國立中山大學96學年度碩士班招生考試試題

科目：工程數學甲【電機系碩士班甲、丁、戊、庚組；丙組選考】 共 1 頁 第 1 頁

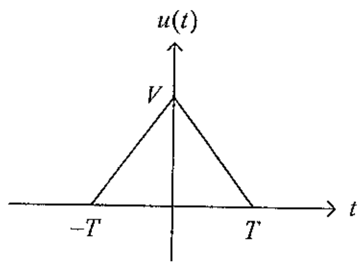
1. (15%) Find the general solution of the following differential equation

$$(y\cos x-\sin 2x)dx+dy = 0.$$

2. (20%) Evaluate $\int_{-\infty}^{\infty}\frac{dx}{(x^2+1)^3}$ by using the residue theorem.

3. (15%) Let S be the surface of the paraboloid $x^2+y^2+z = 2$ lying above the xy-plane. Then the area of S = ______. （只須寫出答案，不須列出算式，計分僅以答案為準。）

4. (20%) （此題只須寫出答案，不須列出算式，計分僅以答案為準。）Consider the signal $u(t)$ described by the following plot



(a) (7%) The Laplace transform of $u(t)$ is defined by $U(s) := \int_0^{\infty}u(t)e^{-st}dt$. So, $U(s)$ = ______.

(b) (6%) The Fourier transform of $u(t)$ is defined by $U(w) := \int_{-\infty}^{\infty}u(t)e^{-jwt}dt$. So, $U(w)$ = ______. （答案必須化成最簡式，否則不予計分。）

(c) (7%) Suppose V and T be chosen such that $\int_{-\infty}^{\infty}U(w)dw = \int_{-\infty}^{\infty}|U(w)|^2dw$. Then the relationship between V and T is ______.

5. (30%) Let $A = \begin{bmatrix} t & 1 & 1 \\ 1 & -1 & 2 \\ -1 & 4 & -5 \\ 1 & 2 & -1 \end{bmatrix}$, where t is an undetermined real parameter.

(a) (6%) Please describe the null space $N(A)$ with respect to the value of parameter t.

From now on, based on the result of (a), let the value of t be chosen so that the homogeneous equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solution.

(b) (8%) Consider the linear equation $A\mathbf{x} = \mathbf{b}$. Let M be a nonsingular matrix so that $MA = A_{rref}$ and $M\mathbf{b} = [\alpha\ \beta\ \gamma\ \delta]^T \in \mathbb{R}^4$, where A_{rref} denotes the reduced row echelon form obtained from matrix A after the application of Gauss elimination. Please find the set of $[\alpha\ \beta\ \gamma\ \delta]^T$ such that equation $A\mathbf{x} = \mathbf{b}$ is solvable, and moreover, write out the solution set, described by S, of the equation.

(c) (6%) Please find $\mathbf{x}^*$ to solve the optimization problem $\min_{\mathbf{x}\in S} f(\mathbf{x})$ with $f(\mathbf{x}) := \|\mathbf{x}\|_2$.

(d) (10%) Let P denote the projection matrix to orthogonally project any vector from $\mathbb{R}^4$ into $R(A)^{\perp}$, the orthogonal complement of the range of A. Please find P and write out the set of all its eigenvalues.

國立中山大學96學年度碩士班招生考試試題

科目：電子學【電機系碩士班甲、乙、戊組】

1. Design a generalized current mirror. Consider the current mirror shown in Figure 1, with parameters $V^+ =$ 5V and $V^- =$ -5V. Neglect base currents and assume $V_{BE} = V_{EB} = 0.6$V. Design the circuit such that $I_{O2} =$ 0.4mA. Please determine I_{REF}, I_{O1}, I_{O3}, I_{O4}, and R_1. (20%)

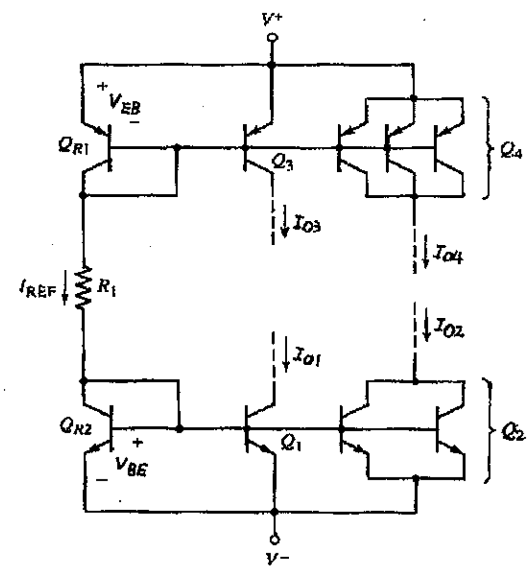


Figure 1.

2. In Figure 2 the small signal equivalent of a bipolar junction transistor with simplified hybrid – π model and including the equivalent Miller capacitance are shown respectively, where the circuit parameters are $R_C = R_L =$ 4 kΩ, $r_\pi =$ 2.6 kΩ, $R_B =$ 200 kΩ, $C_\pi =$ 4 pF, $C_\mu =$ 0.2 pF, and $g_m =$ 38.5 mA/V. Please determine the 3 dB frequency of the current gain for the circuit both with and without the effect of C_M. (20%)

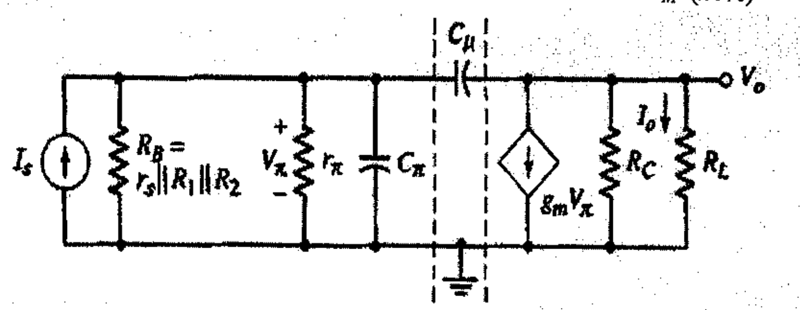


Figure 2(a)

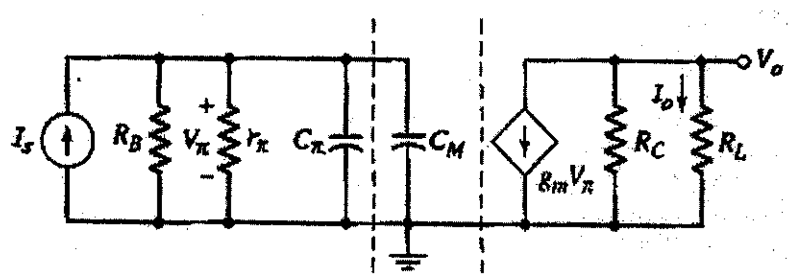


Figure 2(b)

3. The Figure 3a shown below is the Bode plot of open-loop gain for a special circuit to do Stability Analysis.
   (a) Please describe the two methods of the compensation technique as referring Figure 3a. (7%)
   (b). Please explain the fundamental definition of the Gain Margin and Phase Margin. (6%)
   (c). Referring to Figure 3b please do the Stability Analysis if the circuit having a Closed Loop Gain 50db. (7%)

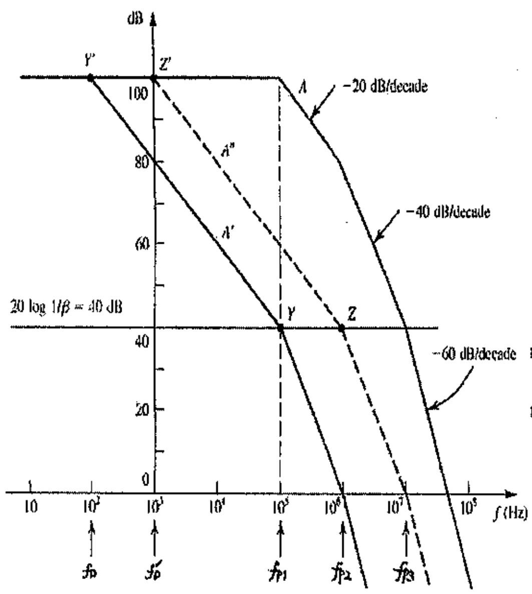


Figure 3(a)

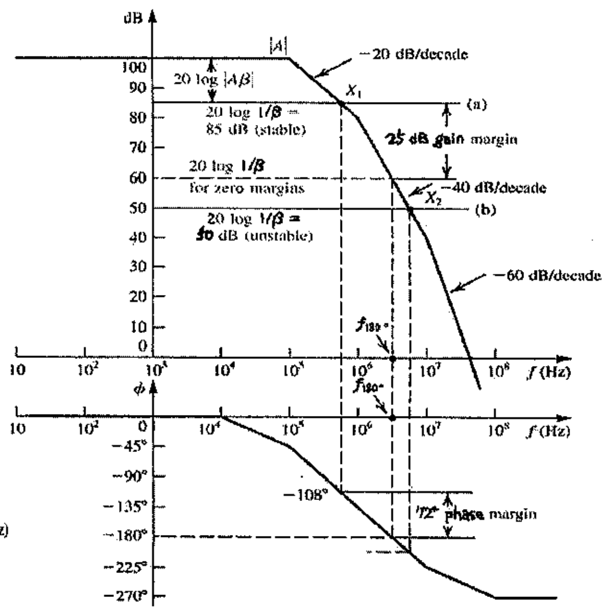


Figure 3(b)

4. Please analyze the circuit of Figure 4 by plotting out the small signal equivalent circuit to determine the small-signal voltage gain V_O/V_S, the input resistance R_{in} and the output resistance $R_{out} = R_{of}$. Note: the transistor has $\beta = 100$. (20%)

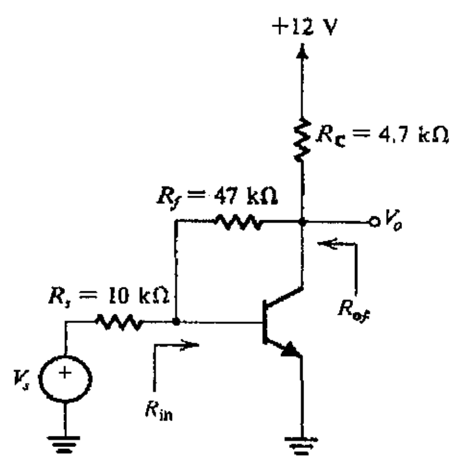


Figure 4.

5. (a) Please draw the cross-section of an enhancement-type NMOS transistor.
   (b) Please do the derivation of the i_D-v_{DS} relationship (i.e. the i_D-v_{DS} expressions) with and without considering the finite output resistance in saturation. (20%)

Parameters:

$\varepsilon_{Si} = 11.7$;

Absorption coefficient of GaAs at $\lambda = 0.75\ \mu m$ is $\alpha \approx 0.7x10^{-4}\ cm^{-1}$.

1) Calculate the dielectric relaxation time constant for a particular semiconductor. (assume an n-type Si semiconductor with a donor impurity concentration of $N_d = 10^{16}\ cm^{-3}$ and $\varepsilon_{Si} = 11.7$.) (20%)

2) To calculate the probability that an energy level 3 kT above E_F is occupied by an electron at 300 K. (20%)

3) (a) To calculate threshold voltage shift due to short channel effects. (10%)

(b) To design the channel width that will limit the threshold voltage shift because of narrow channel effects to a specified value (15%).

Consider an n channel MOSFET with $N_a = 3x10^{16}\ cm^{-3}$ and $t_{OX} = 450$ Å.

Let $L = 1.25\ \mu m$, and assume that the diffused junction depth = 0.5 μm. The fitting parameter that accounts for the lateral space charge width = $\pi/2$.

Assume that we want to limit the threshold shift to $\Delta V_T = 0.2$ V.

4) To calculate the generation rate of electron-hole pairs given an incident intensity of photons. (15%)

Consider gallium arsenide at T = 300 K. Assume the photon intensity at a particular point is $I_v(x) = 0.05\ W/cm^2$ at a wavelength of $\lambda = 0.75\ \mu m$. If excess carrier lifetime; $\tau = 10^{-7}$ s. Absorption coefficient of GaAs at $\lambda = 0.75\ \mu m$ is $\alpha \approx 0.7x10^{-4}\ cm^{-1}$.

5) To calculate (a)the build in potential barrier (10%).

(b) the width of the space charge region in a pn junction when a reverse-bias voltage, V_R, is applied (10%).

Consider a Silicon pn junction at T = 300 K with doping concentrations of $N_a = 10^{16}\ cm^{-3}$ and $N_d = 10^{15}\ cm^{-3}$. Assume that $n_i = 1.5x10^{10}\ cm^{-3}$ and let $V_R = 5V$.

國立中山大學９６學年度碩士班招生考試試題

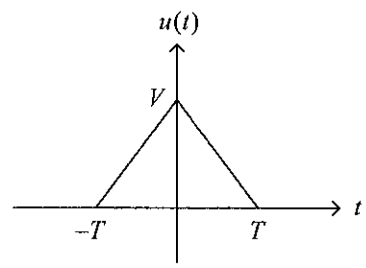
科目：工程數學乙【電機系碩士班乙組】

1. (15%) Find the general solution of the following differential equation

$$(x-y)dx+(x+y)dy = 0.$$

2. (20%) Evaluate $\int_{-\infty}^{\infty} \frac{x\,dx}{(x-1)(x^2+2x+2)}$ by using the residue theorem.

3. (20%)（此題只須寫出答案，不須列出算式，計分僅以答案為準。）Consider the signal $u(t)$ described by the following plot



(a) (7%) The Laplace transform of $u(t)$ is defined by $U(s) := \int_0^{\infty} u(t)e^{-st}dt$. So, $U(s) =$ _______.

(b) (6%) The Fourier transform of $u(t)$ is defined by $U(w) := \int_{-\infty}^{\infty} u(t)e^{-jwt}dt$. So, $U(w) =$ _______.
（答案必須化成最簡式，否則不予計分。）

(c) (7%) Suppose that V and T are chosen such that $\int_{-\infty}^{\infty} U(w)dw = \int_{-\infty}^{\infty} |U(w)|^2 dw$. Then the relationship between V and T is _______.

4. (30%) Let $A = \begin{bmatrix} t & 1 & 1 \\ 1 & -1 & 2 \\ -1 & 4 & -5 \\ 1 & 2 & -1 \end{bmatrix}$, where t is an undetermined real parameter.

(a) (6%) Please describe the null space $N(A)$ with respect to the value of parameter t.

From now on, based on the result of (a), let the value of t be chosen so that the homogeneous equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solution.

(b) (8%) Consider the linear equation $A\mathbf{x} = \mathbf{b}$. Let M be a nonsingular matrix so that $MA = A_{rref}$ and $M\mathbf{b} = [\alpha\ \beta\ \gamma\ \delta]^T \in \mathbb{R}^4$, where A_{rref} denotes the reduced row echelon form obtained from matrix A after the application of Gauss elimination. Please find the set of $[\alpha\ \beta\ \gamma\ \delta]^T$ such that equation $A\mathbf{x} = \mathbf{b}$ is solvable, and moreover, write out the solution set, described by S, of the equation.

(c) (6%) Please find $\mathbf{x}^*$ to solve the optimization problem $\min_{\mathbf{x}\in S} f(\mathbf{x})$ with $f(\mathbf{x}) := \|\mathbf{x}\|_2$.

(d) (10%) Let P denote the projection matrix to orthogonally project any vector from $\mathbb{R}^4$ into $R(A)^{\perp}$, the orthogonal complement of the range of A. Please find P and write out the set of all its eigenvalues.

5. (15%) Let $A \in \mathbb{R}^{n\times n}$ be such that the product operation between A and its transpose is commutative. Please answer the next two questions followed by detailed proofs.

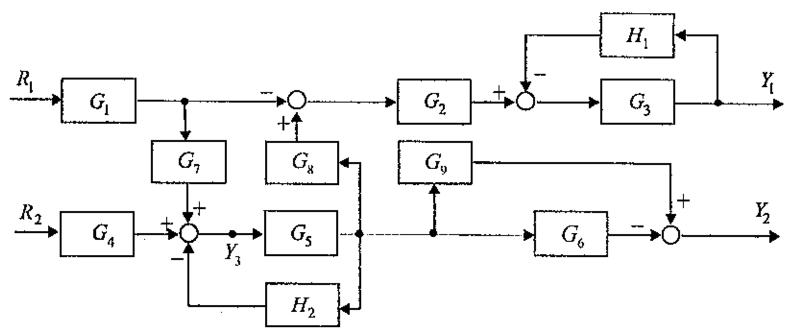
(a) (6%) What is the relationship between $N(A)$ and $N(A^T)$?

(b) (9%) Suppose that α and β are two distinct real eigenvalues of A with $\mathbf{x}$ and $\mathbf{y}$ being the corresponding eigenvectors, respectively. What kind of relationship between $\mathbf{x}$ and $\mathbf{y}$ can be derived from the result of (a) ?

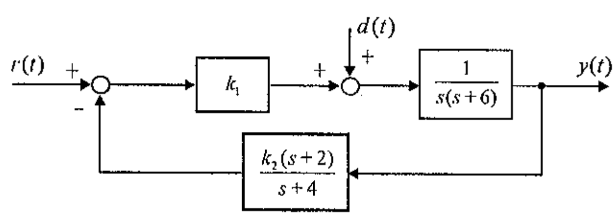
國立中山大學96學年度碩士班招生考試試題

科目：控制系統【電機系碩士班乙組】

1. (20%) Find the transfer function $Y_1(s)/Y_3(s)$ of the following system:



2. The block diagram of a feedback control system is shown below:

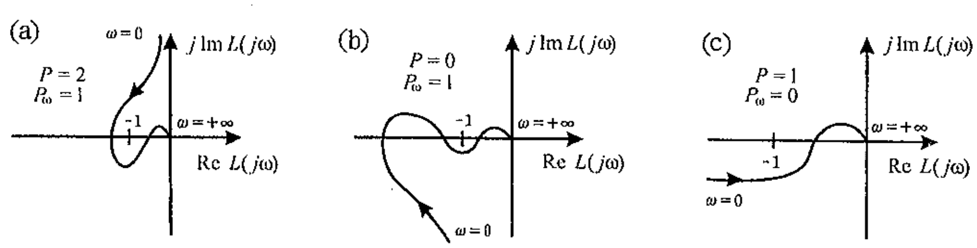


(a)(8%) Suppose that $d(t) \equiv 0$ and $k_2 = 5$. Find the range of k_1 such that all poles of the closed-loop system have a real part less than -1.

(b)(8%) Let the tracking error be defined as $e(t) \triangleq r(t) - y(t)$. If $k_1 = 5$ and $r(t) = 1$, compute the range of k_2 so that the steady-state error $e(t)|_{t\to\infty} \le 0.1$.

(c)(6%) For zero command $r(t) \equiv 0$ and $k_1 = 5$, find the range of k_2 so that the steady-state value of the output $y(t)$ due to a unit step in $d(t)$ is less than 0.01.

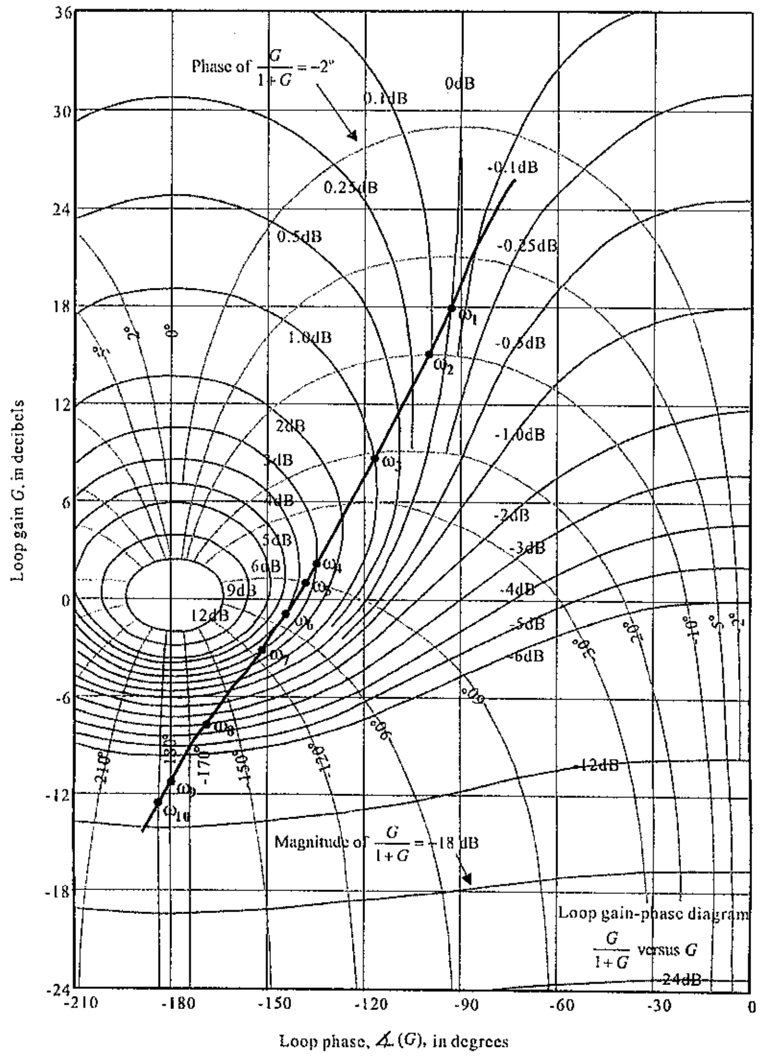
3.(18%) Let "P_ω" be the number of poles of a loop transfer function $L(s)$ on the imaginary axis, "P" be the number of poles of $L(s)$ in the right-half of s–plane. Determine the stability of each system from the Nyquist plot shown below. If the system is unstable, find the number of closed-loop poles in the right-half of s–plane.



4.(18%) Find out if the matrices given in the following can be state-transition matrices. Show your computation of verification, and give reason when you do not think that the matrix is a state-transition matrix.

$$(a)\begin{bmatrix} -e^{-t} & 0 \\ 0 & 1-e^{-t} \end{bmatrix} \qquad (b)\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix} \qquad (c)\begin{bmatrix} e^{-t} & 1-e^{-t} \\ 0 & e^{-t} \end{bmatrix}$$

5. Consider a system whose Nichols chart is shown as follows:



where $\omega_1 = 1, \omega_2 = 3, \omega_3 = 8, \omega_4 = 17, \omega_5 = 20, \omega_6 = 25, \omega_7 = 35, \omega_8 = 50, \omega_9 = 70, \omega_{10} = 78$, (their units are all rad/s).

(a)(2%) Find the maximum peak M_p in the closed-loop frequency response.

(b)(2%) Find the approximate resonance frequency ω_r?

(c)(2%) Find the bandwidth of the closed-loop system.

(d)(4%) Find the phase crossover and gain crossover frequencies of this system.

(e)(4%) Find the phase margin and gain margin of this system.

(f)(3%) Find the closed-loop phase angle at ω_7.

(g)(5%) If the constant loop gain is increased to three times of the original system, is this system still stable? How much gain that you can multiply to the original system before it become unstable?

Problem 1 : MUXs are easily to be used to realize other Boolean functions. An "example" of MUXs is shown in Figure 1, where S_1 and S_0 are selection signals, and I_0, I_2, I_2, I_3 are inputs. Please implement the following Boolean equation with only "one" single minimal-sized MUX (It might not be a 4-to-1 MUX as shown in the figure.) :

$$Z(A,B,C,D) = \sum(0,1,3,6,9,13,14)$$

Note that you don't need to draw the internal design of the MUX. You simply use a block diagram to denote it. (10%)

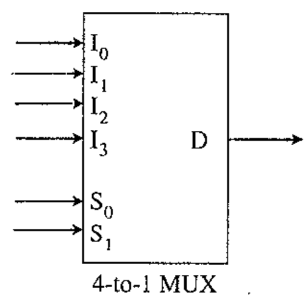


Figure 1: Problem 1 : an example of a MUX - 4-to-1 MUX

Problem 2 : Derive the state table for the circuit in Figure 2 step by step. What is the sequence of input values on wire w to be detected by this circuit? (10%)

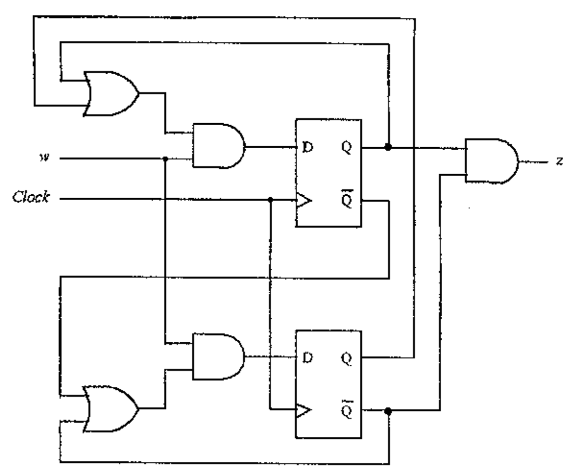


Figure 2: Problem 2

Problem 3 : Figure 3 is one type of FPGA units, called ACT. It is a MUX-based logic block. Show how the function $f = w_2\overline{w_3} + w_1w_3 + \overline{w_2}w_3$ can be implemented using only one ACT unit. (10%)

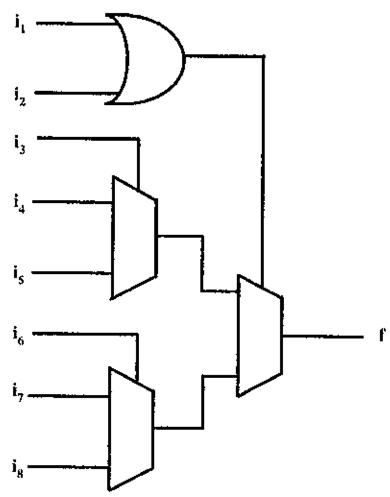


Figure 3: Problem 3

Problem 4 : Design a circuit that generates the 9's complement of a BCD digit, which is composed of $d = D_3D_2D_1D_0$, where D_i for i = 3, 2, 1, 0, is a binary bit. (10%)

Problem 5 : Consider the function $f = \overline{w_2} + \overline{w_1}\ \overline{w_3} + w_1w_3$. Please show how to derive the minterms of f by repeatedly applying Shannon's expansion method to w_3, w_2, w_1, sequentially. (10%)

Problem 6 : Given an adder of which the inputs are A_i, B_i and C_i, the outputs are S_i and C_o, please show how to use only an adder to carry out each of the following Boolean functions individually. Please treat the adder as a black box and ignore the details internally. (20%)
1). XOR
2). XNOR
3). AND
4). OR

Problem 7 : Show a state table for the state-assigned table in Figure 4, using A, B, C, D for the four rows in the table. Give a new state-assigned table using a one-hot encoding. For A use the code $y_4y_3y_2y_1 = 0001$. For the states, B, C, and D, use the codes, 0010, 0100, and 1000, respectively. Synthesize a circuit using D flip-flops. (10%)

Present state y_2y_1	Next state $w=0$ Y_2Y_1	Next state $w=1$ Y_2Y_1	Output z
0 0	1 0	1 1	0
0 1	0 1	0 0	0
1 0	1 1	0 0	0
1 1	1 0	0 1	1

Figure 4: Problem 7

Problem 8 : Referring to the circuit in Figure 5, assume that all of the inputs are randomly generated with values of either "1" or "0". In other words, the probability of "1" or "0" at each input wire is 1/2. Since the switching activity is highly correlated with power consumption of digital circuits, we'd like to estimate the switching activity at each node if possible. Please answer the following questions.

1). What is the relationship between power consumption and the switching activity? Please justify your answer. (10%)

2). What is the switching activity at node j (5%)

3). What is the switching activity at node z (5%)

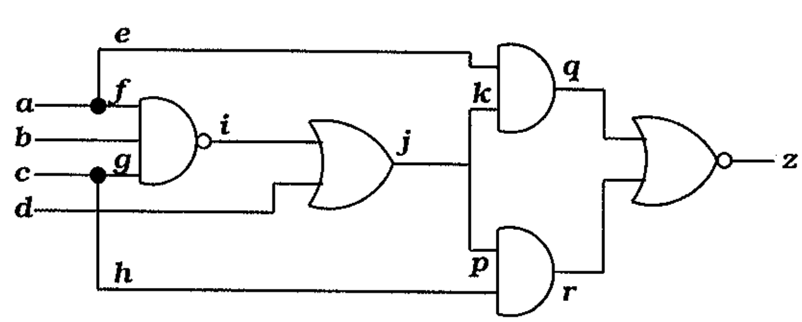


Figure 5: Problem 8 : a circuit to be measured

國立中山大學96學年度碩士班招生考試試題

科目：計算機結構【電機系碩士班丙、庚組】 共 / 頁 第 / 頁

I. (20%) Identify **two** differences between the following terminology pairs.

(1) Computer Networks vs Cluster Computers

(2) Multi-Core Server vs Multi-processor Server

(3) VLIW vs Superscalar

(4) Synchronous DRAM vs Cache DRAM

(5) TLB (Translation Lookaside Buffer) vs Page Table

II. (15%) Consider a hypothetical 32-bit microprocessor having 32-bit instructions composed of two fields: the first byte contains the OP code and the remainder the immediate operand or an operand address. Assume that the local address bus is 32 bits and the local data bus is 16 bits. No time multiplexing between the address and data buses.

(1) What is the maximum directly addressable memory capacity (in bytes)?

(2) What is the minimum bit numbers required for the program counter?

(3) Assume the **direct addressing mode** is applied, how many address and data bus cycles required to fetch an instruction and its corresponding operand or data from memory?

III. (15%) Perform the following three Intel X86 instructions,

```
MOV   AX   0248H
MOV   BX   0564H
CMP   AX   BX
```

and list the Carry Flag(CF), Overflow Flag(OF), Parity Flag(PF), Sign Flag(SF), and Zero Flag(ZF).

IV. (20%) Analysis of Program Structures.

(1) Analyze the following program, and find out how many times the statement "sum ++ " are executed.

```
sum = 0;
For (i = 0; i < n; j ++)
    h = i +1;
    For (j = 0; j < h * h; j++)
        sum ++;
```

(2) Analyze the following program, and find out how many times the statement "A(i, j, k)" are executed.

```
For k = 1 to n
    For i = 0 to k-1
        For j = 0 to k-1
            For i ≠j then A(i, j, k)
        End
    End
End
```

V. (15%) Hamming error correction codes.

(1) How many check-bits are needed if the Hamming error correction code is used to detect single bit errors in a 1024-bit data word? (7%)

(2) For the 8-bit word 00111001, the check bits stored with it would be 0111. Suppose when the word is read from memory, the check bits are calculated to be 1101. What is the data word that was read from memory? (8%)

VI. (15%) Consider a cache and a main memory hierarchy, in which cache = **32K words**, main memory = **128M words**, cache block size = **8 words**, and word size = **4 bytes**.

(1) Show physical address format for Direct Mapping (How many bits in Tag, Block, and Word?)

(2) Show physical address format for **4-way** Set Associative Mapping (How many bits in Tag, Set, and Word?)

(3) Show physical address format for Sector Mapping with **16** blocks per sector. (How many bits in Sector, Block, and Word?)

（需將推導過程敘述清楚）

1. (10%) Given the following facts about Boolean variables *a, b, c,* and *d*, draw its truth table.

   (a and b) or c

   (~ a or d) ^ (b or c)

   a or ~b → c or d

   where operators

   ~: not operator

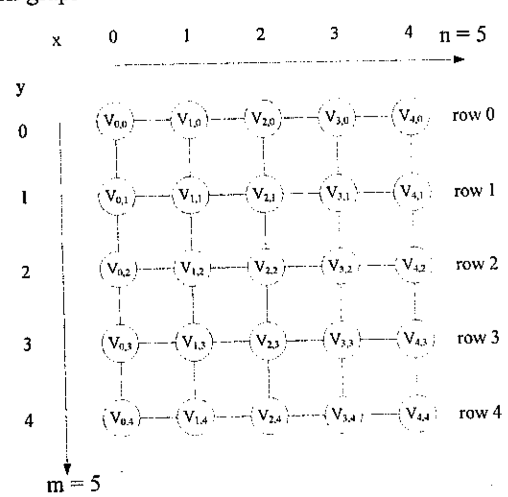
   p→q ≡~ p or q

2. For integers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10,

   (a) (10%) Define a relation $R = \{(a,b) \mid (a \bmod b = 0 \text{ or } a = b + 1) \text{ and } a,b \in I \text{ and } a,b \in [1,10]\}$ where *mod* is a remainder operation, construct the relation matrix for R.

   (b) (10%) Derive a path from 9 to 6 if such path exists. (i.e. path = {(9, i1), (i1, i2), … , (ik, 6)})

3. Given a 2-dimensional mesh graph of *m* rows and *n* columns as follows ($m > 2$, $n > 2$):



   (a) (5%) If both *m* and *n* are **even** numbers, does the mesh graph contain an **Eulerian circuit**? If not exists, why? If exists, construct one.

   (b) (5%) If both *m* and *n* are **even** numbers, does the mesh graph contain a **Hamiltonian circuit**? If not exists, why? If exists, construct one.

   (c) (5%) If both *m* and *n* are **odd** numbers, does the mesh graph contain a **Hamiltonian circuit**? If not exists, why? If exists, construct one.

   (d) (5%) If both *m* and *n* are **odd** numbers, does the mesh graph contain a **Hamiltonian path**? If not exists, why? If exists, construct one.

4. Given a complete graph K_n of *n* distinct vertices,

   (a) (5%) determine the number of complete subgraphs K_{n-1} (of *n-1* vertices) that the complete graph K_n contains.

   (b) (5%) determine the number of complete subgraph K_m (of *m* vertices) that the complete graph K_n contains. ($m \le n$.)

5. (10%) **Prove** that a complete graph K_n of n vertices has a complete subgraph K_{n-1} of $n-1$ vertices. (Give a mathematical proof. Only explanation cannot be treated as a proof.)

6. (10%) A tree of n vertices is a connected graph without any cycles. **Prove** that it has exactly $n-1$ edges.

7. Given a recursive formula $f(n)$ for non-negative integer n as follows,
(*mod* : remainder operation, == :equality test, = : assignment operation)
   if $n = 0, f(n) = 5$
   if $n = 1, f(n) = 12$
   if $n > 1$ *and* n *mod* $2 == 0, f(n) = f(n-1) + f(n-2)$
   if $n > 1$ *and* n *mod* $2 == 1, f(n) = f(n-1) * f(n-2) + 2 * f(n-3)$

   (a) (10%) write a recursive algorithm (a pseudo program) to compute $f(n)$

   (b) (10%) perform recursive evaluation process of $f(3)$ in a **tree structure form of recursive calls f(...).**

國立中山大學96學年度碩士班招生考試試題

科目：資料結構【電機系碩士班丙組選考】

1. [16] Suppose we have implemented a list for storing integers as the diagram shown in Figure 1, in which three integers 70, 15, and 25 have been stored. In the diagram, integers are stored in an array of eight elements, indexed from 0 to 7. Each element in the array may contain one integer and a pointer. A pointer is the index of the next element the current element is linked to. In the implementation, the elements with stored integers form a link, called the data link, and the rest elements form the other link, called the availability link. The variable "count" contains the number of integers currently stored in the array. The variable "data" contains the index of the first element of the data link, and contains -1 if the data link is empty. The variable "available" is the index of the first element in the availability link, and contains -1 if the availability link is empty. The pointer field of the last element in the data link contains -1, and so does in the availability link. When an input integer is inserted, the first element in the availability link is used for storing the integer and the "available" variable points to the successor of the element. When a stored integer is deleted, the corresponding element becomes available for storing new integers and is added to the availability link as the first element of the link. Draw the resulting diagram for each of the following operations:
   a. [4] Insert 20 as the third element in the data link.
   b. [4] Then delete 25.
   c. [4] Then insert 50 as the first element in the data link.
   d. [4] Then delete the second element in the data link.

count	data	available
3	4	0

0	1	2	3	4	5	6	7
	70			15			25
6	7	-1	5	1	2	3	-1

Figure 1

2. [10] Suppose we hash three-digit integers into a 10-entry hash table of ten buckets, N=10, as shown in Figure 2. Each bucket can contain only one integer. Assume that overflow is resolved with open addressing. Suppose we have eight numbers 282, 716, 239, 192, 254, 432, 163, 121 in order. Let the hash function: $hash(x) = x$.

	x
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Figure 2

a. [5] Suppose we use the following linear probe:

$$A_i(x) = (i + hash(x))\%N$$

as the address-generating function. Please show the resulting hash table after the insertion of the eight numbers.

b. [5] Suppose we use the following non-linear function:

$$A_i(x) = (3\times i + 5\times i^2 + hash(x))\%N$$

as the address-generating function. Please show the resulting hash table after the insertion of the eight numbers.

3. [9] Suppose we have a binary tree shown in Figure 3.

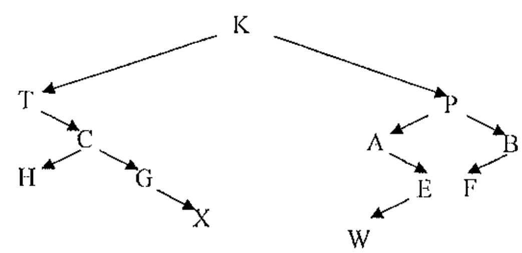


Figure 3

a. [3] List the nodes with the inorder traversal.

b. [3] List the nodes with the preorder traversal.

c. [3] List the nodes with the postorder traversal.

4. [6] Suppose we have a binary search tree shown in Figure 4.

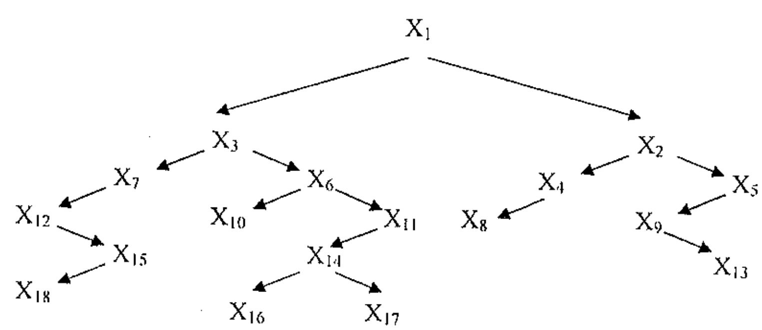


Figure 4

Suppose we replace a node by the maximal node of it's left subtree.

a. [2] What is the resulting binary search tree after the deletion of X_2 from Figure 4?

b. [2] What is the resulting binary search tree after the deletion of X_3 from Figure 4?

c. [2] What is the resulting binary search tree after the deletion of X_1 from Figure 4?

5. [12] Suppose we have the following 15 numbers: 200, 250, 150, 125, 120, 225, 215, 220, 175, 275, 245, 260, 270, 265, 130.

a. [3] Please show the resulting binary search tree after the insertions of these 15 numbers.

b. [9] Please create the AVL tree with these 15 numbers given in order. Please show the resulting AVL trees after the insertions of 215, 270, and 130, respectively.

6. [9] Please find the shortest paths and their lengths from vertex B to each of the other reachable vertices in Figure 5 using the Dijkstra's shortest path algorithm. Please show the result after each iteration. In each iteration, a vertex is chosen and distances are recalculated.

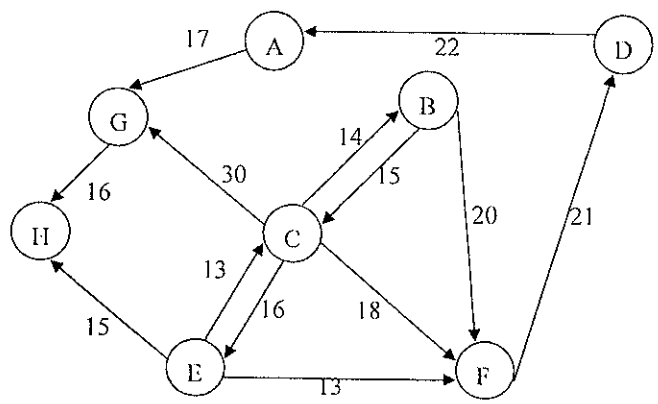


Figure 5

7. [6] Suppose we have a graph shown in Figure 6. Please find a minimum spanning tree by
   a. [3] Using Kruskal's algorithm. Please label an edge by 1, 2, 3, etc., on the obtained tree according to the order of the addition of the edge.
   b. [3] Using Prim's algorithm. Please label an edge by 1, 2, 3, etc., on the obtained tree according to the order of the addition of the edge.

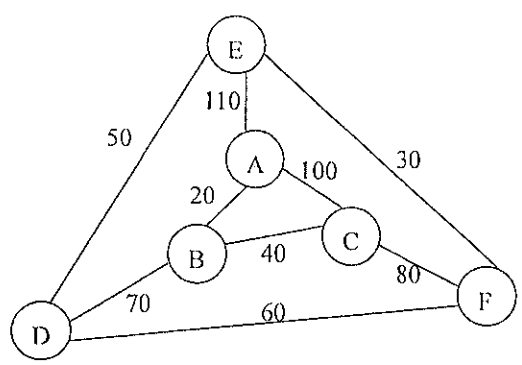


Figure 6

8. [20] Suppose we have a sequence of eight numbers as below:

   13 17 7 21 12 26 2 18

   a. [4] Please sort the numbers in increasing order with the selection sort algorithm. Show the resulting sequence after each iteration. Here an iteration indicates the process of placing a number at its right place. *Hint: You may draw about eight sequences in total.*
   b. [10] Please sort the numbers in increasing order with the heap-sort algorithm. Note that the algorithm has two stages. In the first stage, the numbers are formed into a max-heap. In the second stage, exchanges and reheapings are performed. Show the resulting sequence of the max-heap obtained from the first stage. Also, show the resulting sequence after each iteration in the second stage. An iteration in the second stage means the process of one exchange and the corresponding reheaping. *Hint: You may draw about nine sequences in total.*

c. [6] Please sort the numbers in increasing order with the Quicksort algorithm. Show the resulting sequence after each iteration. Note that the first element is used as the pivot for partitioning a subsequence. Here an iteration means the process of partitioning all the subsequences of the whole sequence. *Hint: You may draw about four sequences in total.*

9. [12] Suppose we have a B-tree shown in Figure 7. Note that each node can have at most 3 pointers. A pointer is denoted by a dot in the figure. A node of (. X .) contains two pointers with the stored value X, and a node of (. X . Y .) contains three pointers with the stored values X and Y.

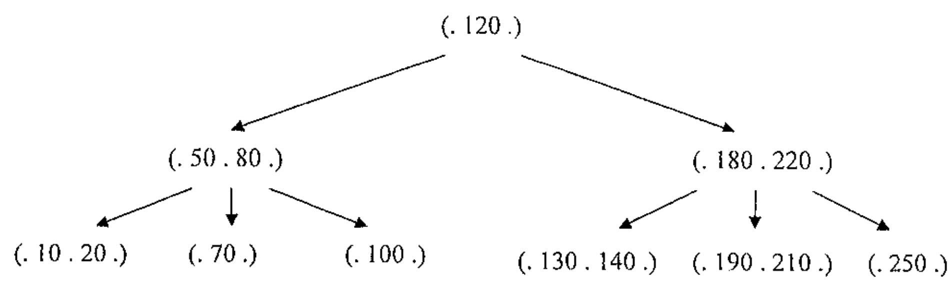


Figure 7

a. [3] Please show the resulting B-tree after the insertion of 30 into Figure 7.

b. [3] Please show the resulting B-tree after the insertion of 160 into Figure 7.

Consider the deletions in a B-tree. Suppose redistribution has a higher priority than merge, and the right sibling has a higher priority than the left sibling. Also, suppose a value in a non-terminal node is replaced by its predecessor.

c. [3] Please show the resulting B-tree after the deletion of 80 from Figure 7.

d. [3] Please show the resulting B-tree after the deletion of 120 from Figure 7.

1. (20%)The circuit of Fig. 1 is operated in steady state. Device 1 absorbs 360 W with unity power factor. Device 2 absorbs 1440 W with a power factor of 0.8 lagging. (a) (10%) Find the value of the capacitor C so that the magnitude of the source current equals 15 A rms. (b) (10%) For the chosen capacitance in (a), what is the power factor of the total load Z?

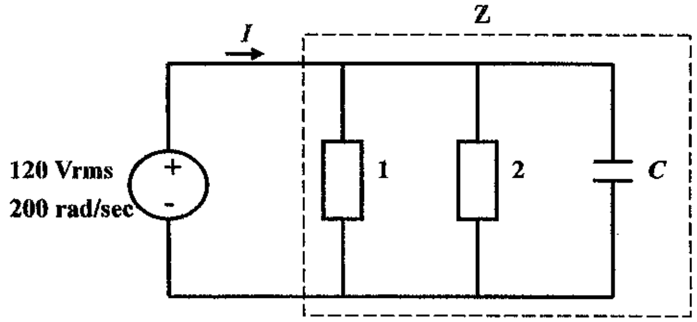


Fig. 1

2. (30%)For the circuit in Fig. 2, the switch was open at $t<0$ and closed at $t=0$. Given $v(0^-)=2$ V and $i(0^-)=1$ A. (a) (20%) Solve for $v(t)$ for $t\geq 0$ using the Laplace transform method. (b) (10%) What is the total energy dissipated by the resistor as time goes to infinity?

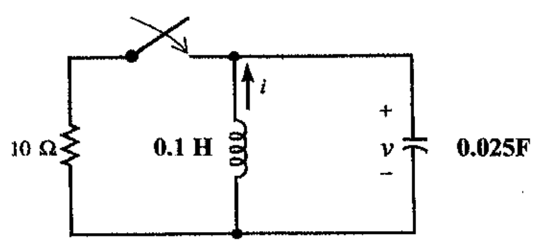


Fig. 2.

3. (a) (10%) Find the equivalent impedance function $Z(s)$ in the circuit of Fig. 3.
   (b) (20%) Find ω and C so that the maximum power is transferred to the 4Ω load.

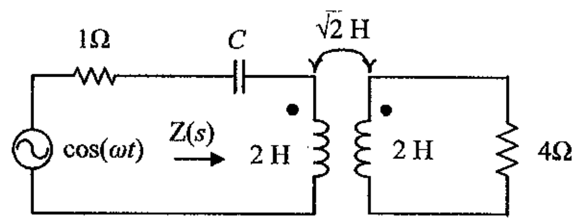


Fig. 3

4. (20%) Assume that the op-amps in Figs. 4-1 and 4-2 are ideal with the supply voltages ±5 Volts. (a) (10%) Given the input $V_i(t)=t-4$ Volt in Fig. 4-1. Plot its output waveform V_o during the period of time $0<t<8$ sec. (b) (10%) Assume the initial voltage at the inverting node of the op-amp in Fig. 4-2 is 1 Volt. Plot its steady-state output waveform V_o for a length of time 0.1 sec.

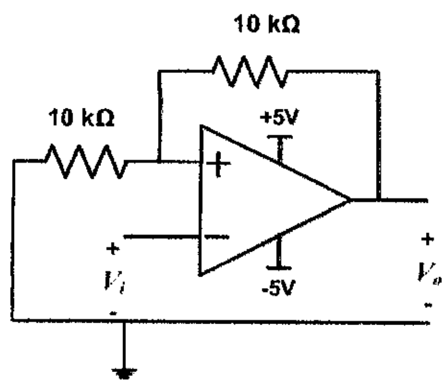


Fig. 4-1

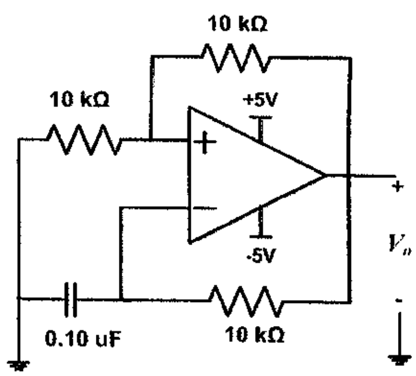


Fig. 4-2

1. Find the Y_{bus} and Z_{bus} of the following network. (20%)

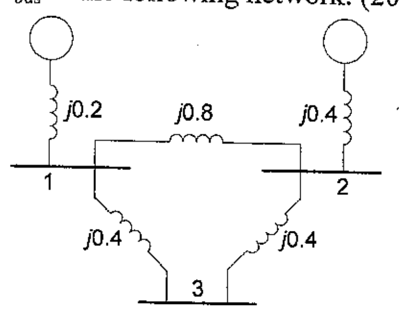


Figure 1

2. (a) Figure 2 shows a power angle curve often used to understand the power system stability problems. Explain why operating point A in Figure 2 is a stable operating point but B is not. P_m is the mechanical power input to a generator and P_e is the electrical output. (10%)

   (b) Explain why Equal-Area Criterion can be used for a quick prediction of stability. (10%)

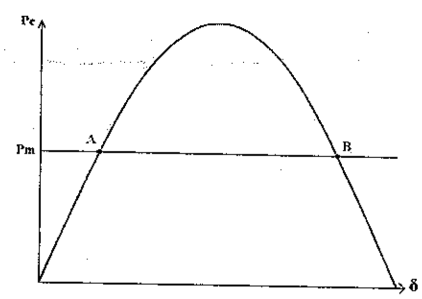


Figure 2

3. (a) Find the sequence components of the following unbalanced 3Φ currents (10%)

$$i_a(t) = 1.6\sqrt{2}\cos(wt + 25^0);$$

$$i_b(t) = 1.0\sqrt{2}\cos(wt + 180^0);$$

$$i_c(t) = 0.9\sqrt{2}\cos(wt + 132^0)$$

   (b) Draw a phasor diagram to show that phasors of i_a, i_b *and* i_c can be expressed by the sequence components obtained in (a). (10%)

4. (a) Explain the equal incremental cost rule and why it can be used to obtain the optimal scheduling of generation. (10%)
   (b) Use the following fuel-cost functions of three thermal power generation units to determine the economic operating point for the three units when the total load is 850 MW. (10%)

$C_1(P_1) = 561 + 7.92P_1 + 0.001562P_1^2$

$C_2(P_2) = 310 + 7.85P_2 + 0.00194P_2^2$

$C_3(P_3) = 78 + 7.97P_3 + 0.00482P_3^2$

$150MW \leqq P_1 \leqq 600MW$

$100MW \leqq P_2 \leqq 400MW$

$50MW \leqq P_3 \leqq 200MW$

5. Consider a balanced three-phase system, prove that although the power in each phase is pulsating, the total instantaneous power is constant and equal to three times the real power in each phase. (10%)

6. Describe the **functions** of the following devices: (10%)
   (i) Differential Relay
   (ii) Circuit Breaker
   (iii) Potential Transformer
   (iv) Shunt Capacitor
   (v) Shunt Reactor

1. (20pts) A charge Q (C) distributes uniformly in a ball of radius 1 m in air. Find
   (a) the distribution of the electric field intensity $\vec{E}$. (10pts)
   (b) the electric potential at a distance r=1/2 m from the center, assuming zero potential at infinity. (5pts)
   (c) the work required to assemble such a charge distribution. (5pts)
2. (20pts) Maxwell generalized the Ampere's law.
   (a) Write the differential form of the time-varying generalized Ampere's law. (5pts)
   (b) ***Derive*** the dimension of the displacement current density term. (5pts)
   (c) If the displacement current vanishes, ***derive*** Kirchhoff's current law.(5pts)
   (d) Copper has a conductivity of 5.8×10^{7} S/m and permittivity of $10^{-9}/(36\pi)$ F/m. If the frequency is 900 MHz, what is the amplitude ratio between the displacement current and conduction current? (5pts)
3. (20pts) Electric field intensity $\vec{E}=\vec{a}_x E_0 e^{-j(\beta_x x+\beta_y y)}$ V/m of 900 MHz exists in free space.
   (a) Find β_x and β_y (including dimension). (10pts)
   (b) Calculate the corresponding magnetic field intensity $\vec{H}$. (5pts)
   (c) If an infinite perfect conductor lies on the xz plane, find the reflected field, $\vec{E}_r$. (5pts)
4. (20pts) A lossless transmission line of characteristic impedance 50 Ω is terminated with a load $Z_L=(50-j100)\Omega$. A 100-MHz TEM wave of amplitude 1 V travels at a speed of $c/2$ toward the load, where c is the speed of light in free space. Determine
   (a) the amplitude of the current at the load. (5pts)
   (b) the position of voltage minimum before the load. (5pts)
   (c) the reflection coefficient and the input impedance at the voltage minimum. (10pts)
5. (20pts) The axis of a long cylindrical conductor of radius a (m) is along the z axis. The conductivity is σ (S/m). A dc source connected to both ends of the conductor results in a current I (A). Find
   (a) the electric field intensity E inside the conductor. (5pts)
   (b) the magnetic field intensity H at $\rho=a^-$ ($\rho<a$, $\rho\to a$, where $\rho=\sqrt{x^2+y^2}$). (5pts)
   (c) the direction of the Poynting vector at $\rho=a^-$. (5pts)
   (d) the length of the conductor if a power of P (W) is dissipated. (5pts)

國立中山大學96學年度碩士班招生考試試題

科目：線性代數【電機系碩士班己組】

1. Find a simple basis for the space generated by the following vectors (15 points)

   V1 = [1, 3, -7], V2 = [2, -1, 0] , V3 = [3, -1, -1] , V4 = [4, -3, 2]

2. Form an orthonormal set from the following vectors (15 points)

   U1 = [1, 1, 1, -1] , U2 = [2, -1, -1, 1] , U3 = [-1, 2, 2, 1]

3. Find an orthonormal set of eigenvectors for the following matrix (15 points)

   A = [C1, C2, C3], where C1 = $[7, -16, -8]^t$, C2 = $[-16, 7, 8]^t$,

   C3 = $[-8, 8, -5]^t$, t is the matrix transpose.

4. Find the QR decompositions of the following matrix (15 points)

   A = [C1, C2, C3], where C1 = $[1, 0, 1]^t$, C2 = $[2, 1, 4]^t$,

   C3 = $[4, 1, 6]^t$, t is the matrix transpose.

5. Solve the matrix differentiation equation dX(t)/dt = AX(t), (20 points)

   Where:

   A = [C1, C2, C3], C1 = $[5, -1, 1]^t$, C2 = $[4, 0, -2]^t$, C3 = $[3, -3, 1]^t$,

   X(0) = $[2, -2, 2]^t$, t is the matrix transpose.

6. Is the following quadratic form positive definite? (20 points)

   $$F = 2x_1^2 + x_2^2 + 6x_3^2 + 2x_1x_2 + x_1x_3 + 4x_2x_3$$

1. Let the random variables X and Y be independent and *Gaussian*, with zero mean and variance of σ^2.

$$f_X(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp[-\frac{x^2}{2\sigma^2}],\quad f_Y(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp[-\frac{y^2}{2\sigma^2}]$$

If a new random variable Z is defined by

$$Z=a\frac{X}{Y}$$

(a) Find the *conditional probability distribution function* of Z given Y, $F_{Z|Y}(z|y)$, and $f_{Z|Y}(z|y)$, in terms of $F_X(.)$ and $f_X(.)$, respectively. (15%) (b) What is the *probability density function* of Z, $f_Z(.)$ (10%)?

2. For all nonnegative integers k and m, let

$$P(X=k,Y=m)=(1-v_1)(1-v_2)v_1^k v_2^m;$$

X and Y might be the numbers of photoelectrons counted at two photodetectors.

(a) Find $P(X=k)$ and $P(Y=m)$. (10%)

(b) Find $P(X+Y\le p)$, where p is any positive integer. (10%)

3. Let X and Y be two random variables with jointly Gaussian distribution being defined as

$$f_{XY}(x,y)=\frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}}e^{-Q(x,y)}$$

where

$$Q(x,y)=\frac{1}{2\sigma^2(1-\rho^2)}[x^2-2\rho xy+y^2]$$

(a) For ρ=0, find the joint probability density function of random variables of

$$Z=X^2+Y^2$$

and

$$W=X$$ (10%)

(b) Compute $f_Z(z)$ from the result of (a). (5%)

4. The probability $P(A)$ of any event A defined on a sample space, Ω, can be expressed in terms of conditional probabilities, $P(A|B_n)$. Where B_n are mutually exclusive events, n = 1,2, ..., N, whose union equals Ω, that is

$$B_n\cap B_m=\varnothing,\ n\neq m$$

$$\bigcup_{n=1}^{N}B_n=\Omega \text{ (Universal Space)},$$

(a) Prove that

$$P(A)=\sum_{n=1}^{N}P(A|B_n)P(B_n)$$ (10%)

(b) In the automated manufacturing of memory chips, company *Z* produces one defective chip for every five good chips. The defective chips (DC) have a time of failure *X* that obeys the PDF (*Probability Distribution Function*)

$$F_X(x|DC) = (1-e^{-x/2})u(x) \text{ ,(} x \text{ in month)}$$

while the time for failure for the good chips (GC) obeys the PDF

$$F_X(x|GC) = (1-e^{-x/10})u(x) \text{ ,(} x \text{ in month)}$$

A chip is purchased. What is the probability that the chip will fail before six months of use? (10%)

5. Let $\Gamma(\alpha)$ be a *gamma function*, which is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx$$

for $\alpha>0$. It is known that the *gamma function* has the following properties, e.g., $\Gamma(\alpha)=(\alpha-1)\Gamma(\alpha-1)$, $\Gamma(n+1)=n!$, $\Gamma(1/2)=(\pi)^{1/2}$ and $\Gamma(1)=1$. Let us consider the *random variable* X has a *gamma distribution* with *probability density function* to be defined by

$$f_X(x) = \begin{cases} \dfrac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta}, & if \quad x>0 \\ 0 \ , & otherwise \end{cases}$$

where $\alpha>0$ and $\beta>0$.

(a) For $\alpha=1/2$ and $\beta=2$, please evaluate the *mean* μ = E[X], and *variance* $\sigma_X^2 = E[(X-E[X])^2]$ (12%).

(b) For $\alpha=1$ we have the so-called *exponential distribution*, again, find the *mean* μ and σ_X^2 for *random variable* X (8%).

Communications Theory

1. (16%) Select a suitable modulation scheme (choose from Binary ASK (or ON-OFF keying), Binary FSK and Binary PSK) for the following applications.
   Note: Justification of your answer is required.
   (a) (4%) Data transmission from a satellite over a noisy radio link; satellite has limited power capability.
   (b) (4%) Multiplexed voice transmission over coaxial cable; primary objective is to transmit as many signals as possible over a single cable.
   (c) (4%) Point-to-point voice communication (short distance, single channel) over a twisted pair of wires.
   (d) (4%) Record high fidelity music on tape using a recorder having a usable bandwidth from 500 Hz to 50 kHz.

2. (15%) For a band-limited signal $x(t)$ with bandwidth W radians/second (i.e., $X(\omega) = 0$, for $|\omega| > W$), it is well known that the signal can be exactly recovered from equally spaced samples $x(nT), -\infty < n < \infty$. One signal which can be defined using only the sample values is

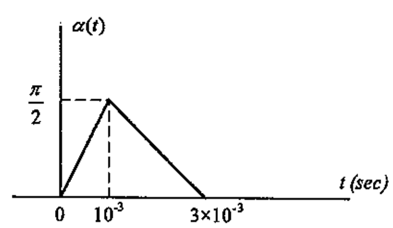
$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT).$$

   (a) (8%) Find an expression for $X_s(\omega)$ in a form which clearly show that the signal $x(t)$ can be recovered, to within a constant, by passing $x_s(t)$ through the following ideal low-pass filter:

$$H(\omega) = \begin{cases} A, & |\omega| \le W, \\ 0, & |\omega| > W. \end{cases}$$

   (b) (7%) Determine the constant A and any restriction on T which will make the exact recovery possible.

3. (20%) Consider the angle modulated wave $cos(\omega_c t + \alpha(t))$, where $\alpha(t)$ is shown below.



   (a) (5%) What is the maximum frequency deviation?
   (b) (15%) Now consider the composite wave

$$e(t) = cos\omega_c t + cos(\omega_c t + \alpha(t)).$$

   Let $e(t) = \text{Re}\{a(t)e^{j\phi(t)}e^{j\omega_c t}\}$. Draw the locus of $a(t)e^{j\phi(t)}$ for the time interval $0 \le t \le 3 \times 10^{-3}$ seconds. What is the maximum of the phase deviation $\phi(t)$?

4. (25%) Additive white Gaussian noise is generally used in communication simulations and for signal analysis.

   (a) (6%) If a noise time-sequence $w[n]$ of length N is generated from a random number generator, and the autocorrelation function $R_w(k)$ of $w[n]$ is calculated. For a reasonably large N, try to roughly sketch the autocorrelation function $R_w(k)$, and justify your answer.

   (b) (6%) Repeat Part (4a) if $-\infty < n < \infty$.

   (c) (6%) Shannon's famous formula for the capacity of an additive white Gaussian noise channel with bandwidth W is given by

$$C = W \log_2(1 + \frac{P}{N_0 W}) \text{ bits/sec.}$$

   Suppose that a channel with bandwidth $W = 5000$ Hz is available. Calculate the minimum required received signal-to-noise ratio, i.e., $P/(N_0 W)$, to attain a data rate of 40000 bits/sec.

   (d) (7%) Now suppose that we want to digitally communicate a message with bandwidth 2500 Hz over the channel in Part (4c). If the message is sampled at the Nyquist rate (samples/sec), how many *bits per sample* are available for quantizing the message amplitude?

5. (10%) For uniform pulse-code modulation (PCM), if the amplitude interval of input signal, $[-x_{max}, x_{max}]$ is divided into N equal subintervals, each of length $\Delta = 2x_{max}/N$. Assume that the density function of the input signal in each subinterval is uniform, show that the quantization distortion (or quantization noise) is $D = \Delta^2/12$.

6. (14%) Answer the following questions:

   (a) (4%) What is the main objective of using a matched filter?

   (b) (4%) What is the main objective of using an equalizer?

   (c) (6%) what are the functional differences between source coding and channel coding in communications systems?

~End~