1} - (0.3)p_{n-2}$ , where  $n \geq 2$ . Since  $p_0 = 0$  and  $p_1 = 1$ , if the first case (of a new outbreak) is recorded on December 27, 2019, when did the probability for the occurrence of a new case decrease to less than 0.01 for the first time?
- [10%] How many ways are there to place 20 marbles of the same size in six distinct jars if each marble is a different color?



# 國立中山大學 111 學年度 碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學【資工系碩士班乙組】

## — 作答注意事項 —

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# 國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學【資工系碩士班乙組】

題號：434002

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 1 頁第 1 頁

1. (18%) Find the inverse of the following matrix?

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 0 & 2 & 1 \end{bmatrix}$$

2. (14%) Let  $A$  denote the matrix

$$A = \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation given by  $T(\vec{x}) = A\vec{x}$ .

2.1 (6%) Describe  $T$  geometrically.

2.2 (8%) Compute  $A^{2011}$ .

3. (16%) Solve the following differential equations. If an explicit solution cannot be found, leave the solution in an implicit form.

3.1 (8%)  $y' = ye^x - 2e^x + y - 2$

3.2 (8%)  $ty' = t^4 - 2y, y(1) = 0$

4. (20%) Consider the even function

$$f(x) = \begin{cases} x, & -\pi \leq x \leq 0, \\ -x, & 0 < x \leq \pi \end{cases}$$

The Fourier series expansion of  $f(x)$  on  $[-\pi, \pi]$  has the form

$$f \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

Find the  $a_0$  and  $a_1$ .

5. (16%) Use the Laplace transform method to solve

$$y'' - 4y' = 2\delta(t), \quad y(0) = y'(0) = 0$$

where  $\delta(t)$  is the delta function. Determine the solution for all values of  $t$ .

6. (16%) Use the method of variation of parameters to find a particular solution of

$$y'' - 2y' + y = e^t$$

# 國立中山大學 111 學年度 碩士班暨碩士在職專班招生考試試題

科目名稱：作業系統【資工系資安碩班碩士班】

## — 作答注意事項 —

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- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

# 國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：作業系統【資工系資安碩班碩士班】

題號：485001

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 1 頁第 1 頁

INSTRUCTIONS: If any question is unclear or you believe some assumptions need to be made, state your assumptions clearly at the beginning of your answer.

## 1. [Operating System: 80%]

- (1) Processes will never finish in a deadlock. (20%)
  - a. Please show and explain the four representative conditions that hold simultaneously in a system for a deadlock situation to occur.
  - b. Please explain what are the prevention, avoidance, detection, and recovery of deadlock?
- (2) Suppose we have the following set of processes all of which arrive the system at time 0, with the length of the CPU burst given in milliseconds. (15%)
  - a. What are the completion time and average waiting time of these processes by using the first-come, first-served scheduling algorithm?
  - b. What are the completion time and average waiting time of these processes by using the shortest-job-first scheduling algorithm?
  - c. Suppose a smaller priority number stands for a higher priority. What is the completion time of these processes by using a non-preemptive priority scheduling algorithm?

Process	Burst Time	Priority
$P_1$	10	2
$P_2$	1	3
$P_3$	4	4
$P_4$	2	5
$P_5$	6	1

- (3) Please show and explain the system models of nonvirtual machine and virtual machine, and then explain type 1 hypervisor and type 2 hypervisor. (15%)
- (4) What are the segmentation and paging of memory management. (10%)
- (5) What is the redundant array of independent disks (RAID)? Please explain the following levels of RAID: RAID 0 to RAID 6. (10%)
- (6) Module-management system, module loader and unloader, driver-registration system, and conflict-resolution mechanism are four components of Linux, please explain what they do in such system. And, please explain the basic idea of completely fair scheduler (CFS). (10%)

## 2. [Security: 20%]

- (1) Explain what is Cyber Kill Chain? (5%)
- (2) Explain what is a Fileless Malware attack? (4%)
- (3) How to achieve confidentiality, integrity, and non-repudiation by the technique of Digital Envelope based on Public Key Infrastructure? (5%)
- (4) What is a Honeypot system in cybersecurity? (3%)
- (5) What is the concept of Defense-in-Depth in cybersecurity? (3%)

# 國立中山大學 111 學年度

## 碩士班暨碩士在職專班招生考試試題

科目名稱：離散數學與演算法【資工系資安碩班碩士班】

### — 作答注意事項 —

考試時間：100 分鐘

- 考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
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- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

# 國立中山大學 111 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：離散數學與演算法【資工系資安碩班碩士班】

題號：485002

※本科目依簡章規定「不可以」使用計算機(問答申論題)

共 1 頁第 1 頁

*There are 10 problems in this test. Note that you should write down detailed steps for the solution to each problem; otherwise, no credits for that problem will be given.*

1. [10%] If  $R$  is a ring with unity and  $a, b$  are units of  $R$ , prove that  $ab$  is a unit of  $R$  and that  $(ab)^{-1} = b^{-1}a^{-1}$ .
2. [10%] In how many ways can Alice place 36 different books on five shelves so that there is at least two books on each shelf? (For any of these arrangements consider the books on each shelf to be placed one next to the other, with the first book at the left of the shelf.)
3. [10%] Let  $S$  be a set of six positive integers the maximum of which is at most 14. Prove that the sums of the elements in all the nonempty subsets of  $S$  cannot all be distinct.
4. [10%] A company buys 22 vending machines, each of them is to be assigned to one of six sub-divisions. Each sub-division will get at least two vending machines. In how many ways can these assignments be made?
5. [10%] If a first case of chickenpox is recorded in a country, let  $p_n$  denote the probability that a least one case is reported during the  $n$ th week after the first recorded case in case that the percentage of the population that has received at least one dose of the chickenpox vaccine achieves 85%. The country records provide evidence that  $p_n = p_{n-1} - (0.25)p_{n-2}$ , where  $n \geq 2$ . Since  $p_0 = 0$  and  $p_1 = 1$ , if the first case (of a new outbreak) is recorded on November 23, 2020, when did the probability for the occurrence of a new case decrease to less than 0.008 for the first time?
6. [10%] In how many ways can one parenthesize the product  $stuvwxyz$ ?
7. [10%] A wheel of fortune has the numbers from 1 to 16 painted on it in a random manner. Show whether that regardless of how the numbers are situated, there are three consecutive numbers, whose total is 23 or more, on the wheel or not.
8. (Algorithm points) [10%] Please express heapsort algorithm and analyze its time complexity in detail.
9. (Algorithm points) [10%] Let  $T = (V, E)$  be a complete  $n$ -ary tree of height  $h$  with  $q$  leaves. Prove that  $q \leq n^h$  and  $h \geq \lceil \log_n q \rceil$ .
10. (Algorithm points) [10%] Given a knapsack with maximum capacity  $W$  and a set  $S$  consisting of  $n$  items, where each item  $i$  has weight  $w_i$  and benefit value  $b_i$ . Please describe an algorithm based on dynamic programming for packing the knapsack to achieve a maximum total benefit value of the picked items and analyze the time complexity of the algorithm. ( $w_i, b_i$ , and  $W$  are integer values.)