國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：計算機結構【資工系碩士班甲組、乙組】

題號：4078

NOTE: Please simply your computation into decimal numbers whenever possible.

1 (15%) You are going to enhance a machine, and there are two possible improvements: either make multiply instructions run four times faster than before, or make memory access instructions run two times faster than before. You repeatedly run a program that takes 200 seconds to execute. Of this time, 50% is used for multiplication, 20% for memory access instructions, and 30% for other tasks. Determine the speedup for the corresponding cases. Copy Table 1 to your answer sheet and fill your answers.

Enhancement cases:

C1: Improve only multiplication

C2: Improve only memory access

C3: Improve both multiplication and memory access

Table 1.

Enhancement case	Speedup
C1	
C2	
C3	

2 (5%) The dynamic power of a processor is related to the capacitive load (C), operation frequency (F) and voltage (V) with this equation $Power=CFV^2$. Suppose we developed a new, simpler processor that has 80% of the capacitive load of the more complex older processor. Further, we can reduce the voltage of the new processor by 20% and let the new processor run at 20% faster frequency. What's the dynamic power of the new processor (P_{new}) with respect to the dynamic power of the old processor (P_{old})?

3 (10%) The following C procedure `swap` is compiled into the corresponding MIPS assembly code. Assume that the parameters `v` and `k`, and variable `temp` are in registers `$a0`, `$a1` and `$t0`, respectively. Other registers that can be used are `$t1` and `$t2`. C procedure:

```
void swap(int v[], int k)
{
   int temp;
   temp = v[k];
   v[k] = v[k+1];
   v[k+1] = temp;
 }
```

MIPS assembly code:

```
Swap:  sll   $t1, $a1,  2
       add   $t1, $a0, OPA
       lw    $t0, 0($t1)
       lw    $t2, 4(OPB)
       sw    OPC, 0($t1)
       sw    OPD, 4($t1)
       jr    $ra
```

Please determine proper values for the operands (OPA, OPB, OPC, OPD). Copy the following table (Table 3) to your answer sheet and fill in the operand values.

Table 3.

Operand	Value
OPA	
OPB	
OPC	
OPD	

4 (10%) Figure 4.1 is a basic multiplication hardware and Figure 4.2 is an improved version of the basic one.

Figure 4.1: Basic multiplication hardware

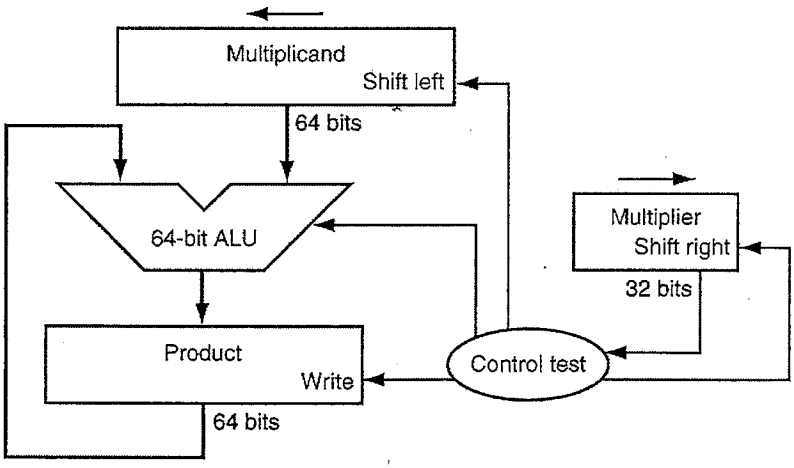
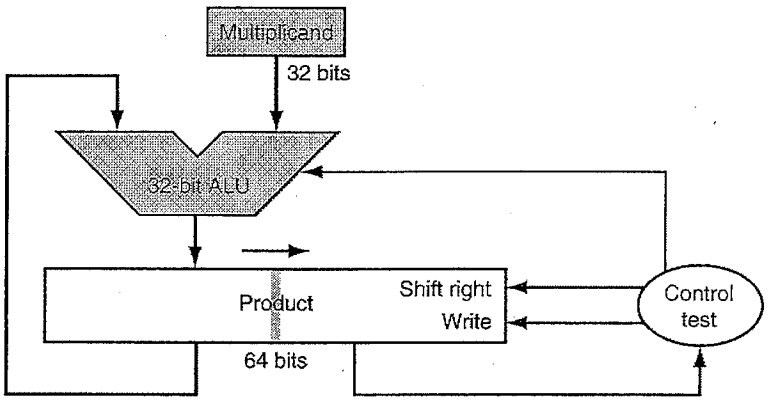


Figure 4.2: Improved multiplication hardware



The following statements are possible observations for the improvement:

S1: The multiplicand register, ALU, Multiplier register and the Product register are all 32 bits wide;

S2: The Product register always shifts right two bits in each iteration;

S3: The original Multiplier register disappears and is now placed in the right half of the Product register initially;

S4: The left side of the Product register is initially set to zero.

S5: The Multiplicand register is initially set to zero.

Please copy the following table (Table 4) to your answer sheet and determine if these statements are true or false.

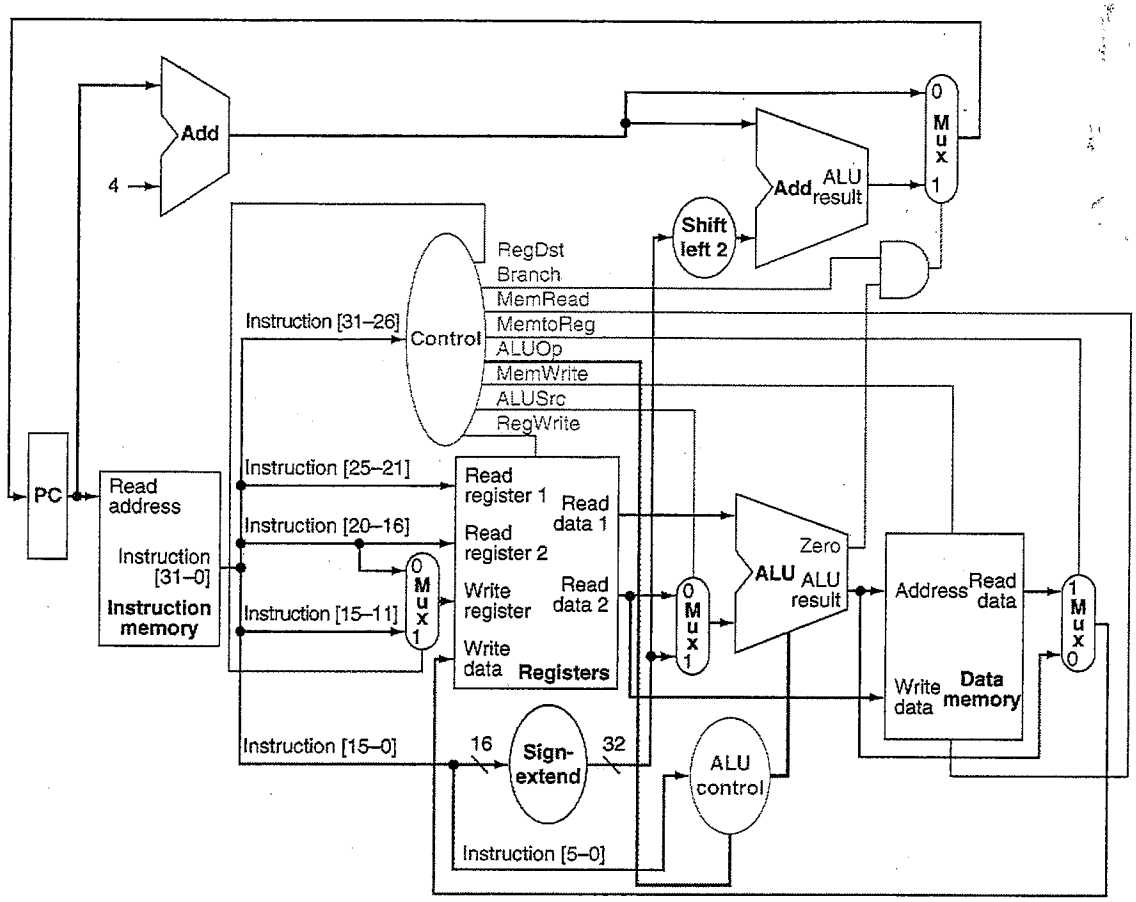
Table 4:

Statement	True (T) or False (F)
S1	
S2	
S3	
S4	
S5	

5 (20%) The following figure is a simple implementation of a MIPS subset (`R-type, lw, sw, beq`). Copy Table 5 to your answer sheet and fill in the corresponding control signal values (0, 1 or X) for instructions `lw` (load word) and `beq` (branch on equal).

Table 5:

Instruction	ALUSrc	MemtoReg	RegWrite	MemWrite	RegDst
`lw`					
`beq`					



6 (10%) There are three small caches, each consisting of four one-word blocks. The first cache is direct mapped, the second cache is two-way set associative, and the third cache is fully associative. Find the numbers of misses for each organization given the following sequence of block addresses: 8, 0, 8, 6, 0. Copy Table 6 to your answer sheet and fill in the answers. Assume that the least recently used replacement policy is used for the fully associative cache and the set-associative cache.

Table 6:

Cache organization	Number of misses
Direct mapped	
Two-way set associative	
Fully associative	

7 (Total 15%) Suppose we want to sum 100,000 numbers on a single-bus multiprocessor computer. Let's assume we have 10 processors. The first step would be to split the set of numbers into subsets of the same size. All processors start the program by running the following loop that sums their subset of numbers, where Pn is its processor index (0~9):

sum[Pn] = 0;

```
for (I = 10000 * Pn; I < 10000 * (Pn+1); I = I + 1)
    sum[Pn] = sum[Pn] + A[I]; // sum the assigned areas
```

The next step is to add these many partial sums, so we divide to conquer. Half of processors add pairs of partial sums, then a quarter add pairs of the new partial sums, and so on until we have the single, final sum. We want each processor to have its own version of loop counter variable *I*, so we must indicate that it is a "private" variable. In this problem, the two processors must synchronize before the "consumer" processor tries to read the result from the memory location written by the "producer" processor; otherwise, the consumer may read the old value of the data. Here is the code (*half* is also a private variable):

```
half = 10; //10 processors in 1-bus multiprocessor.
repeat
        synch(); // wait for partial sum completion.
        if (half%2 != 0 && Pn == 0)
            sum[0] = sum[0] + sum[half - 1]; //the case for odd number of
                                             //summing processor
        half = half / 2; // dividing line on who sums.
        if (Pn < half) sum[Pn] = sum[Pn] + sum[Pn+half];
until (half == 1); // exit with final sum in sum[0].
```

Question: according to the algorithm, find out what operations are executed by the designated processor during the designated repeat-loop iteration. (Ex: NOP or sum[0] = sum[0] + sum [4]). Copy Table 7 to your answer sheet and fill in the answers.

Table 7:

Processor index	Repeat-loop iteration	Operation
Pn=2	first	
Pn=2	second	
Pn=0	third	

8 (15% total) Consider a two-bit adder with its two operands A_i and B_i where i=0, 1 (0 is the least significant bit) and the carry-in bit C_0 and carry-out bit C_2.

8.1 (5%) Derive the Boolean equation for the carry-out bit C_2 using only A_i, B_i (where i=0, 1) and C_0.

8.2 In a carry-lookahead adder, there are two additional signals defined, $G_i=A_i \cdot B_i$ (generate), $P_i=A_i+B_i$ (propagate). Derive the Boolean equation for the carry-out bit C_2 using only G_i, P_i (where i=0, 1) and C_0.

8.3 (5%) What are the advantages of the Boolean equation in Problem 8.1 over the Boolean equation in Problem 8.2?

國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：工程數學【資工系碩士班乙組】

題號：4079

1. (15%) Please find the solution of y for the first order differential equation.

$$y' + \frac{1}{x}y = 3x^2, \quad \text{and } y(1) = 5$$

2. (15%) Please find the particular solution of y for the second order differential equation.

$$y'' + 2y' - 3y = 4e^x$$

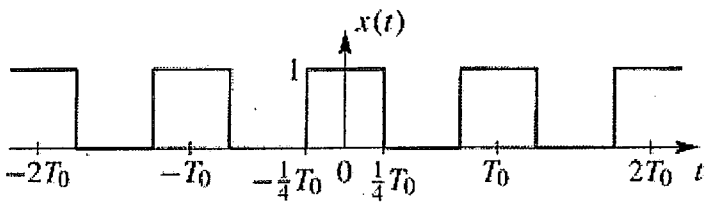
3. (18%) A signal $x(t)$ is a periodic with period T_0=8 and can be expressed as $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/8)kt}$. The Fourier coefficients of this particular signal are given by the integral $\frac{1}{8}\int_{-4}^{0}(4+t)e^{-j(2\pi/8)kt}dt$

3.1 (6%) In the integral expression for a a_k above, the integrand and the limits define the signal $x(t)$. Determine an equation for $x(t)$ that is valid over one period.

3.2 (6%) Using the result from (3.1), draw a plot of $x(t)$ over the range $-8 \le t \le 8$ seconds. Label your plot carefully.

3.3 (6%) Determine the DC value of $x(t)$.

4. (12%) Considering the periodic function $x(t)$ plotted in following Figure,



4.1 (6%) Find the DC value a_0 and other Fourier coefficients a_k for $k \neq 0$ in the Fourier series representation of $x(t)$.

4.2 (6%) Define a new signal as $y(t)=2x(t-T_0/2)$. Use the time shifting property to write down the Fourier series coefficient b_0 and b_k for $k \neq 0$ for the periodic signal $y(t)$ without evaluating any integrals.

5. (20%) Find the inverse Laplace transform of $G(s) = \dfrac{s}{(s+3)(s^2+4s+5)}$

6. (20%)

6.1 (10%) Find the LU-factorization of the matrix $A = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 3 \\ 2 & -10 & 2 \end{bmatrix}$,

6.2 (10%) Use the result of 6.1 to solve $\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 3 \\ 2 & -10 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 11 \\ -12 \end{bmatrix}$

國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：作業系統與資料結構【資工系碩士班甲組】

題號：4080

※ There are totally ten questions, each with 10 points. Please detail your answer for each question in the answer sheet.

1. Let us consider a set of processes as follows, where they have arrived in the order of P1, P2, P3, P4, and P5 all at time 0:

Process	Burst time	Priority
P1	10	3
P2	1	1
P3	2	3
P4	1	4
P5	5	2

   (a) (4%) Please draw four Gantt charts to show the execution of all processes based on the following scheduling algorithms: First-come first-served (FCFS), round robin (RR with quantum = 1), shortest job first (SJF), and non-preemptive priority (where a smaller priority number indicates a higher priority).
   (b) (4%) Calculate the turnaround time of each process in the four scheduling algorithms.
   (c) (2%) Which of the algorithms results in the minimum average waiting time? Note that you should give your calculation.

2. (a) (8%) What are the four conditions to result in a deadlock? Please detail each condition in your answer.
   (b) (2%) Will a deadlock necessarily arise when any of the four conditions occur? Why or why not? Please explain your reason.

3. In virtual-memory management, demand paging plays an important role.
   (a) (2%) Please explain what is demand paging?
   (b) (4%) To support demand paging, what are the two types of hardware will you require? Note that you should give the functionality of each hardware.
   (c) (4%) Suppose that you design demand paging for an operating system. Assume that the memory-access time is 200 nanoseconds and the average page-fault time is 75 microseconds. If you want the effective access time for demand paging no larger than 230 nanoseconds, what will be the expected page-fault rate? You need to give your calculation.

4. (10%) In a multimedia system, SCAN-EDF is a popular disk scheduling scheme. Suppose that you have the following file requests (request, deadline, cylinder):
   (A, 150, 25), (B, 201, 112), (C, 399, 95), (D, 94, 31), (E, 295, 185), (F, 78, 85), (G, 165, 150), (H, 125, 101), (I, 300, 85), (J, 210, 90).
   Considering that the disk head is currently on cylinder 49 and moving toward cylinder 50, please show how the disk head moves to satisfy all requests using SCAN-EDF.

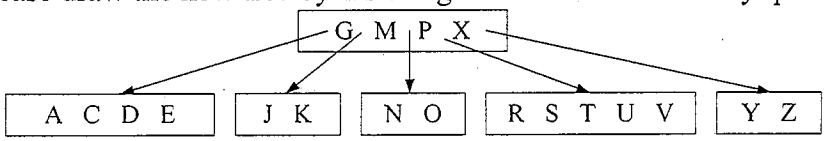
5. (10%) Authentication is useful for proving that a message has not been modified during transmission. The message-authentication code and the digital-signature algorithm are two popular authentication solutions. Please explain how they operate.

6. (10%) In a local-area network, sites may want to transmit data over a link simultaneously. If nothing is done, this will cause serious collision. CSMA/CD and token passing are two popular techniques developed to avoid the aforementioned collision. Please explain how CSMA/CD and token passing operate.

7. (a) (8%) Given n objects to be sorted, please give the worst-case and the average-case running time of the following algorithms: Insertion sort, merge sort, heap sort, and quick sort.
   (b) (2%) One guy said that he has invented a comparison-based sorting algorithm, called Supersort, with the worst-case running time of $O(n)$. Do you think that Supersort is possible? Please give your reason.

8. (10%) Suppose that a 12000-character article is composed of only alphabets I, J, K, L, M, and N, which appear 5400, 1560, 1440, 1920, 1080, and 600 times, respectively. Please draw the Huffman coding trees for fixed-length codeword and variable-length codeword.

9. Consider a B-tree as follows, where the minimum degree t for this B-tree is three.
   (a) (2%) How many keys can be held in each node? Please give your calculation.
   (b) (2%) Please draw the new tree by inserting B.
   (c) (2%) Please draw the new tree by inserting Q in the tree formed by question (b).
   (d) (2%) Please draw the new tree by inserting L in the tree formed by question (c).
   (e) (2%) Please draw the new tree by inserting F in the tree formed by question (d).



10. Consider the adjacency matrix of an undirected graph as follows:

	A	B	C	D	E	F
A	0	36	0	0	0	25
B	36	0	12	0	6	11
C	0	12	0	15	3	0
D	0	0	15	0	20	0
E	0	6	3	20	0	9
F	25	11	0	0	9	0

   (a) (5%) Please show the order in which the edges are added to the minimum spanning tree using the Kruskal's algorithm.
   (b) (5%) Assuming that vertex A is the root, please show the order in which the edges are added to the minimum spanning tree using the Prim's algorithm.
   (Notice: In your answer, you can use weights to represent edges.)

國立中山大學101學年度碩士暨碩士專班招生考試試題

科目：離散數學【資訊工程學系碩士班】

題號：4081

There are 9 problems in this test. Write down detailed steps for the solution to each problem. Otherwise, no credits for that problem will be given.

1. (10%) Assume that a sequence of numbers is defined by $x_0 = 0$, $x_1 = 1$, and $x_n + 2x_{n-1} = 15x_{n-2}$ for $n > 1$. Find the generating function for the sequence, and then find an explicit expression for x_n.

2. (10%) Solve the following recurrence equation for $T(n)$, $n > 0$.

$$T(n) = \begin{cases} 0, & \text{if } n = 1 \\ T(\lfloor n/2 \rfloor) + \log n, & \text{if } n > 1. \end{cases}$$

3. (10%) Assume that, for any two people x and y, x is a friend of y if and only if y is a friend of x. Show that, in any group of two or more people, there are always two people with exactly the same number of friends inside the group.

4. (10%) Let G be a simple graph. A *path* of G is a sequence of distinct vertices $v_0, v_1, \ldots, v_k$ such that v_{i-1} and v_i are adjacent for each $i = 1, 2, \ldots, k$. The *length* of the path $v_0, v_1, \ldots, v_k$ is k. The *degree* of a vertex is the number of edges incident to that vertex. Show that if the minimum degree of G is greater than or equal to k, then G has a path whose length is at least k.

5. (15%) A *lattice path* from (x_0, y_0) to (x_n, y_n) in the xy plane is defined as a sequence of points $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ such that each $x_{i+1} = x_i + 1$, and each $y_{i+1} = y_i \pm 1$, $i = 1, 2, \ldots, n-1$. How many lattices paths are there from $(0, 1)$ to $(10, 3)$? How many of them do not touch or cross the x axis?

6. (10%) Fibonacci numbers are defined as $f_0 = 0$, $f_1 = 1$ and $f_n = f_{n-1} + f_{n-2}$ for $n > 1$. Show that f_{3k} is even, for every positive integer k.

7. (10%) Let $S = \{1, 2, \ldots, n\}$.

   (a) Prove that if n is even then any $n/2 + 1$ subset of S contains two numbers whose sum is $n + 1$.

   (b) In general, determine the value k such that any k subset of S contains two numbers whose sum is $n + 1$.

8. (10%) A graph G can be embedded in a plane if it can be drawn in the plane in such a way that no edges intersect. Similarly, a graph G can be embedded in the surface of a sphere if it can be drawn in the surface in such a way that no edges intersect. Show that, for any graph G, G can be embedded in a plane if and only if G can be embedded in the surface of a sphere.

9. (15%) A *theorem* of a propositional calculus is a proposition that is always true. Determine which of the followings are theorems. Justify your answers.

   (a) $(x \Rightarrow x)$

   (b) $\neg(x \Leftrightarrow x)$

   (c) $(((x \Rightarrow y) \wedge (\neg(x) \Rightarrow y)) \Rightarrow y)$

   Note that the symbol $\Rightarrow$ denotes `imply`, $\Leftrightarrow$ denotes `if and only if`, $\wedge$ denotes `and`, $\vee$ denotes `or`, and $\neg$ denotes `not`.