科目：工程數學甲【通訊聯招甲組、電機系甲、丁、戊、庚組、通訊所乙組；電機系丙組選考】

1. (15%) Solve the differential equation

$$\frac{dy_1}{dx}+3y_1+3\frac{dy_2}{dx} = 2e^{-2x}$$
$$\frac{dy_2}{dx}+2y_2-y_1 = 1$$
with $y_1(0)=y_2(0)=0$.

2. (15%) Use the power series method to solve the following equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y = 0$$

where n is a nonnegative integer.

3. (20%) Let $L: V \to W$ be a one-to-one and onto, hence invertible, linear transformation between vector spaces V and W. Denote its inverse by L^{-1}, hence L^{-1} is a transformation from vector space W to vector space V.

(a) Show that L^{-1} is also linear. (8%)

(b) Let $\{\mathbf{z}_1,\cdots,\mathbf{z}_r\}$ be a basis for W. Show that $\{L^{-1}(\mathbf{z}_1),\cdots,L^{-1}(\mathbf{z}_r)\}$ is a basis for V. (12%)

4. (14%) Given matrices $A\in\mathbb{R}^{m\times p}$ and $B\in\mathbb{R}^{m\times q}$, with A being full column rank, consider the linear equation $A\mathbf{x}+B\mathbf{y}=\mathbf{0}$. By choosing a vector $\mathbf{y}\in\mathbb{R}^q$, we may consider to solve vector $\mathbf{x}\in\mathbb{R}^p$ to satisfy the equation.

以下小題僅需依序寫下答案即可，不需做任何推導。

(a) What is the condition on $\mathbf{y}$ so that the equation is consistent, i.e. it is solvable? (4%)

By knowing that every full column rank matrix has a left inverse, let $C\in\mathbb{R}^{p\times m}$ be such that $CA=I$.

(b) If $\mathbf{y}$ is chosen so that the considered equation is consistent, what is the general form of the solution to the equation? (4%)

If $\mathbf{y}$ is chosen so that the considered equation is inconsistent, then the concept of *projection* plays an important role in studying the solution of the equation. Let U and V be two subspaces of $\mathbb{R}^m$ such that $\mathbb{R}^m=U\oplus V$ and let P and Q be the projection matrices that project vectors of $\mathbb{R}^m$ onto U along V and onto V along U, respectively. It means that, $\forall\mathbf{r}\in\mathbb{R}^m$, $P\mathbf{r}\in U$, $Q\mathbf{r}\in V$, and $\mathbf{r}=P\mathbf{r}+Q\mathbf{r}$ is the unique decomposition associated with the two projections.

(c) Let $U=R(A)$, the range of A, and let $V=N(C)$, the null space of C, what is P? (6%)

5. (36%) Each of the following problems has only one correct answer. You will obtain 3 points for each question if you give the correct answer. However, you will get -1 points for each question if the answer is not correct. Zero point will be deducted if you give no answer. The lowest point in this set of problems you may obtain is zero. For instance, if you give one correct answer, four wrong answers, and no answers for the rest of the problems, then you will obtain zero point for this problem set. Note that $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ are the unit vectors in the positive direction of the Cartesian x, y, z coordinate system respectively. If $z=x+jy$ is a complex variable, then x, y are real numbers, and "j" denotes the pure imaginary number $(0,1)$.

第五大題請於答案卷「是非、選擇題專用區」作答。

【5.1 於該區第(1)格作答、5.2 於該區第(2)格作答……，依此類推】

5.1 Consider the following three systems, where $y[n]$ or $y(t)$ denotes the system output, and $x[n]$ or $x(t)$ is the system input:

I. $y[n] = x[-n], \quad -\infty < n < \infty$

II. $$y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n+1], & n \leq -1 \end{cases}$$

III. $y(t) = [\sin(6t)]x(t)$

Which of the following statements is correct?

(a) I is time-invariant, II is linear, III is memoryless
(b) I is causal, II is stable, III is memoryless
(c) I is stable, II is linear, III is causal
(d) I is memoryless, II is stable, III is linear
(e) None of the above statements are correct

5.2 Suppose that the Fourier transform of the function $f(t) = te^{-4t^2}$ is $F(\omega) = ae^{-\omega^2/b}$, then which of the following statements is correct?

(a) a is a real number, and $b > 20$
(b) a is an imaginary number, and $14 < b < 18$
(c) Both a and b are real numbers
(d) $a = j\sqrt{\pi}\omega / c$, $c > 10$, $b < 20$
(e) None of the above statements are correct

5.3 Let $f_1(t) = e^{\sin 3t}$, $f_2(t) = \sin|t|$, $f_3(t) = \sin(\sin t)$, $-\infty < t < \infty$. Which of the following statements is correct?

(a) functions $f_1(t)$ and $f_2(t)$ are periodic, $f_3(t)$ is not periodic
(b) functions $f_1(t)$ and $f_3(t)$ are periodic, $f_2(t)$ is not periodic
(c) functions $f_1(t)$ and $f_2(t)$ are not periodic, $f_3(t)$ is periodic
(d) functions $f_1(t)$, $f_2(t)$, and $f_3(t)$ are all periodic
(e) None of the above statements are correct

5.4 Consider a second-order rational fraction

$$f(z) = \frac{1 - 1.7z^{-1}}{(1 - 0.8z^{-1})(1 - 1.25z^{-1})}$$

Suppose that the region of convergence is $0.8 < |z| < 1.25$, then the inverse z-transform of $f(z)$ is

(a) $x[n] = 2(0.8)^n u[n] + (1.25)^n u[-n-1]$
(b) $x[n] = \{2(0.8)^n - (1.25)^n\} u[n]$
(c) $x[n] = 2(0.8)^n u[-n-1] + (1.25)^n u[n]$
(d) $x[n] = \{2(0.8)^n - (1.25)^n\} u[-n-1]$
(e) None of the above statements are correct

5.5 If one wants to compute the outflow per unit volume of a vector field $\mathbf{v} = (x^2/z)\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + yz\hat{\mathbf{k}}$ at a point $P = (2, 9, -1)$, which of the following is the correct way to do?

(a) Compute the curl of $\mathbf{v}$ at P, i.e., curl $\mathbf{v} \triangleq \nabla \times \mathbf{v} = -\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$
(b) Compute the divergence of $\mathbf{v}$ at P, i.e., div $\mathbf{v} \triangleq \nabla \bullet \mathbf{v} = 5$
(c) Compute curl(curl of $\mathbf{v}$) at P, i.e., $\nabla \times (\nabla \times \mathbf{v}) = 8\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$
(d) Compute the Laplacian of $\mathbf{v}$ at P, i.e., $\nabla^2 \mathbf{v} = -10\hat{\mathbf{i}}$
(e) None of the above methods are correct

科目：工程數學甲【通訊聯招甲組、電機系甲、丁、戊、庚組、通訊所乙組；電機系丙組選考】

5.6 The directional derivative of a function $f(x,y,z)=x^2+yz+xz^2$ in the direction of the vector $\mathbf{v}=\hat{\mathbf{i}}-2\hat{\mathbf{j}}$ at a point $P=(1,0,1)$ is denoted as α. Then

(a) $\alpha\le 0$ (b) $4\le\alpha\le 10$ (c) $14\le\alpha\le 16$ (d) $0<\alpha\le 2$
(e) None of the above answers are correct

5.7 Let a position vector be $\hat{\mathbf{P}}(\tau)=(\cos\tau)\hat{\mathbf{i}}+\sqrt{15}\ \tau\hat{\mathbf{j}}+(\sin\tau)\hat{\mathbf{k}}$. $\hat{\mathbf{P}}(\tau)$ will generate a curve as τ varies from a to b, i.e., $a\le\tau\le b$. Suppose that the length of the curve from $a=\pi/2$ to $b=3\pi/2$ is L, then

(a) $L=2\pi$ (b) $L=16\pi$ (c) $5\pi\le L\le 8\pi$ (d) $12\pi\le L\le 14\pi$
(e) None of the above answers are correct

5.8 It is well known that the work done by an applied force along a specific trajectory can be computed by using line integral. Assume that the force is $\hat{\mathbf{F}}(x,y,z)=x^2\hat{\mathbf{i}}+y^2\hat{\mathbf{j}}+z^2\hat{\mathbf{k}}$, and the trajectory is specified by $C: x=\cos t,\ y=\sin t,\ z=t,\ 0\le t\le\pi$. Suppose that the work done by this force $\hat{\mathbf{F}}$ along the curve C is β/γ. Then

(a) $20\le\beta\le 25,\ 1\le\gamma\le 5$ (b) $33\le\beta\le 38,\ 6\le\gamma$ (c) $26\le\beta\le 32,\ 1\le\gamma\le 3$
(d) $\beta\le 25,\ 6\le\gamma\le 10$ (e) None of the above answers are correct

5.9 Let $f(z)=(1-x)^3+j\,y^3$, where $z=x+jy$ is a complex variable. We also let $f'(z)\triangleq df(z)/dz$. Which of the following statements is correct?

(a) $f'(z)$ exists only at $z=1$
(b) $f'(z)$ exists only at $z=0$
(c) $f'(z)$ does not exist $\forall\, z$ in the complex plane
(d) $f'(z)$ exists except at $z=1$
(e) None of the above statements are correct

5.10 Let $z=x+jy$ be a complex variable. Which of the following is an open set?

(a) $|z+1-j|\le 2$ (b) $\mathrm{Im}\,z=2$, where Im stands for imaginary part (c) $|z+2-j3|\ge 1$
(d) $0\le\arg z\le\pi/3,\ z\ne 0$, where "arg" stands for argument
(e) None of the above sets are open

5.11 Let z be a complex number. Which of the following statements is correct?

(a) $|\cos z|^2\le 1$
(b) $\sin\overline{z}$ is analytic at $z=0$, where $\overline{z}$ is the complex conjugate of z
(c) $Arg(z_1z_2)=Arg(z_1)+Arg(z_2)$ $\forall\, z_1$ and z_2 in the complex plane, where "Arg" stands for the principal value of arg z
(d) $|e^{z^2}|\le e^{|z|^2}$
(e) None of the above statements are correct

5.12 Compute the value of $\oint_C \tan z\,dz=\sigma+j\rho$, where C stands for the circle $|z|=2$ in counterclockwise direction. Then which of the following statements is correct?

(a) $\sigma=0,\ \pi\le\rho\le 2\pi$
(b) $\sigma=0,\ \rho=0$
(c) $0\le\sigma\le 2\pi,\ \rho=0$
(d) $\sigma=0,\ -4\pi\le\rho\le -2\pi$
(e) None of the above statements are correct

科目：電子學【通訊聯招甲組、電機系甲、乙、戊組、通訊所乙組】

1. (10%) A voltage amplifier, when connected to a 10kΩ source, has an overall gain of 1667 V/V. When a second identical amplifier is connected in parallel to the same source, the corresponding gain for each is found to be 909 V/V. Find the input resistance of the amplifier.

2. (10%) What is the highest frequency of a triangle wave of 20-V peak-to-peak amplitude that can be reproduced by an op amp whose slew rate is 10 V/μs? For a sine wave of the same frequency, what is the maximum amplitude of output signal that remains undistorted?

3. (10%) For the circuit shown in Fig. 1, diodes D_1 through D_4 are identical. Each has n=1 and is a "1-mA diode". For small input signals, find the value of small-signal transmission v_o/v_i for I=10μA. If I=1mA, what is the largest possible output signal for which the diode currents deviate by at most 10% of their dc values?

4. (10%) For a 0.8 μm CMOS fabrication process: V_{tn} = 0.8V, V_{tp} = –0.9 V, $\mu_n C_{ox}$ = 90 μA/V^2, $\mu_p C_{ox}$ = 30 μA/V^2, C_{ox} = 1.9 fF/μm^2, ϕ_f = 0.34 V, γ = 0.5 V$^{1/2}$, V_A (*n*-channel devices) = 8L (μm), and $|V_A|$ (*p*-channel devices) = 12L (μm). Find the small-signal model parameters (g_m, r_o, and g_{mb}) for both an NMOS and a PMOS transistor having W/L = 20 μm/2 μm and operating at I_D = 100 μA with $|V_{SB}|$ = 1V. Also find the overdrive voltage at which each device must be operating.

5. (10%) For the circuit shown in Fig. 2, v_{sig} is a small sine-wave signal with zero average. The transistor β is 100. (a) Find the value of R_E to establish a dc emitter current of about 0.5 mA. (b) Find R_C to establish a dc collector voltage of about +5V. (c) For R_L = 10 kΩ and the transistor r_o = 200 kΩ, determine the overall voltage gain.

6. (17%) From Fig. 3, R_{B1}=50Ω, R_{C2}=15kΩ, and I_B=0.5mA. The transistors have β=100, C_μ=2pF, and f_T=400MHz, neglect r_x and r_o. Calculate the midband gain A_M and the 3dB frequency f_H.

7. (16%) From Fig. 4, R_E=150Ω, R_C=7.5kΩ, I_B=1mA. Find the voltage gain and input resistance of the amplifier assuming that β=150.

8. (17%) From Fig. 5, the circuit is a feedback circuit. The load resistance is R_L. Assume the op amp is ideal. (a) Identify the feedback topology. (b) Find the expression for A_f. (c) Find the expression for β.

科目：電子學【通訊聯招甲組、電機系甲、乙、戊組、通訊所乙組】

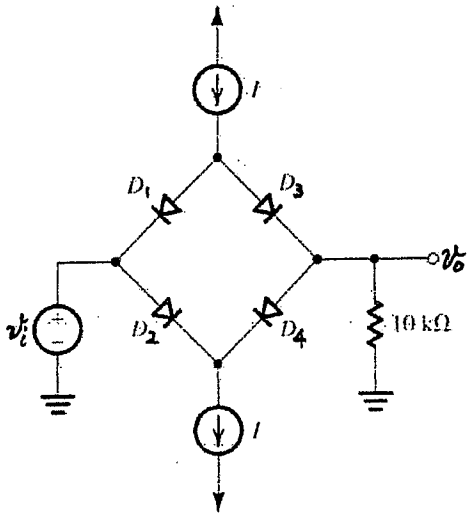

Figure 1

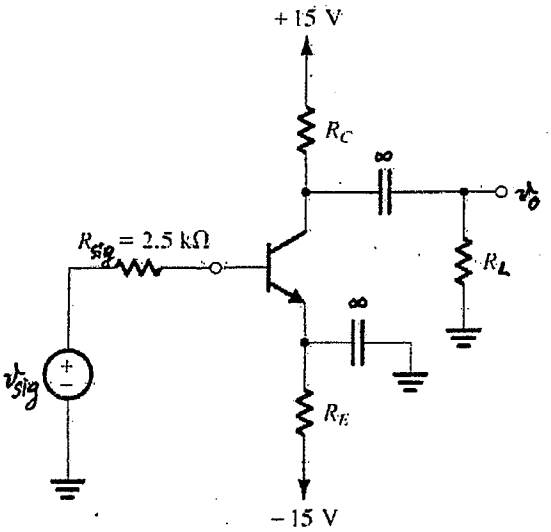

Figure 2

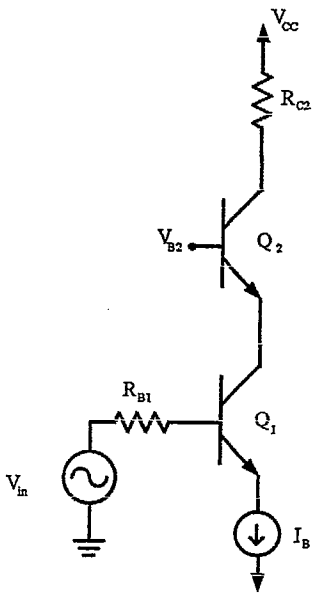

Figure 3

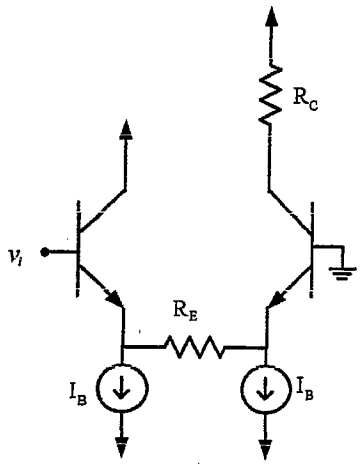

Figure 4

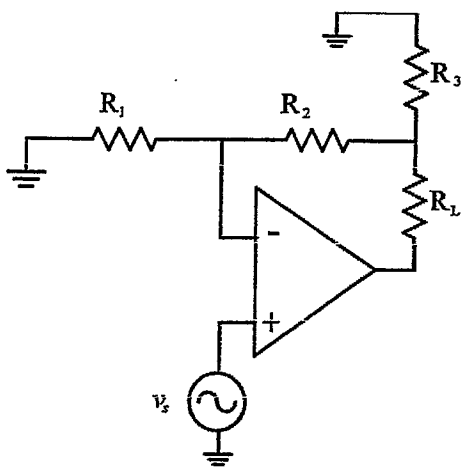

Figure 5

科目：半導體概論【電機系碩士班甲組】V

1) Assume that the Fermi energy level for a particular material is 6.25 eV and that the electrons in this material follow the Fermi-Dirac distribution function. calculate the temperature at which there is a 1 percent probability that an energy state 0.3 eV below the Fermi energy level will not contain an electron. (10%).

2) (a) To calculate the build in potential barrier (10%) (b) the width of the space charge region in a pn junction when a reverse-bias voltage is applied (20%). Consider a Silicon pn junction at T=300 K with doping concentrations of $N_a=10^{16}cm^{-3}$ and $N_d=10^{15}cm^{-3}$. Assume that $n_i=1.5x10^{10}\ cm^{-3}$ and let $V_R=5V$.

($\varepsilon_{Si}=11.8\ \varepsilon_O$; $\varepsilon_O=8.85x10^{-14}$ F/cm; $q=1.6x10^{-19}$ C)

3) To calculate the effective Richardson constant. Consider the tungsten-silicon diode with a barrier height of $\phi_{Bn}=0.67$ V and $J_{sT}\approx 6x10^{-5}$ A/cm^2 at room temperature.(15%)

4)In the energy band diagrams, for a Ge ($10^{14}\ cm^{-3}$) doped silicon, the Fermi level, E_F is located at (a) close to E_C , (b) close to E_V, (c) $E_F = E_{Fi}$ (d) close to the middle gap; above E_{Fi} (e) close to the middle gap; below E_{Fi} , (f) none of above. Where E_{Fi} is the intrinsic Fermi level. To choose and explain the answer.(15%)(單選)

5) With increasing temperature from 10 K to 600 K and neglecting the change of energy band gap, the Fermi level of a doped semiconductor will (a) unchanged (b)move toward conduction band edge E_C (c) move toward valence band edge E_V (d) move to the middle gap (e) move toward band edge and then move toward middle edge (f) none of above. To choose and explain the answer..(15%)(單選)

6) Brief description on the future trend of semiconductor industry in Taiwan. (15%)

科目：工程數學乙【電機系碩士班乙組】

1. (15%) Solve the differential equation

$$\begin{aligned} \frac{dy_1}{dx}+3y_1+3\frac{dy_2}{dx} &= 2e^{-2x} \\ \frac{dy_2}{dx}+2y_2-y_1 &= 1 \end{aligned} \qquad \text{with} \quad y_1(0)=y_2(0)=0.$$

2. (15%) Use the power series method to solve the following equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y = 0$$

where n is a nonnegative integer.

3. (20%) Let $L: V \to W$ be a one-to-one and onto, hence invertible, linear transformation between vector spaces V and W. Denote its inverse by L^{-1}, hence L^{-1} is a transformation from vector space W to vector space V.

(a) Show that L^{-1} is also linear. (8%)

(b) Let $\{\mathbf{z}_1,\cdots,\mathbf{z}_r\}$ be a basis for W. Show that $\{L^{-1}(\mathbf{z}_1),\cdots,L^{-1}(\mathbf{z}_r)\}$ is a basis for V. (12%)

4. (26%) Given matrices $A\in\mathbb{R}^{m\times p}$ and $B\in\mathbb{R}^{m\times q}$, with A being full column rank, consider the linear equation $A\mathbf{x}+B\mathbf{y}=\mathbf{0}$. By choosing a vector $\mathbf{y}\in\mathbb{R}^q$, we may consider to solve vector $\mathbf{x}\in\mathbb{R}^p$ to satisfy the equation.

(a) Show that there exists a matrix, called C, in $\mathbb{R}^{p\times m}$ with $rank(C)=p$ such that $CA=I$. (Matrix C is called a left inverse of matrix A.) (6%)

以下小題僅需依序寫下答案即可，不需做任何推導。

(b) What is the condition on $\mathbf{y}$ so that the equation is consistent, i.e. it is solvable? (4%)

(c) If $\mathbf{y}$ is chosen so that the considered equation is consistent, what is the general form of the solution to the equation? (4%)

If $\mathbf{y}$ is chosen so that the considered equation is inconsistent, then the concept of *projection* plays an important role in studying the solution of the equation. Let U and V be two subspaces of $\mathbb{R}^m$ such that $\mathbb{R}^m=U\oplus V$ and let P and Q be the projection matrices that project vectors of $\mathbb{R}^m$ onto U along V and onto V along U, respectively. It means that, $\forall\mathbf{r}\in\mathbb{R}^m$, $P\mathbf{r}\in U$, $Q\mathbf{r}\in V$, and $\mathbf{r}=P\mathbf{r}+Q\mathbf{r}$ is the unique decomposition associated with the two projections.

(d) Let $U=R(A)$, the range of A, and let $V=N(C)$, the null space of C, what is P? (6%)

(e) If $R(A)\perp N(C)$, what is the relationship between A and C? (6%)

科目：工程數學乙【電機系碩士班乙組】

第五大題請於答案卷「是非、選擇題專用區」作答。

【5.1 於該區第(1)格作答、5.2 於該區第(2)格作答……，依此類推】

5. (24%) Each of the following problems has only one correct answer. You will obtain 3 points for each question if you give the correct answer. However, you will get -1 points for each question if the answer is not correct. Zero point will be deducted if you give no answer. The lowest point in this set of problems you may obtain is zero. For instance, if you give one correct answer, four wrong answers, and no answers for the rest of the problems, then you will obtain zero point for this problem set. Note that $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ are the unit vectors in the positive direction of the Cartesian x, y, z coordinate system respectively. If $z = x + jy$ is a complex variable, then x, y are real numbers, and "j" denotes the pure imaginary number $(0,1)$.

5.1 Consider the following three systems, where $y[n]$ or $y(t)$ denotes the system output, and $x[n]$ or $x(t)$ is the system input:

$$\textbf{I.}\quad y[n] = x[-n],\quad -\infty < n < \infty$$

$$\textbf{II.}\quad y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n+1], & n \leq -1 \end{cases}$$

$$\textbf{III.}\quad y(t) = [\sin(6t)]x(t)$$

Which of the following statements is correct?

(a) I is time-invariant, II is linear, III is memoryless
(b) I is causal, II is stable, III is memoryless
(c) I is stable, II is linear, III is causal
(d) I is memoryless, II is stable, III is linear
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(b) a is an imaginary number, and $14 < b < 18$
(c) Both a and b are real numbers
(d) $a = j\sqrt{\pi}\omega / c$, $c > 10$, and $b < 20$
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(a) Functions $f_1(t)$ and $f_2(t)$ are periodic, $f_3(t)$ is not periodic
(b) Functions $f_1(t)$ and $f_3(t)$ are periodic, $f_2(t)$ is not periodic
(c) Functions $f_1(t)$ and $f_2(t)$ are not periodic, $f_3(t)$ is periodic
(d) Functions $f_1(t)$, $f_2(t)$, and $f_3(t)$ are all periodic
(e) None of the above statements are correct

5.4 Consider a second-order rational fraction

$$f(z)=\frac{1-1.7z^{-1}}{(1-0.8z^{-1})(1-1.25z^{-1})}$$

Suppose that the region of convergence is $0.8<|z|<1.25$, then the inverse z-transform of $f(z)$ is

(a) $x[n]=2(0.8)^n u[n]+(1.25)^n u[-n-1]$
(b) $x[n]=\{2(0.8)^n-(1.25)^n\}u[n]$
(c) $x[n]=2(0.8)^n u[-n-1]+(1.25)^n u[n]$
(d) $x[n]=\{2(0.8)^n-(1.25)^n\}u[-n-1]$
(e) None of the above statements are correct

5.5 Let $f(z)=(1-x)^3+jy^3$, where $z=x+jy$ is a complex variable. We also let $f'(z)\triangleq df(z)/dz$. Which of the following statements is correct?

(a) $f'(z)$ exists only at $z=1$
(b) $f'(z)$ exists only at $z=0$
(c) $f'(z)$ does not exist $\forall z$ in the complex plane
(d) $f'(z)$ exists except at $z=1$
(e) None of the above statements are correct

5.6 Let $z=x+jy$ be a complex variable. Which of the following is an open set?

(a) $|z+1-j|\le 2$ (b) $\mathrm{Im}\, z=2$, where Im stands for imaginary part (c) $|z+2-j3|\ge 1$
(d) $0\le \arg z\le \pi/3$, $z\neq 0$, where "arg" stands for argument
(e) None of the above sets are open

5.7 Let z be a complex number. Which of the following statements is correct?

(a) $|\cos z|^2\le 1$
(b) $\sin\overline{z}$ is analytic at $z=0$, where $\overline{z}$ is the complex conjugate of z
(c) $Arg(z_1z_2)=Arg(z_1)+Arg(z_2)$ $\forall z_1$ and z_2 in the complex plane, where "Arg" stands for the principal value of arg z
(d) $|e^{z^2}|\le e^{|z|^2}$
(e) None of the above statements are correct

5.8 Compute the value of $\oint_C \tan z\, dz=\sigma+j\rho$, where C stands for the circle $|z|=2$ in counterclockwise direction. Then which of the following statements is correct?

(a) $\sigma=0,\ \pi\le\rho\le 2\pi$
(b) $\sigma=0,\ \rho=0$
(c) $0\le\sigma\le 2\pi,\ \rho=0$
(d) $\sigma=0,\ -4\pi\le\rho\le -2\pi$
(e) None of the above statements are correct

PROBLEM 1 (TOTAL: 30 MARKS)

Consider five transfer functions $G_1(s)$ to $G_5(s)$ whose poles (marked by 'x') and zeros (marked by 'o') are shown in Figures 1-(1) to 1-(5), respectively. Your task is to match the step responses in Figure 2 (next page) to these transfer functions.

Instructions:

- For this problem, NO justification is required for your answers.
- Every transfer functions in Figure 1 matches at MOST one step response in Figure 2.
- Every step responses in Figure 2 matches at MOST one transfer function.
- Write your answers in the provided answer booklet. Do NOT write your answers on the exam papers. Make sure you label your answers clearly (for example, "$G_1(s)$: *Response* (A)") so that the examiner can unambiguously identify how you match the transfer functions with the responses. No mark will be given to ambiguous answers.
- A correct match worths 6 marks.

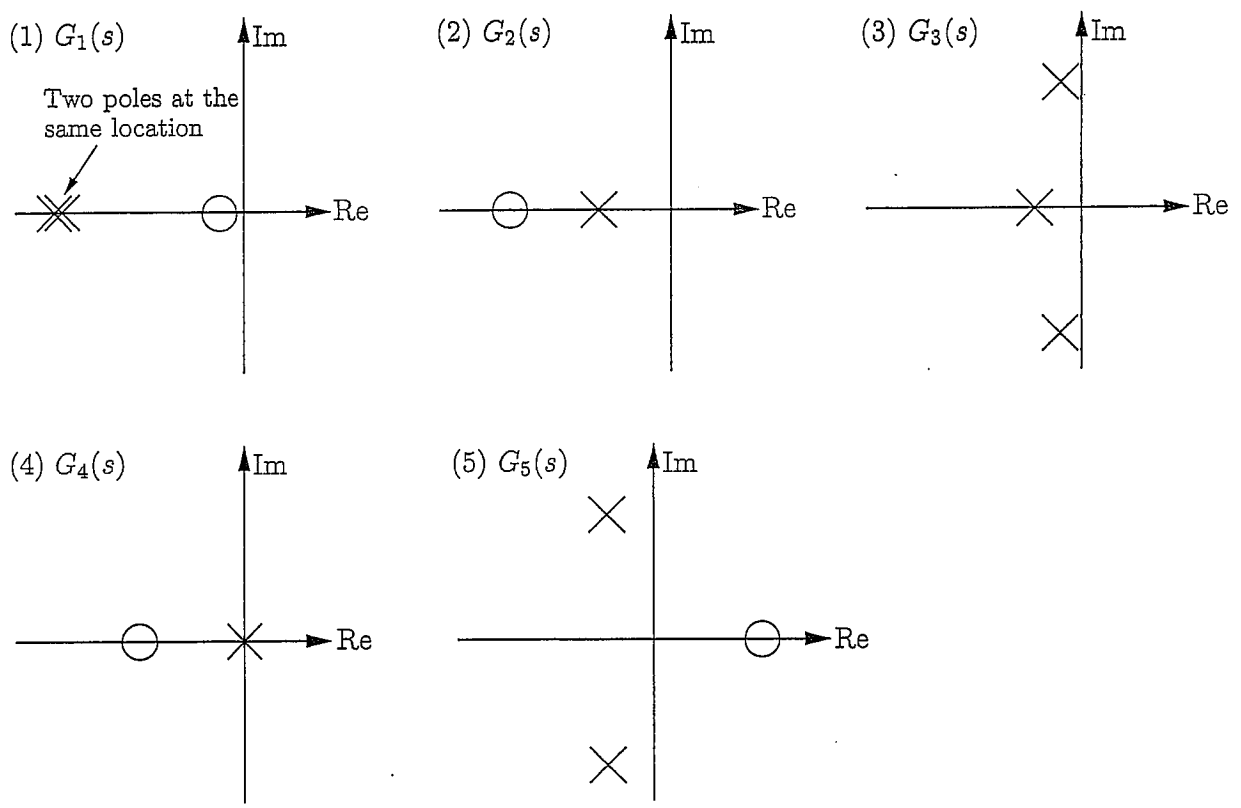


Figure 1: Locations of poles and zeros of transfer functions $G_1(s)$ to $G_5(s)$.

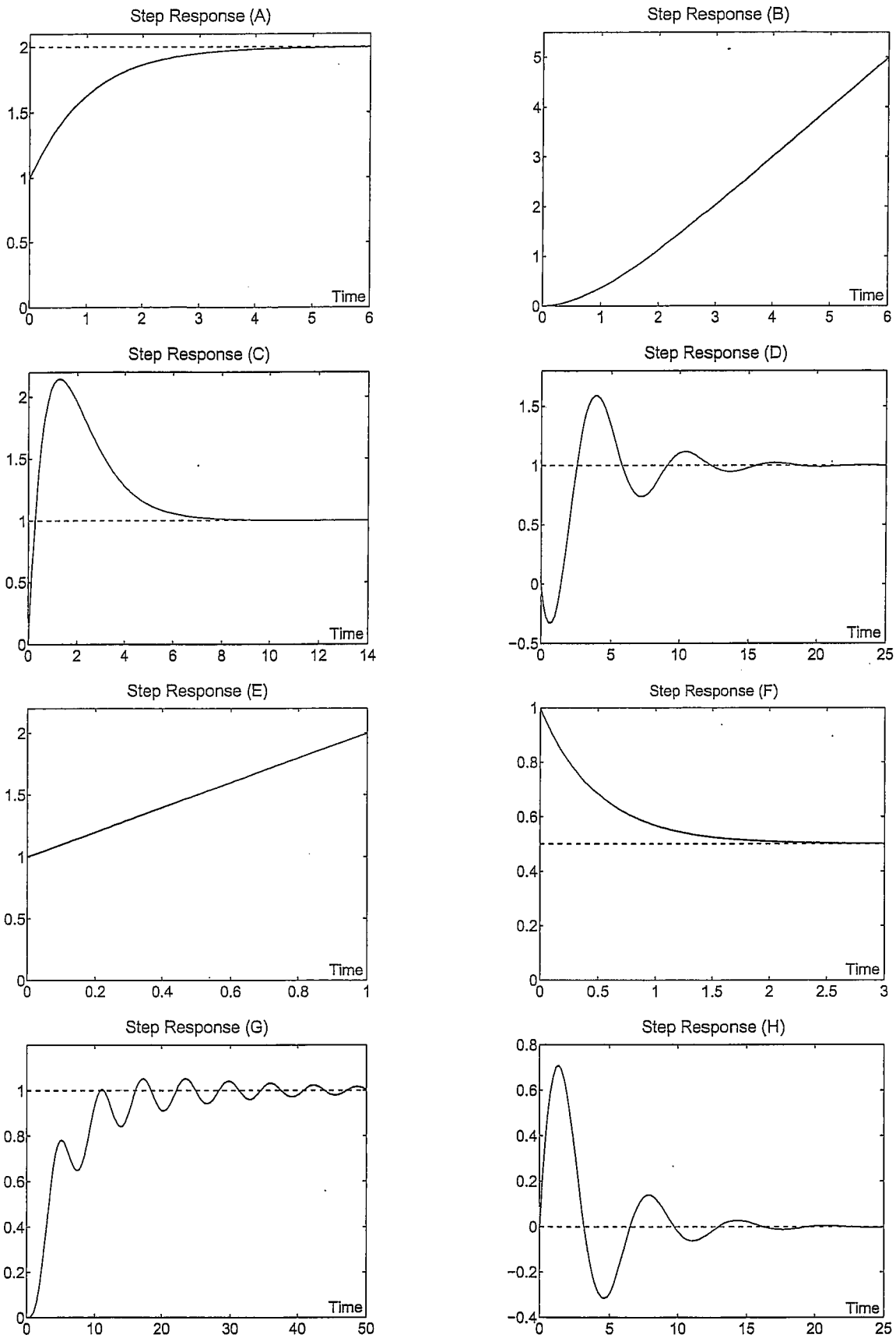


Figure 2: Step Responses.

PROBLEM 2 (TOTAL: 30 MARKS)

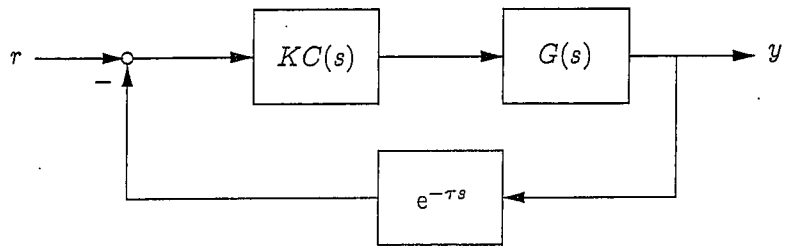


Figure 3: A feedback system

Consider the feedback system shown in Figure 3, where $G(s)$ is the transfer function for a plant to be controlled, $C(s)$ represents a dynamical compensator, and K is a constant. A (constant) feedback signal transmission delay τ is modelled in the frequency domain by the block $e^{-\tau s}$, as shown in the figure. It is known that

$$G(s) = \frac{1}{s^2 - 1},$$

whilst $C(s)$ has the form

$$C(s) = \frac{s + \alpha}{s + \beta},$$

where α and β are two positive constants. The Bode plots of $C(s)G(s)$ are given in Figure 4 (page 4).

(2.1) (2 marks) What is the value of α/β? Justify your answer.

(2.2) (8 marks) Suppose $\tau = 0$. Is the feedback system stable when $K = 1$? Justify your answer.

(2.3) (10 marks) Suppose $\tau = 0$. Find the range of K for which the feedback system is stable.

(2.4) (10 marks) Let $K = 10$. The Nyquist curve for $KC(s)G(s)$ is shown in Figure 5 (page 5). Find the largest $\bar{\tau}$ such that the feedback system is stable for any τ in the range $[0, \bar{\tau})$. Note that $|10C(j\omega)G(j\omega)| = 1$ when $\omega = 1.13307$.

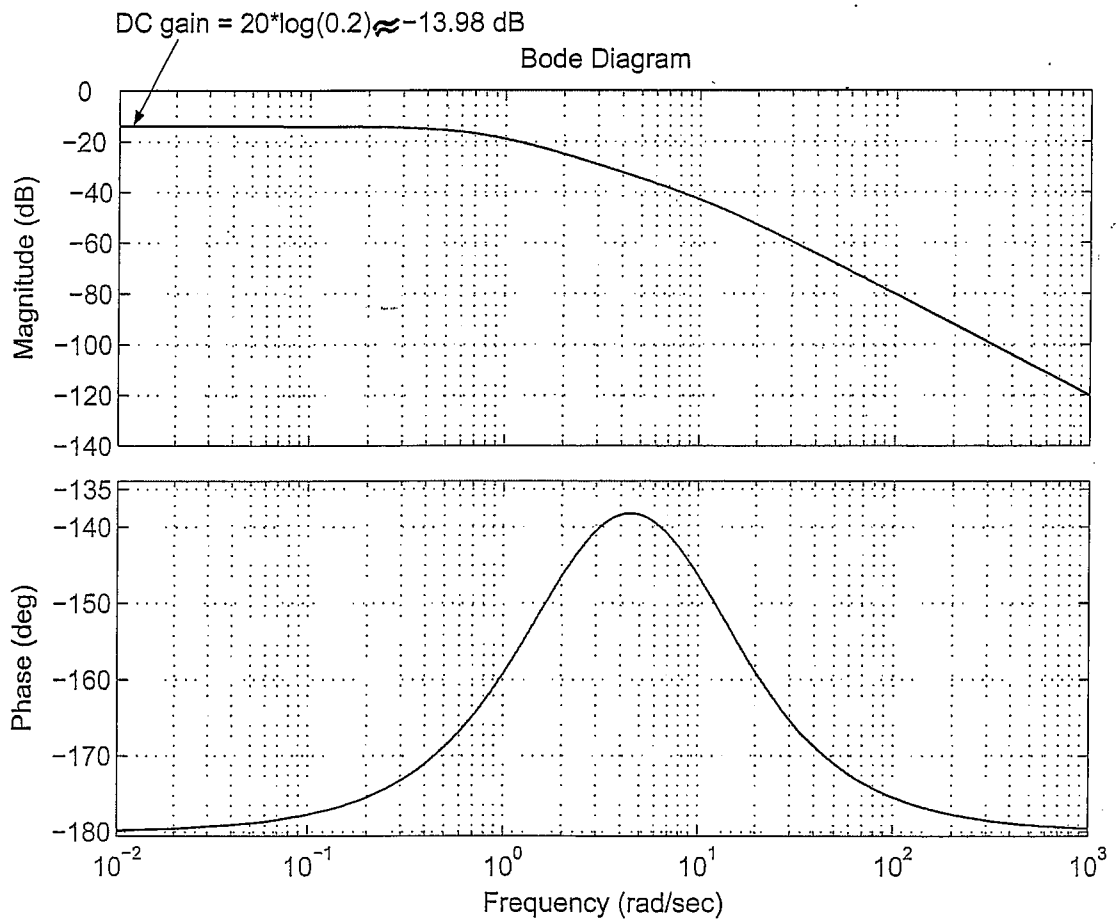


Figure 4: Bode plot of $C(s)G(s)$.

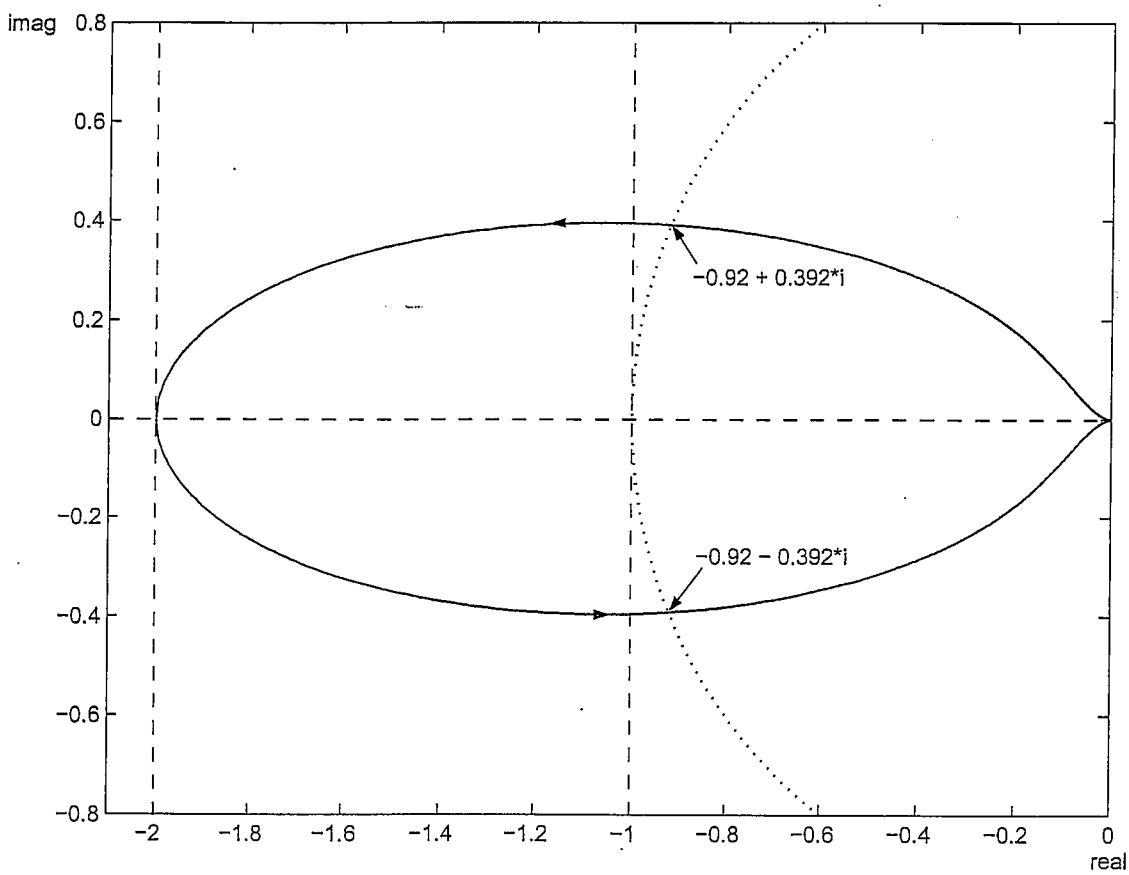


Figure 5: Nyquist curve of $10C(s)G(s)$. The dotted curve is a part of the unit circle centered at the origin.

PROBLEM 3 (TOTAL: 40 MARKS)

Consider the single degree-of-freedom robot arm illustrated in Figure 6. By variation of the joint angle θ, this arm is intended to undertake "pick-and-place" operations, where the robot arm picks up an object from one position and places it at another. With this objective in mind, the joint angle θ is to be actuated via a DC motor, which in turn is to be regulated via a SISO feedback controller to achieve zero steady state error angular position control for *any* step reference angle command θ_r. The aim here is to design the feedback controller to achieve this.

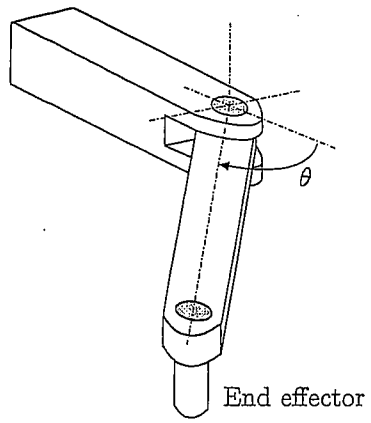


Figure 6: Robot arm (Problem 3).

The motor and the moveable portion of the arm can be modelled via the transfer function

$$\frac{\theta(s)}{V(s)} = G(s) = \frac{K}{Js^2 + Ls},$$

which maps input armature voltages $V(s)$ to armature angular positions $\theta(s)$. Nominally, $K = 5\,Nm\,V^{-1}$, $J = 20\,(kg)m^2$, and $L = 100\,Nms(rad)^{-1}$. The controller to be designed is required to implement three dominant closed loop poles at $s = -6, -7, -8$ rad/s. The feedback control loop is illustrated in Figure 7.

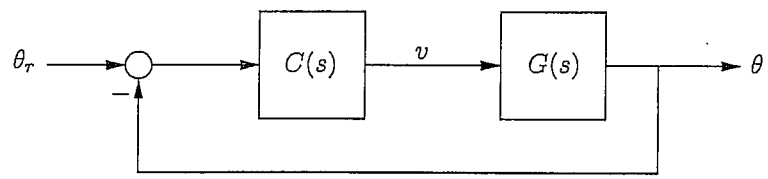


Figure 7: Robot arm feedback control system, where $C(s)$ denotes the controller to be implemented.

(3.1) (5 marks) Determine what constraints (if any) are required on the controller in order for the closed loop system to successfully implement the stated tracking requirements.

(3.2) (15 marks) Design a controller that implements the desired closed loop poles and tracking requirements stated above. You must show all working.

During the course of normal operation, the combined moment of inertia J of the motor and arm can vary depending on the load carried by the end effector of the robot arm.

(3.3) (5 marks) Show that the closed loop characteristic equation can be rewritten as

$$1 + \frac{1}{J}\left(\frac{Ls + C(s)K}{s^2}\right) = 0, \tag{1}$$

where $C(s)$ is the controller to be implemented.

(3.4) (15 marks) Using equation (1) with your controller $C(s)$ designed in (3.2), construct a rough root-locus sketch of the closed loop poles as a function of $\frac{1}{J}$. Using this sketch, explain the likely effects of variations in J on the closed loop transient response.

END OF THE EXAMINATION

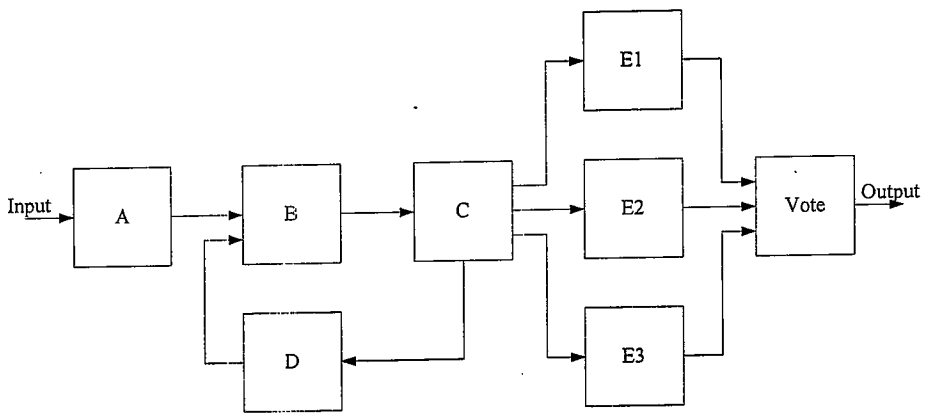
科目：離散數學【電機系碩士班丙組選考】

1. (10%) Prove with truth table that $\sim((a \vee b) \wedge (\sim b)) \vee a$ is true.

2. (15%) Given *knows* relationships among 6 persons { *John, Mary, Peter, Jimmy, Tom, Robert* } as follows:
   knows(John, Mary), knows(Mary, John), knows(Peter, Jimmy), knows(Tom, Robert), knows(Robert, Mary), knows(Mary, Jimmy), knows(Peter, John), knows(Mary, Peter)
   (a) (5%) Write the relation matrix for *knows* relationships.
   (b) (5%) Draw the directed graph to represent these *knows* relationships.
   (c) (5%) Derive transitive closure of *knows* relation matrix.

3. (15%) In an undirected complete graph with ***n* distinct vertices**,
   (a) (5%) How many complete subgraphs are contained in the graph?
   (b) (5%) How many ways can the undirected complete graph be oriented into a directed complete graph? (i.e. Orient each edge in chosen direction for all edges.)
   (c) (5%) Given two selected vertices S and T, how many paths of length n – 1 with distinct vertices from vertex S to vertex T are contained in the graph?

4. (15%) A system is composed of the following structure:



A run from input to output consists of:

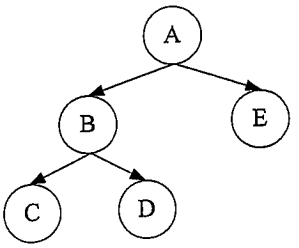
- executing A for once,
- executing the cycle of B, C, and D for 3 times,
- executing identical functions E1, E2, and E3 at the same time, and vote their results as output.
  - When the input to E1, E2, and E3 are correct, and if all three E's are correct or any two of E's are correct, the output is correct.

Modules A, B, C, D, and E (has three E instances E1, E2, and E3) have error probabilities P_A, P_B, P_C, P_D, and P_E, respectively, for each run of the corresponding module. Write the formula of the probability that the output is correct.

5. (15%) A tree is a connected graph without any cycles. A cut set in a connected graph is a set of edges in a graph such that the removal of the cut set from the graph will evolve into two disconnected subgraphs.
   (a) (10%) Prove that when adding one more edge into a tree, it becomes a graph that is not a tree.
   (b) (5%) Prove that any minimal cut set of a tree consists of only 1 edge.

科目：離散數學【電機系碩士班丙組選考】

6. (15%) Given a binary tree, write a recursive algorithm to output a sequence of (traversed node label, subtree size of the node) for all tree nodes. For example, given the following tree, the algorithm will output: (C, 1), (D, 1), (B, 3), (E, 1), (A, 5)



7. (15%) A quicksort algorithm of sequence of distinct values is formulated as follows:

```
Algorithm quicksort(sequence S, s, e) // s is the stating index, e is the ending index
begin
    p = select a value from the sequence;
    permute the sequence S into a new sequence S where p is at S[i], S[j] < p for j < i, and S[k] > p for k > i;
    if (s < i -1) quicksort(S, s, i-1);
    if (i+1 < e) quicksort(S, i+1, e);
end.
```

Prove that the recursive quicksort algorithm permutes the sequence S into an ascending sequence S. (i.e. after applying quicksort(S,1,n) for a length-n sequence S, S[i] < S[j] for i < j)

Hint: Utilize mathematical induction.

科目：資料結構【電機系碩士班丙組選考】

1. [5] Suppose we have a byte-addressable machine, i.e., each byte is accessed via an address. Let the locations of an array be allocated in a row-major manner, and each element takes one byte. What is the address of the element B[5][15][25] in an array declared as B[10][20][30]? Assume that the address of the first byte of the array is 5000.
2. [5] Assume we want to use a singly linked list to store characters. Assume that the first character 'g' is stored at address 1500, the second character 'o' is stored at address 500, the third character 'o' is stored at address 1000, and the fourth character 'd' is stored at address 100. Let the address of the first character is stored in the variable named head. Please show the contents of head and all the nodes of the list. Note that you have to show the pointers explicitly, not just using arrows.
3. [5] Please convert the infix arithmetic expression $a-(b+d\times e)/(a\times d-f)-b$ into the postfix expression.
4. [5] Consider the graph of Figure 1. Please display the nodes of the graph using depth-first search, starting with node 1. Assume that nodes with smaller numbers are selected earlier than the ones with larger numbers during the search.
5. [5] Usually, there are two ways to represent a binary tree: array representation and linked representation. For the binary tree of Figure 2, please represent it using array representation (starting from index 1).
6. [5] Usually, there are three ways to display the nodes of a binary tree: preorder, inorder, and postorder. Please display the preorder sequence of Figure 2.
7. [5] The array shown below is a max heap with 12 integers. Insert 72 into the array and make it into a max heap again. Show the resulting array.

	100	85	65	55	60	50	30	20	25	10	40	12			
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

8. [5] The array shown below is a max heap with 12 integers. Remove 95 from the array and make it into a max heap again. Show the resulting array.

	95	70	65	55	60	50	35	20	25	10	45	15			
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

9. [5] Consider the symmetric Mix-Max heap (SMMH) shown in Figure 3. Please insert 7 into it and draw the resulting SMMH.
10. [5] Consider the symmetric Mix-Max heap (SMMH) shown in Figure 3. Perform a

delete-min operation and draw the resulting SMMH.

11. [5] Consider the AVL search tree shown in Figure 4. Please insert 107 into it and draw the resulting AVL search tree.
12. [5] Consider the AVL search tree shown in Figure 4. Assume that a node to be deleted is replaced by the largest integer of its left subtree. Please delete 55 from it and draw the resulting AVL search tree.
13. [5] Consider the B-tree of order 3 shown in Figure 5. Please insert 40 into it and draw the resulting B-tree.
14. [5] Consider the B-tree of order 3 shown in Figure 5. Assume that a node to be deleted is replaced by the largest integer of its left subtree. Please delete 85 from it and draw the resulting B-tree. Note that a node always considers its right sibling first for combination.
15. [10] Consider the array shown below. Make the array into a max heap, without creating or using other arrays, and show the resulting array.

	20	55	15	25	10	50	30	80	90	60	40	65			
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

16. [10] Suppose we have a binary tree each of its nodes containing an upper-case character. We know that the preorder sequence of the binary tree is JLKHICGDBFE, and the inorder sequence is KLICHJDBGEF. Please draw this binary tree.
17. [10] Suppose we have 12 integers: 25, 10, 50, 30, 90, 60, 40, 80, 20, 55, 15, 65. Construct a binary search tree by inserting the integers, starting from 25, one after another. Show the resulting binary tree for these 12 integers.

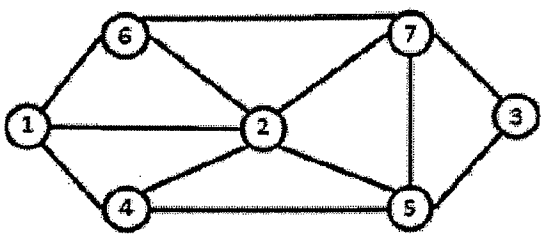


Figure 1

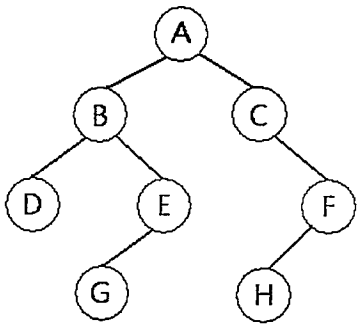


Figure 2

科目：資料結構【電機系碩士班丙組選考】

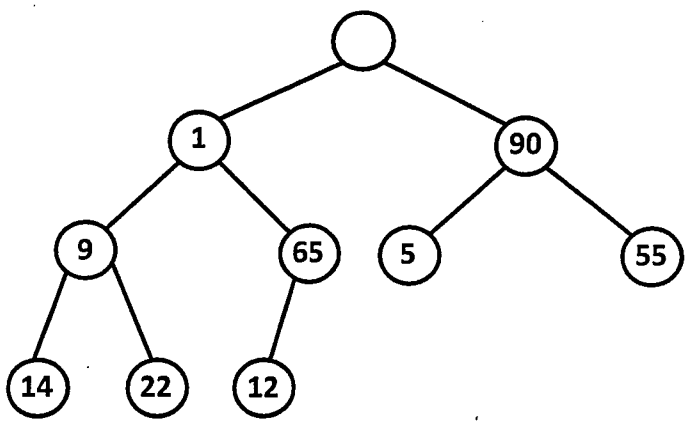


Figure 3

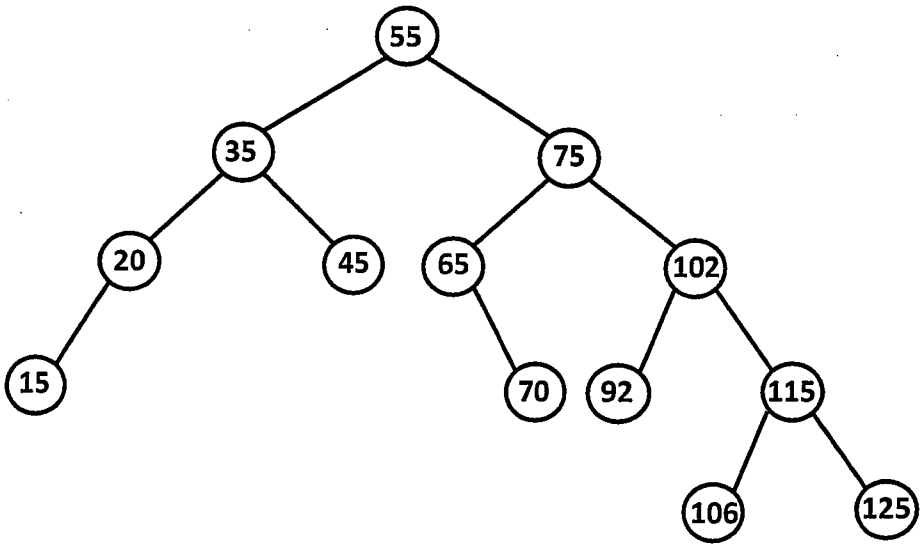


Figure 4

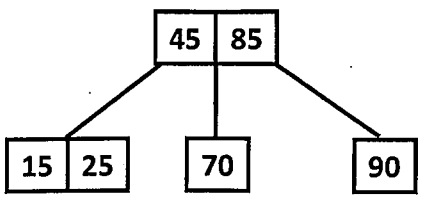


Figure 5

科目：數位電路【電機系碩士班丙組選考、庚組】

[Problem 1] By Boolean algebra to determine and prove whether or not the following expressions are valid, i.e. whether the left- and right-hand sides represent same function. (10%)

a. $\overline{x1}\cdot x2\cdot\overline{x3}+\overline{x1}\cdot x2\cdot x3+x1\cdot\overline{x2}\cdot\overline{x3}=x2+x1\cdot\overline{x3}$

b. $(x1+x3)(\overline{x1}+\overline{x2}+\overline{x3})(\overline{x1}+x2)=(x1+x2)(\overline{x2}+x3)(\overline{x1}+x3)$

[Problem 2] Determine and descript the each functional behaviors of the circuits (a) (b) (c) (d) in Figure 1. (20%)

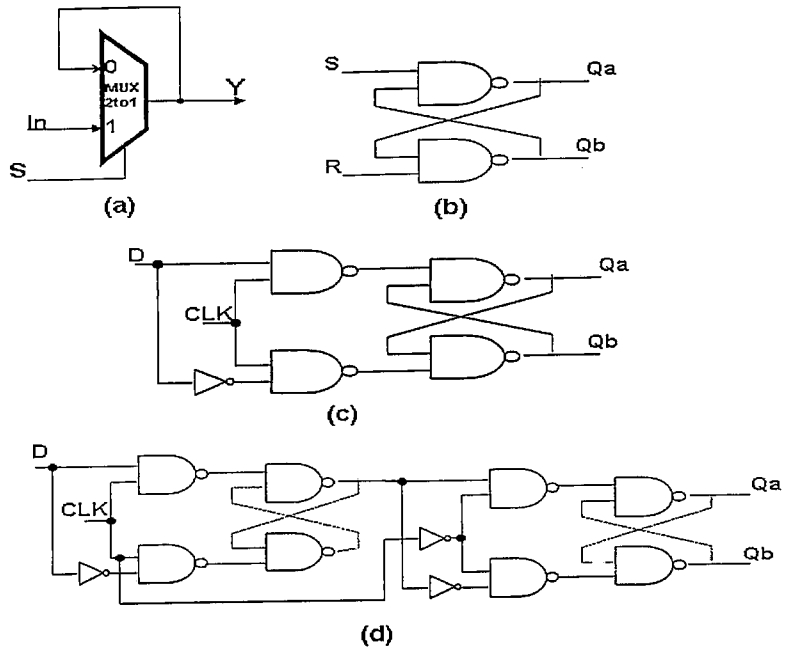


Figure 1.

[Problem 3] Consider the logical circuit shown in Figure 2, and implement it as a CMOS gate. (10 %)

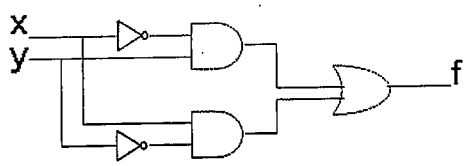


Figure 2. The logical circuit

[Problem 4] Given the 8-bits adder (named Add8), the 2-to-1 8-bits multiplexers (named MUX8_2to1) and the basic gates such as NOT, AND, OR, NAND, and NOR, you are asked to design an ALU in function block diagrams, which must match the following requirements:

(1) Support add, sub, and sgt (set on great than) functions. Their operation selection bits (op_sel) are as follows: add(00), sub(10), sgt(11),

(2) Report the result status in sign, zero, overflow, and carry bits. (20%)

[Problem 5] Design a 32-bit adder using the basic gates such as NOT, AND, OR, NAND, NOR and XOR. Let the basic gates take the same delay time as 1*d*. The delay time of the critical path of the 32-bit adder must be limited under 16d. (20%)

[Problem 6]

(a) Write the code for a positive-edge-triggered D flip-flop with synchronous reset in VHDL or Verilog HDL. (5%)

(b) In VHDL or Verilog HDL, derive and write a circuit by the state-assigned table as table 1. (15%)

Present state y_2y_1	Next state Y_2Y_1				Output s
	ab = 00	01	10	11	
00	00	01	01	10	0
01	00	01	01	10	1
10	01	10	10	11	0
11	01	10	10	11	1

Table 1. The state-assigned table

科目：計算機結構【電機系碩士班丙組、庚組】

I. (20%) Identify <u>two</u> differences between the following computer terminology pairs.
(1) (5%) Isolated I/O vs Memory-Mapped I/O
(2) (5%) Hamming Code vs RAID
(3) (5%) DDR-SDRAM vs Rambus DRAM
(4) (5%) Traditional Superscalar vs IA-64 Architectures

II. (20%) Consider a hypothetical 32-bit microprocessor having 32-bit instructions composed of two fields: the first byte contains the OP code and the remainder the immediate operand or an operand address. Assume that the local address bus is 32 bits and the local data bus is 16 bits. No time multiplexing between the address and data buses.
(1) (5%) What is the maximum directly addressable memory capacity (in bytes)?
(2) (5%) What is the minimum number of bits required for the program counter?
(3) (10%) Assume **direct addressing mode** is applied, how many address and data bus cycles required to fetch an instruction and its corresponding operand or data from main memory?

III. (20%) In a multiprogramming environment, since the system throughput (Ws) is too complicated to estimate, it is common by using CPU throughput (Wp) to evaluate a computer performance.
(1) (10%) Derive an equation for Wp, if given the following parameters.
I_c = Average Instruction count of a program
CPI = Cycles per Instruction
τ = CPU cycle time (second)
(2) (10%) If Wp = MIPS × K, Find out K = ?
<Note>: MIPS = Million Instructions per Second.

IV. (20%) A non-pipelined processor has a clock rate of 2.5 GHz and an average CPI (cycles per instruction) of 4. An upgrade to this processor introduces a new processor with five-stage pipeline. However, due to internal pipeline delays, such as latch delay, the clock rate of the new processor has to be reduced to 2 GHz and an average CPI of 1.
(1) (10%) What is the speedup achieved for a typical program with 100 instructions?
(2) (10%) What is the MIPS rate for the two processors, respectively?
<Note>: MIPS = Million Instructions per Second.

V. (20%) A two-level memory system has eight (page 0 to page 7) virtual pages on a disk to be mapped into four page frames (PFs) in the main memory. Assume each page has 4 words. The i-th page in disk consists of word address from **4i** to **4i+3**, where i = 0, 1, 2, …, 7. A certain program trace generated the following word address.

0, 1, 2, 8, 10, 15, 16, 7, 9, 4, 5, 13, 14, 30, 31, 11, 12, 14, 15, 19, 20, 22, 23, 3, 4, 5, 6

(1) (10%) Show the successive virtual pages residing in the four page frames using LRU. Assuming that the main memory PFs are empty initially. Compute the Hit Ratio.
(2) (10%) Repeat the above using circular FIFO. Assuming that the main memory PFs are initially loaded with virtual pages **0**, **1** and **2** , and the pointer points to an empty slot position. Compute the Hit Ratio.
<Note>: LRU = Least-Recently Used, FIFO = First-In-First-Out.

科目：電路學【電機系碩士班丁組】 ✓

1.(20%) A 220V_{rms}, 60Hz voltage source $v_i(t)$ is applied to a RL load as shown in Fig. 1, where R=7.54Ω, L=20mH.

(A)(6%) Find instantaneous power, average power, and reactive power on the resistance R.

(B)(6%) Find instantaneous power, average power, and reactive power on the inductance L.

(C)(8%) Design a power factor correction scheme to increase the power factor of the load side to 0.95, and explain the impact of the power factor correction on the supply side.

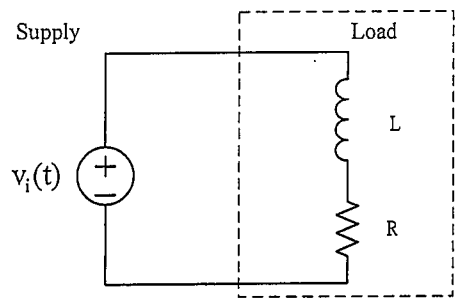


Fig. 1

2.(10%) Calculate total average power drawn by the three phase load Z_L of Fig. 2. Load Z_L is composed of a resistance 7.54Ω and an inductance 10mH in series connection. The source voltage is assumed as :

$v_a(t)=100\cos(377t)$ V,

$v_b(t)=90\cos(377t-120)+10\cos(-377t-120)$ V,

$v_c(t)=90\cos(377t+120)+10\cos(-377t+120)$ V.

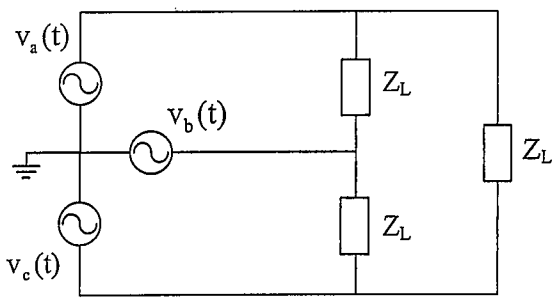


Fig. 2

3.(15%) In Fig. 3, assume $i(t)=I_m\cos(\omega t)$ A, and $\omega>0$.

(A)(5%) Find the input impedance Z.

(B)(5%) What value of ω will cause v(t) to be zero?

(C)(5%) What value of ω will cause v(t) to be maximum?

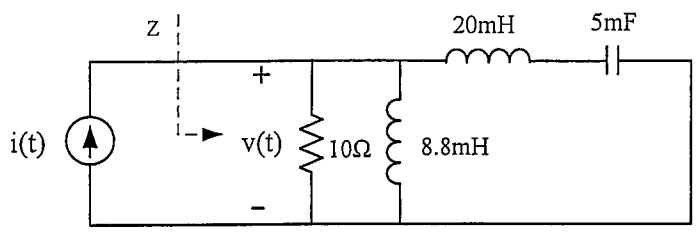


Fig. 3

科目：電路學【電機系碩士班丁組】

4.(15%) In Fig. 4, assume $v_i(t)=100\cos(3000t)$ V and the coupling coefficient of mutual inductance k is 0.5. Find a minimum capacitor C to achieve maximum power transfer on the 5Ω resistance, and the associated maximum power.

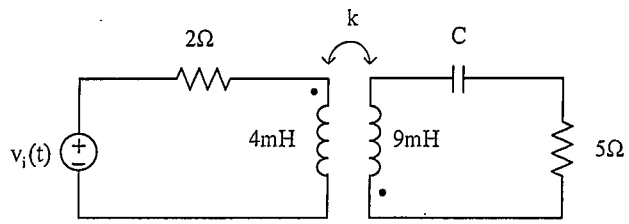


Fig. 4

5.(15%) In Fig. 5, the OPAMP is ideal and operates in the linear region. Input voltage $v_i(t)=\cos(1000t)u(t)$ V. The initial voltages on capacitors are zero, and u(t) represents the unit step function.

(A)(10%) Find the complete response of $v_o(t)$.

(B)(5%) Using phasor approach to solve $v_o(t)$ and explain solution relationship with (A).

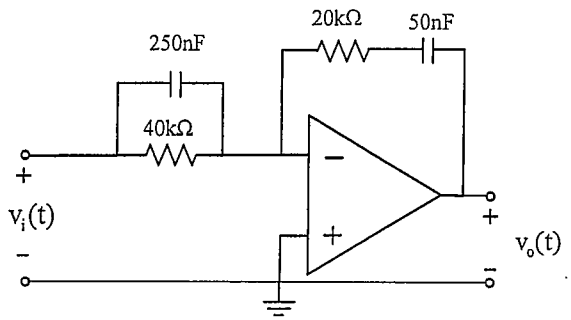


Fig. 5

6.(10%)Find z (impedance) and y (admittance) parameters of the two-port network in Fig. 6.

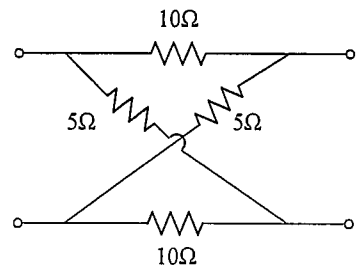


Fig. 6

7.(15%) In Fig. 7, $v_s(t) = 200\cos(2\pi t)$ V and all diodes are ideal. Please draw $i_s(t)$ and $v_o(t)$ waveforms and find their associated effective (or rms) values.

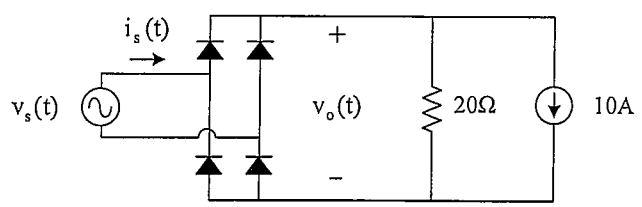


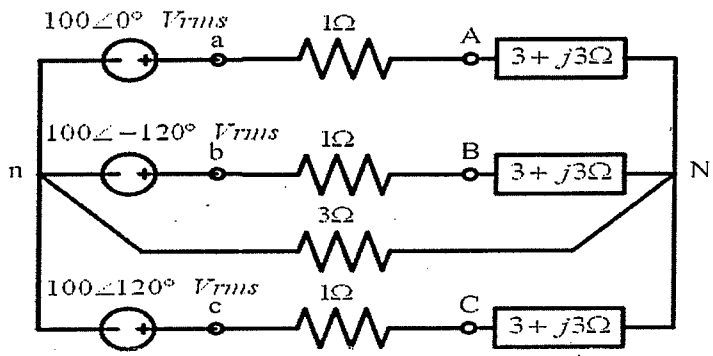
Fig. 7

科目：電力系統【電機系碩士班丁組】

答案請依序填於答案卷「是非、選擇題專用區」，計算過程仍需明列於「非選擇題作答區」，每空格 10 分。

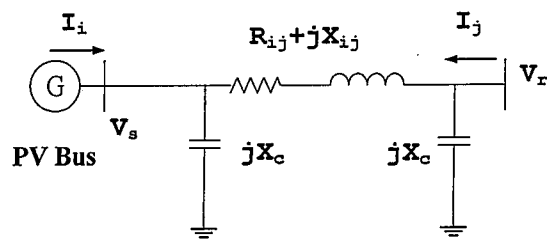
1. (20%) Two loads Z_1 and Z_2 are in parallel across a source of $200\angle 0^0 V_{rms}$.
If Z_1 draws 8kW at a power factor of 0.8 lagging and $Z_2 = 4 - j3\Omega$, find
the reactive power of load Z_1, (1) , and
the complex power delivered by the source (2).

2. (20%) For a balanced 3Φ , 4w system



find the phase current I_{aA} (3) , and the total power delivered (4).

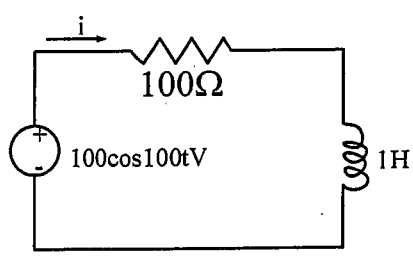
3. (20%)



Let Vs = $1.02\angle 0^0 V_{rms}$, Vr = $1.0\angle -5^0 V_{rms}$, R = 0.0152, X=0.119, Xc=-j0.01

All units are in p.u.. Calculate the Ploss (5) , and the Qloss (6)

4. (40%) For a circuit as shown
we know V(t)=100cos100t volts, Find
a) the power delivered by the source (7)
b) power factor *pf* = (8)
c) to change *pf* to 0.95, add a capacitor C = (9)
d) to compare the current of (b)(c), the current reduction is (10) % ?

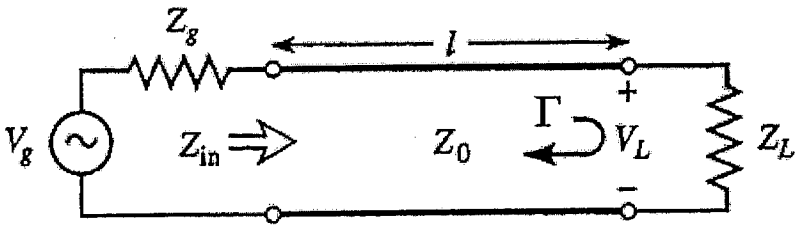


科目：電磁學【通訊聯招碩士班甲組、電機系碩士班戊組、通訊所碩士班乙組】√

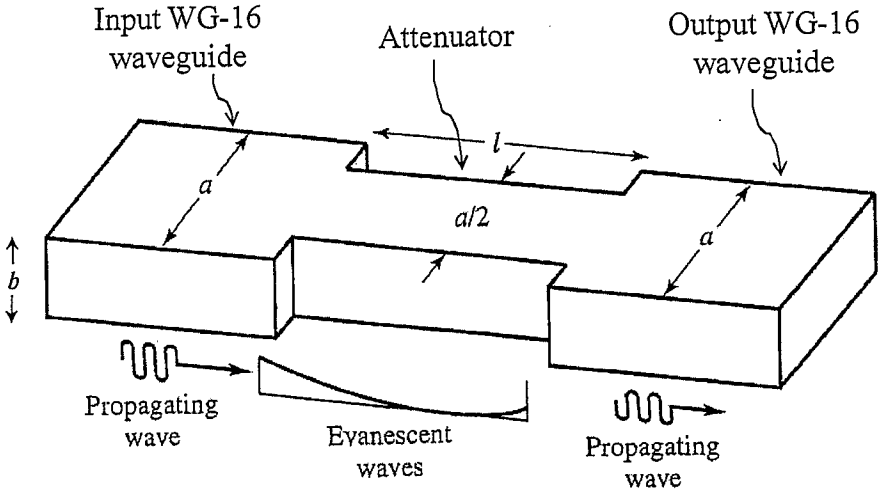
1. (25%) The inner conducting spherical shell of radius a is held at a potential V_1 and the outer shell of radius b at V_2. The region between these two concentric shells is filled with an insulating material. Determine the potential distribution between the shells.

2. (25%) Determine the inductance per unit length of an air coaxial transmission line with a solid inner conductor of radius R_i and a very thin outer conductor of inner radius R_o. ($R_o/R_i = 5$ and $\mu_o = 4\pi \times 10^{-7}\, H/m$)

3. The transmission line circuit shown below has $V_g = 1\angle 0^\circ$, $Z_g = 50\ \Omega$, $Z_0 = 50\ \Omega$, $Z_L = 30 - j20\ \Omega$, and $l = 0.6\ \lambda$. Calculate the power delivered to the load (denoted as P_L) using three different ways as follows.
   (a) (10 %) Find the reflection coefficient (Γ) by which P_L can be further calculated.
   (b) (10 %) Find the input impedance (Z_{in}) by which P_L can be further calculated.
   (c) (10 %) Find the load voltage (V_L) by which P_L can be further calculated.



4. (a) (10%) A standard WG-16 air-filled rectangular waveguide for X-band applications has the dimensions: a = 2.29 cm and b = 1.02 cm. If this WG-16 waveguide operates only in the dominant mode (TE_{10} mode), what is the allowable operating-frequency range?
   (b) (10%) An attenuator connected between the input and output WG-16 waveguides is made using a section of rectangular waveguide with a width of $a/2$ and a length of l, and this waveguide section operates below its dominant mode cutoff frequency. At the operating frequency of 12 GHz, determine the required length, l, of this waveguide section to achieve an attenuation of 100 dB between the input and output WG-16 waveguides.



科目：線性代數【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】 √

1. (10%) Consider a matrix A defined by its diagonalization as follows:

$$A = CDC^{-1} = \begin{bmatrix} 3 & 2 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 6 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 6 & 5 \end{bmatrix}^{-1},$$

where the columns of matrix C are the eigenvectors of A. Find the eigenvalues and the corresponding eigenvectors of A^T.

2. (10%) Find the singular value decomposition of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$.

3. For the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(a) (5%) Find $2A^4 + 3A^3 - 41A^2 - 111A + 5I$.

(b) (5%) Find $\left|AA^T\right|$.

(c) (5%) Find $\left|\left(A^{-2}\right)^T\right|$.

4. (15%) Suppose W_1 and W_2 are the subspace of V.

Prove that $V = W_1 \oplus W_2 \leftrightarrow \dim(V) = \dim(W_1 + W_2) = \dim(W_1) + \dim(W_2)$.

5. (15%) Please <u>verify</u> that the inverse matrix of $A+BCD$ equals to

$$(A+BCD)^{-1} = A^{-1} - A^{-1}B\left(C^{-1} + DA^{-1}B\right)^{-1} DA^{-1}$$

6. (20%) Given the following matrices:

(i) $\begin{bmatrix} 2 & i \\ -i & 1 \end{bmatrix}$　　(ii) $\begin{bmatrix} 1+i & 3 \\ 3 & 5 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 1+i & 1 \\ 1-i & 2 & -i \\ 1 & i & 5 \end{bmatrix}$　　(iv) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(a) (5%) Which are skew-symmetric?

(b) (5%) Which are Hermitian?

(c) (5%) Which are unitary?

(d) (5%) Which are positive definite?

7. (a) (5%) Apply the Gram-Schmidt process to the following vectors to form a set of orthonormal bases.

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad u_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \qquad u_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

(b) (5%) Find the orthogonal projection of the vector $\begin{bmatrix} 2 & 1 & -3 & 2 \end{bmatrix}^T$ on the space spanned by u_1, u_2, and u_3.

(c) (5%) Find the QR decomposition of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

科目：機率【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】 ✓

1. Random variables X and Y have respective density functions defined as

$$f_X(x) = \frac{1}{a}[u(x) - u(x-a)]$$

$$f_Y(y) = bu(y)e^{-by}$$

for $a>0$ and $b>0$, and $u(x)$ is denoted as the *unit step* function of x.

(a) Find and sketch the density function of Z=X+Y if X and Y are statistically independent. (7%)

(b) What is the moment generating function of Z. (8%)

2. (***Linear amplifier with cutoff***) Let us consider X to be the random variable with zero-mean and unit variance Gaussian distribution, $N(0, 1)$, and consider a non-linear device with transmittance, $y = g(x)$, to be defined as

$$g(x) = 0, \quad |x| \geq 1$$
$$g(x) = x, \quad -1 < x < 1.$$

Given that

$$f_X(x) = \frac{1}{2\pi}\exp[-\frac{x^2}{2}]$$

(a) Please find the probability density function of $Y=g(X)$, $f_Y(y)$. (10%).

(b) (***Infinite roots***) If we consider the *periodic extension* of the transmittance, that is the extended g(x) is defined as

$$g(x) = \sum_{n=-\infty}^{\infty} (x-2n)rect(\frac{x-2n}{2}),$$

where

$$rect(\frac{x}{2}) = \begin{cases} 1, & |x| \leq 1 \\ 0, & otherwise. \end{cases} \qquad (10\%)$$

Find $f_Y(y)$.

3. Let $Y = A\cos(\omega t + \Theta)$ where A, ω and t are constants, and Θ is a uniform random variable in the interval $(0, 2\pi)$. The random variable Y results from sampling the amplitude of a sinusoidal with random phase Θ. (a) Find $E[Y]$ (8%) and (b) $E[Y^2]$ (7%).

4. Let X be a random variable and $A = \{X \leq b\}$.

(a) Find the conditional probability distribution function, $F_{X|A}(X|A)$ (10%).

(b) What is the conditional density function $f_{X|A}(X|A)$? (5%)

科目：機率【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】

5. Let $Z = X_1 + X_2$ with $f_{X_1}(x), f_{X_2}(x)$ and $f_Z(z)$ denoting the probability functions of X_1 and X_2, and Z, respectively.

(a) Show that $\Phi_Z(\omega) = \Phi_{X_1}(\omega)\Phi_{X_2}(\omega)$ (10%)

(b) If $f_{X_1}(x) = f_{X_1}(x) = \frac{1}{\sqrt{2\pi}}\exp\{-(\frac{x-\mu_X}{2})^2\}$, please find $\Phi_Z(\omega)$ and $f_Z(z)$ (10%)

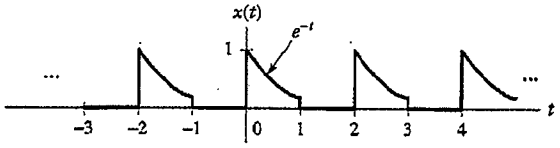
6. *Show that* (a) $E[Y] = E[E[Y|X]]$ (7%). (b) $E[Z|X] = E[E[Z|X,Y]|X]$ (8%).

(*Note:* $E[Y] = \int_{-\infty}^{\infty} y f_Y(y)dy$)

科目：通訊理論【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】 √

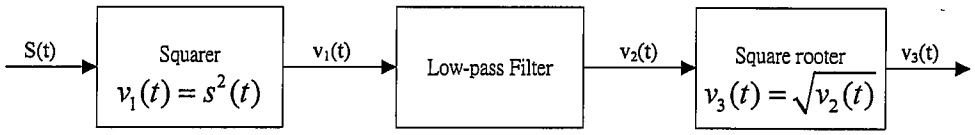
通訊理論 （Communications Theory）

1. (15 points) A signal is given as follows.



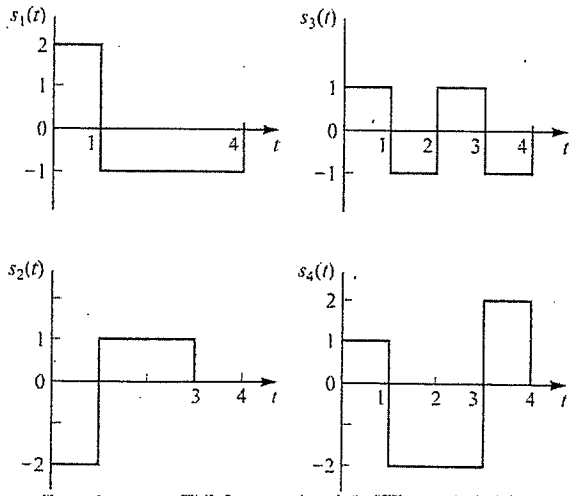
(a) (7 points) Explain whether the signal is an energy signal or a power signal.
(b) (8 points) Find the Fourier representation of the signal.

2. (20 points) An AM signal $s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$ is considered in the following systems:
(a) (10 points) If the signal $s(t)$ is used as input to a square-law detector whose transfer characteristic is defined by $v_0(t) = a_1 v_i(t) + a_2 v_i^2(t)$, where a_1 and a_2 are constants, $v_i(t)$ is the input, and $v_0(t)$ is the output . Find the conditions for which the message signal $m(t)$ may be recovered from $v_0(t)$.
(b) (10 points) The signal $s(t)$ is applied to the system below. Assuming that $|k_a m(t)| < 1$ for all t and the message signal $m(t)$ is limited to the spectral interval $-W \le f \le W$ and that the carrier frequency $f_c > 2W$, show that $m(t)$ can be obtained from the square-rooter output $v_3(t)$.



3. (15 points) An FM signal with a frequency deviation of *10 kHz* at a modulation frequency of *5 kHz* is applied to two frequency multipliers connected in cascade. The first multiplier doubles the frequency and the second multiple triples the frequency. Determine the frequency deviation and the modulation index of the FM signal obtained at the second multiplier output.

4. (20 points) Gram-Schmidt Orthogonalization Procedure
(a) (10 points) Suppose that we have a set of finite energy signal waveforms $\{s_i(t), i = 1, 2, ..., M\}$. Please describe the Gram-Schmidt orthogonalization procedure to construct a set of orthonormal waveforms.
(b) (5 points) Consider the four waveforms shown in the following figure. Determine the dimensionality of the waveforms and a set of orthonormal functions.
(c) (5 points) From (b), use the orthonormal functions to represent the four waveforms by vectors $\mathbf{s}_1$, $\mathbf{s}_2$, $\mathbf{s}_3$, and $\mathbf{s}_4$.



科目：通訊理論【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】

5. (20 points) Explanations

(a) (5 points) Please describe the Nyquist sampling theorem for a band-limited signal of finite energy.

(b) (5 points) Please describe the Nyquist criterion for distortionless baseband transmission in the absence of noise.

(c) (5 points) Consider a random process $X(t)$ that is initiated at $t=-\infty$. Let $X(t_1)$, $X(t_2)$, ..., $X(t_k)$ denote the random variables obtained by observing the random process $X(t)$ at times t_1, t_2,..., t_k, respectively. The joint distribution function of this set of random variables is $F_{X(t_1),X(t_2),...,X(t_k)}(x_1, x_2, ..., x_k)$. Please describe the condition that the random process $X(t)$ is said to be stationary in the strict sense.

(d) (5 points) Please describe the conditions that the process $X(t)$ is said to be Ergodic in the mean.

6. (10 points) Signal Constellation

For the QAM signal constellation shown in the following figure, determine the optimum decision boundaries for the detector, assuming that the SNR is sufficiently high so that errors only occur between adjacent points.