## 科目:半導體概論【電機系碩士班甲組】

共/頁第/頁

#### Physical constants:

K= 1.38x10 <sup>-23</sup> J/K	e=1.60x10 <sup>-19</sup> C	$\epsilon_{\rm O} = 8.85 \times 10^{-12}  {\rm F/m}$
$h = 6.625 \times 10^{-34} \text{ J-s}$	Silicon; Eg=1.12 eV (300 K)	$\epsilon_{\rm Si} = 11.7 \; \epsilon_{\rm O}$
Silicon; $m_n^* = 1.08 m_0$ ; $m_h^* = 0.56 m_0$		

1. To calculate the open-circuit voltage of a silicon pn junction solar cell. Consider a silicon pn junction at  $T=300~{\rm K}$  with the following parameters: (20%)

Na =  $5 \times 10^{18}$  cm<sup>-3</sup>; Nd =  $1 \times 10^{16}$  cm<sup>-3</sup>;  $D_n$ = 25 cm<sup>2</sup>/s;  $D_p$ =10 cm<sup>2</sup>/s;  $\tau_{no}$ = $5 \times 10^{-7}$  s;  $\tau_{po}$ =  $10^{-7}$  s;  $\tau_{i}$ =  $1.5 \times 10^{10}$  cm<sup>-3</sup>; Let the photocurrent density be  $J_L$  =  $I_L/A$  = 15 mA/cm<sup>2</sup>.

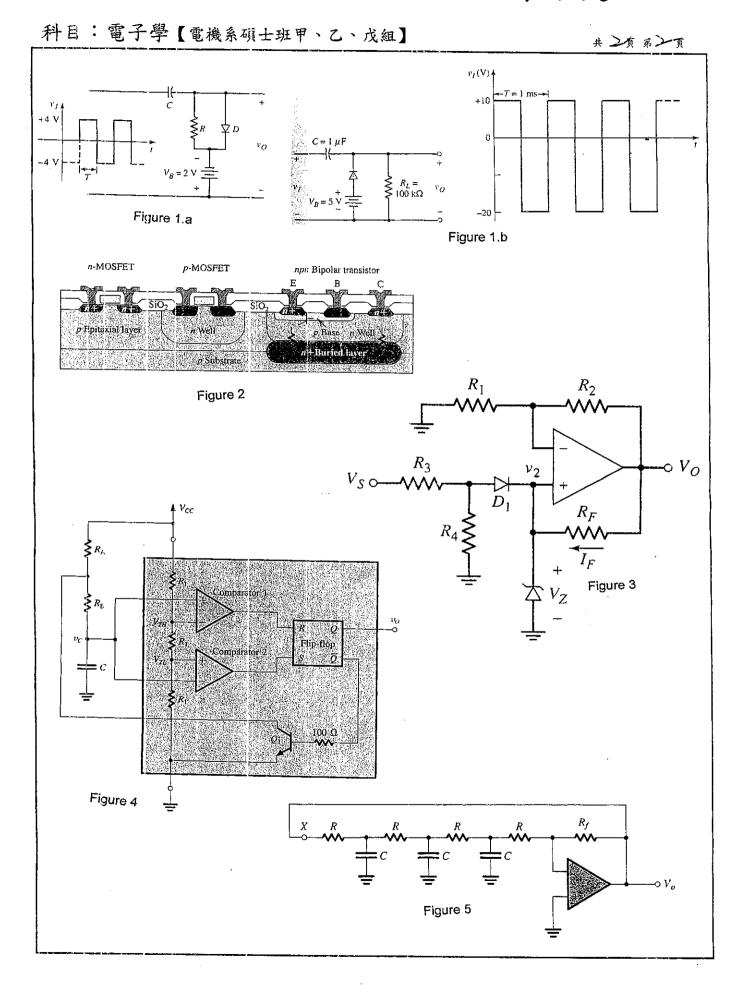
- Plot and explain the I-V characteristic of a tunnel diode with a reverse bias voltage. (10%)
- 3. For a non-degenerate Silicon n-type semiconductor, calculate the maximum doping concentration of Arsenic (ionization energy = 0.05 eV in Silicon) at T = 300 K. (10%)
- 4. Derive to obtain the electron effective mass under an applied electric field. (15%)
- 5. For a undoped Si semiconductor; What is the possible type; n or p ? Calculate (8%) and explain. (7 %)
- 6. To calculate the actual Schottky barrier height in a metal-semiconductor diode for zero applied bias. Consider a contact between metal ( $\phi_m = 4.55$  V) and n-type Silicon doped to Nd =10<sup>16</sup> cm<sup>-3</sup> at T = 300 K; the electron affinity for silicon;  $\chi = 4.01$  V. (10%)
- 7. Consider a Silicon pn junction at T = 300 k with doping concentrations of Na =  $10^{16}$  cm<sup>-3</sup> and Nd =  $10^{15}$  cm<sup>-3</sup>. To calculate (a) the space charge region width in n-type;  $X_n$  (b) the electric potential in the space charge region, at  $x_n$ =0.5  $\mu$ m; (c) breakdown voltage.

(the maximum electric field in pn junction;  $E_{crit} = 1 \times 10^5 \text{ V/cm}$ ) (20%)

### 科目:電子學【電機系碩士班甲、乙、戊組】

共之頁第/頁

- 1. Determine and sketch the steady output voltage  $V_o$  for the circuits in Figure 1.a and Figure 1.b respectively. Their input signals are given by its figure respectively and assume the cut-in voltage  $V_r = 0$ V and on resistance  $r_f = 0$  for the diode. 2\*10%
- 2. As far as the devices shown in Figure 2 are concerned,
  - (a). Please draw and explain what Base-Width Modulation is and what Channel Length Modulation is. 10%
  - (b). Please plot the concentration profile of the minority carrier in (i) a cut-off, (ii) a forward-active, and (iii) a saturated npn bipolar transistor respectively. 10%
- 3. Please use a Zener diode with a breakdown voltage of 5.6V to design an Op-amp voltage reference source as shown in Figure 3 with an output of 10.0V. Assume the voltage regulation will be within specifications if the Zener diode is biased between 1-1.2mA. In other words, please design the values of R<sub>1</sub>, R<sub>2</sub>, R<sub>3</sub>, R<sub>4</sub> and R<sub>F</sub>.
- 4. Please design an astable circuit as shown in Figure 4 Using a 680pF capacitor to obtain a square wave with a 50-kHz frequency and a 75% duty cycle. Specify the values of  $R_A$  and  $R_B$ . 2\*10%
- 5. For the circuit in Figure 5,
  - (c). Please break the loop at node X and find the loop gain.
  - (d). For R =  $10k\Omega$ , please find C and  $R_{\rm f}$  to obtain sinusoidal oscillations. 2\*10%



# 科目:工程數學甲【電機系碩士班甲、丁、戊、庚組(含丙組選考)】共/頁第/頁

 (35%)填空題,計分僅以最後答案為準,不考慮計算過程。答案請寫在答案卷計算題部份, 註明小題號,按(1)、(2)、...、依序列出。

Part I (25pts). The Fourier transform (FT) of x(t) is defined as

 $X(j\omega) \triangleq \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$ . Let w(t) = 1 be a constant function. Define

$$y(t) = \begin{cases} 1, & t \ge 0 \\ -1, & t < 0 \end{cases} \text{ and unit step function } u(t) = \begin{cases} 1, & t \ge 0 \\ 0, & t < 0 \end{cases}$$

- (a) Find the FTs:  $W(j\omega) = (1)$ ,  $Y(j\omega) = (2)$  and  $U(j\omega) = (3)$
- (b) Express the FT  $Z(j\omega) = (4)$  of  $z(t) = \int_{-\infty}^{t} x(\tau)d\tau$  in terms of  $X(j\omega)$ .
- (c) Evaluate the integral  $\int_{-\infty}^{\infty} U(j\omega)d\omega =$  (5)

Part II (10pts). The spherical coordinates  $(r, \theta, \phi)$  is related to the rectangular coordinates by  $r = \sqrt{x^2 + y^2 + z^2}$ ,  $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ ,  $\phi = \tan^{-1}(y/x)$ . Let  $\vec{F} = \vec{a}_r 1 + \vec{a}_\theta 2 - \vec{a}_\theta 3$ .

- (a) Find the surface integral  $\int_S \vec{F} \cdot d\vec{s} = \underline{\qquad}$  (b) where S is a portion of an upper sphere (z > 0) with  $\theta \ge \pi/3$ . The radius of the sphere is 2.
- (b) Find  $F_x = (7)$ , the x-component of  $\vec{F}$  at a point whose spherical coordinates is  $(2, \pi/4, \pi/4)$ .
- 2. (30%) Let  $V = \mathbb{R}^{n \times n}$ ,  $W = \{ \mathbf{w} \in V \mid \mathbf{w} = \mathbf{w}^T \}$ , and let L be a transformation from V to W. Define a scalar-valued function  $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}$  by  $\langle \mathbf{u}, \mathbf{v} \rangle := tr(\mathbf{u}^T \mathbf{v})$ .
  - (a) (4%) Show that L defined by, for any  $\mathbf{v} \in V$ ,  $L(\mathbf{v}) := \frac{1}{2}(\mathbf{v} + \mathbf{v}^T)$  is linear and onto.
  - (b) (4%) Show that  $(V, \langle \cdot, \cdot \rangle)$  is an inner product space.
  - (c) (6%) Derive the orthogonal complement  $W^{\perp}$ .

Now let's consider n=2 case. Let E and F be the orthonormal bases, generated from the ordered basis  $\hat{E} = \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$  for V and the ordered basis  $\hat{F} = \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} a & b \\ b & c \end{bmatrix}$  for W, respectively. For simplicity, assume that all entries of vectors in E and F are nonnegative.

- (d) (8%) Derive the matrix A to represent L with respect to the orthonormal bases E and F.
- (e) (8%) Find the set of all vectors  $\mathbf{v} \in V$ , denoted by  $\mathbf{v} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  with  $a, b, c, d \in \mathbb{R}$ , having the property that the angle between  $\mathbf{v}$  and its image  $\mathbf{w}$  in W is  $45^\circ$ , i.e.  $\measuredangle(\mathbf{v}, \mathbf{w}) = \pi/4$ .
- 3. (15%) Evaluate  $\oint_C \frac{3z^3+2}{(z-1)(z^2+9)} dz$ , by using the residue theorem,

where  $C: \{z\}=4$ .

4.(20%) Find the general solution of the differential equation:  $x^2(y'-1) = y^2$ 

## 科目:工程數學乙【電機系碩士班乙組】

共 | 頁第 | 頁

 (25%)填空題,計分僅以最後答案為準,不考慮計算過程。答案請寫在答案卷計算題部份, 註明小題號,按(1)、(2)、...、依序列出。
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- (a) Find the FTs:  $W(j\omega) = (1)$ ,  $Y(j\omega) = (2)$  and  $U(j\omega) = (3)$
- (b) Express the FT  $Z(j\omega) = (4)$  of  $z(t) = \int_{-\infty}^{t} x(\tau) d\tau$  in terms of  $X(j\omega)$ .
- 2. (40%) Let  $L:V\to W$  be a linear transformation between vector spaces V and W with  $\dim V=p$  and  $\dim W=q$ , and let  $E=[\mathbf{v}_1,\cdots,\mathbf{v}_p]$  and  $F=[\mathbf{w}_1,\cdots,\mathbf{w}_q]$  be two ordered bases for V and W, respectively. Let A be the matrix representation of L with respect to bases E and F.
  - (a) (10%) Show that L is onto if and only if matrix A is full row rank.

Now let's restrict  $V = \mathbb{R}^{n \times n}$  and  $W = \{ \mathbf{w} \in V \mid \mathbf{w} = \mathbf{w}^T \}$ , and let L be a transformation from V to W. Define a scalar-valued function  $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}$  by  $\langle \mathbf{u}, \mathbf{v} \rangle := tr(\mathbf{u}^T \mathbf{v})$ .

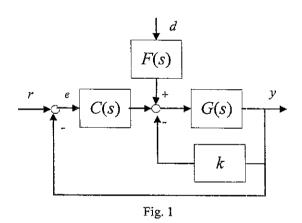
- (b) (4%) Show that L defined by, for any  $v \in V$ ,  $L(v) := \frac{1}{2}(v + v^T)$  is linear and onto.
- (c) (4%) Show that  $(V, \langle \cdot, \cdot \rangle)$  is an inner product space.
- (d) (6%) Derive the orthogonal complement  $W^{\perp}$ .

Now let's consider n=2 case. Let E and F be the orthonormal bases, generated from the ordered basis  $\hat{E} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$  for V and the ordered basis  $\hat{F} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} a & b \\ b & c \end{bmatrix}$  for W, respectively. For simplicity, assume that all entries of vectors in E and F are nonnegative.

- (e) (8%) Derive the matrix A to represent L with respect to the orthonormal bases E and F.
- (f) (8%) Find the set of all vectors  $\mathbf{v} \in V$ , denoted by  $\mathbf{v} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  with  $a, b, c, d \in \mathbb{R}$ , having the property that the angle between  $\mathbf{v}$  and its image  $\mathbf{w}$  in W is 45°, i.e.  $\measuredangle(\mathbf{v}, \mathbf{w}) = \pi/4$ .
- 3. (15%) Find the general solution of the differential equation:  $x^2(y'-1) = y^2$
- 4. (20%) Find the Laplace transform of the function:  $(1 \cos t)/t$ .

For the closed-loop system shown in Fig. 1, please answer the following questions.

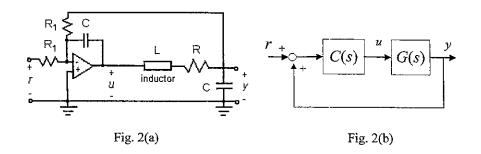
- 1. (20%) Suppose  $G(s) = 1/(s^2 + 4s + 1)$ , F(s) = 1/(s + 2) and C(s) = 4 + (4/s), please find the range of k for stabilizing the system. Also find the transfer functions Y(s)/D(s) and Y(s)/R(s), where Y(s), D(s) and R(s) are the Laplace Transforms of Y(s), Z(s) and Z(s) and Z(s) are the Laplace Transforms of Z(s).
- 2. (10%) Suppose  $G(s) = (s-1)/(s^2+4s+1)$ , F(s) = 1 and C(s) = 1/(s-1). Is the closed-loop system stable? Explain your answer.
- 3. (20%) Suppose G(s) = 1/(s+2), k=0 and F(s) = 1/(s+4), and disturbance d is known to be a sinusoidal signal of frequency  $1/(2\pi)$  Hz. Please design a second-order controller which has the form  $C(s) = (2s+b)/(s^2 + a_1s + a_2)$  to achieve perfect asymptotic disturbance rejection (that is, disturbance d does not affect output y in steady state).

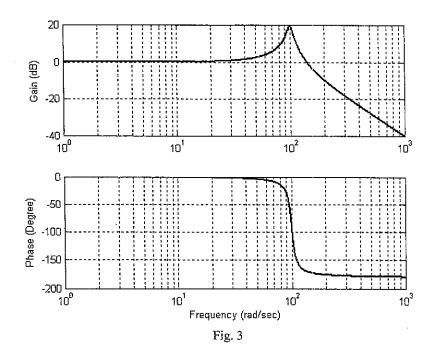


4. (20%) Consider a plant G(s), whose response to a unit step function is  $y(t) = \int_0^t (1 - e^{t-\tau}) d\tau$ . Please determine the transfer function G(s). Is it BIBO (Bounded-Input-Bounded-Output) stable? Why?

The block diagram of the circuit in Fig. 2(a) is plotted in Fig. 2(b). Please answer the following questions.

- 5. (10%) Please find the transfer functions C(s) and G(s).
- 6. (20%) Figure 3 plots the frequency response of G(s). Determine the value of the product  $R_1C$  to obtain a gain margin of 20 dB. In this case, given a positive unit step input r, what is output y in steady state?





1. (15%) Please answer the following questions briefly.

(a) Compare the differences between RAID 3 and RAID 5. (5%)

(b) What is superscaler? What is superpipeline? (5%)

(c) What are the differences among UMA, NUMA, and CC-NUMA? (5%)

2. (15%) Consider a computer system that contains an I/O module controlling a simple keyboard/printer teletype. The following registers are contained in the processor and connected directly to the system bus:

INPR: Input Register, 8 bits
OUTR: Output Register, 8 bits

FGI: Input Flag, 1 bit
FGO: Output Flag, 1 bit
IEN: Interrupt Enable, 1 bit

Keystroke input from the teletype and printer output to the teletype are controlled by the I/O module. The teletype is able to encode an alphanumeric symbol to an 8-bit word and decode an 8-bit word into an alphanumeric symbol.

(a) Describe how the processor, using the first 4 registers above, can achieve I/O with the teletype. (10%)

(b) Describe how the function can be performed more efficiently by also employing IEN. (5%)

3. (20%) Consider a machine with a byte addressable main memory of 2<sup>16</sup> bytes and block size of 8 bytes. Assume that a direct mapped cache consisting of 32 lines is used with this machine.

(a) How a 16-bit memory address is divided when accessing the cache? (5%)

(b) Into what line would bytes with each of the following addresses be stored? (5%) 0001 0001 0001 1011

1100 0011 0011 0100

1101 0000 0001 1101

1010 1010 1010 1010

(c) Suppose the byte with address 0001 1010 0001 1010 is stored in the cache. What are the addresses of the other bytes stored along with it? (5%)

(d) How many total bytes of memory can be stored in the cache? (5%)

4. (10%) Describe a simple technique for implementing an LRU (Least Recently Used) replacement algorithm in a four-way set associative cache.

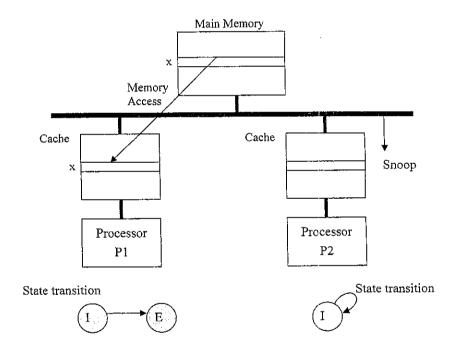
5. (15%) Consider a computer system with both segmentation and paging. When a segment is in memory, the space overhead of the segment comes from two sources: internal fragmentation and page table entries for the segment. Assume that the segment size is s, and each page table entry occupies 8 bytes. What page size (in bytes) minimizes such overhead?

6. (10%) A computer has a cache, main memory, and a disk used for virtual memory. If a referenced word is in the cache, 20 ns are required to access it. If it is in main memory but not in the cache, 60 ns are needed to load it into the cache, and then the reference is started again. If the word is not in main memory, 12 ms are required to fetch the word from disk, followed by 60 ns to copy it to the cache, and then the

reference is started again. The cache hit ratio is 0.9 and the main-memory hit ratio is 0.6. What is the average time in ns required to access a referenced word on this system?

7. (15%) Consider a situation in which two processors in an SMP configuration, over time, require access to the same line of data from main memory. Both processors have a cache and use the MESI protocol. Initially, both caches have an invalid copy of the line. The following figure depicts the consequence of a read of line x by processor P1. If this is the start of a sequence of accesses, draw the subsequent figures for the following sequence:

- (a) P2 reads x. (3%)
- (b) P1 writes to x (for clarity, label the line in P1's cache x'). (3%)
- (c) P1 writes to x (label the line in P1's cache x"). (4%)
- (d) P2 reads x. (5%)



Hint:

I: Invalid, E: Exclusive. Both of them indicate the state of a cache entry, and there are two other states for each cache entry.

#### 科目:資料結構【電機系碩士班丙組選考】

共之頁第一頁

- 1. (30%) Please answer the following questions briefly and to the point.
  - (a) (3%) Give two advantages of linked lists over arrays and explain why.
  - (b) (3%) Describe how to use array to implement a stack and a queue, respectively.
  - (c) (3%) Describe how to use linked list to implement a stack and a queue, respectively.
  - (d) (3%) What is a heap? How to implement a heap using an array?
  - (e) (3%) What is binary search? Can we implement binary search using linked list?
  - (f) (3%) What are advantages and disadvantages of hashing?
  - (g) (3%) Why don't we use one MIN-heap and one MAX-heap together instead of one MIN-MAX heap?
  - (h) (3%) What are advantages and disadvantages of AVL-trees?
  - (i) (3%) Give two representations for graphs.
  - (j) (3%) What are the main advantages with 2-3 trees over binary search trees?
- 2. (15%) Suppose you are given the following 10 numbers:

90, 60, 70, 100, 10, 50, 40, 20, 30, 80.

- (a) (5%) Construct a binary search tree for these numbers presented in the given order. Please draw the resulting tree.
- (b) (6%) Please show the in-order sequence, pre-order sequence, post-order sequence, respectively, of the obtained tree of Problem 2(a).
- (c) (4%) Please delete 60 from the obtained tree of Problem 2(a) and show the resulting tree.
- 3. (10%) Suppose you are given the following 10 numbers:

95, 65, 75, 105, 15, 55, 45, 25, 35, 85.

- (a) (5%) Construct a MIN-heap for these numbers presented in the given order. Please draw the resulting heap.
- (b) (5%) Please delete 15 from the obtained heap of Problem 3(a) and show the resulting heap.
- 4. (10%) Suppose you are given the following 10 numbers:

55, 45, 25, 35, 85, 95, 65, 75, 105, 15.

- (a) (5%) Construct a 2-3 tree for these numbers presented in the given order. Please draw the resulting tree.
- (b) (5%) Construct an AVL-tree for these numbers presented in the given order. Please draw the resulting tree..

#### 科目:資料結構【電機系碩士班丙組選考】

共2頁第2頁

- 5. (10%) Suppose N numbers are placed in array elements a[1], a[2], ..., and a[N]. Please write pseudo-code for sorting these N numbers by insertion sort. Note that after sorting, a[1], a[2], ..., and a[N] contain numbers in increasing order.
- 6. (10%) Suppose you are given the following 10 numbers:

55, 45, 25, 35, 85, 95, 65, 75, 105, 15.

- (a) (5%) Please sort these numbers in increasing order using merge sort. Please show necessary steps such that the use of merge sorting technique can be recognized.
- (b) (5%) Please sort these numbers in increasing order using quick sort. Please show necessary steps such that the use of quick sorting technique can be recognized.
- 7. (10%) Suppose you are given the following numbers:

20, 62, 31, 14, 1, 25, 3, 19, 11

and the following hash function:

 $H(x) = x \mod 11.$ 

You are asked to store these numbers by hashing. Let the size of the hash table be 11. Please build and show the hash tables using the following overflow handling techniques:

- (a) (5%) Linear probing.
- (b) (5%) Chaining.
- 8. (5%) Suppose you are given an undirected graph with five nodes a, b, c, d, and e. Also the graph has eight edges e<sub>ac</sub>, e<sub>ad</sub>, e<sub>ae</sub>, e<sub>bc</sub>, e<sub>bd</sub>, e<sub>be</sub>, e<sub>cd</sub>, e<sub>de</sub> with weights 300, 80, 50, 70, 75, 200, 90, and 60, respectively. Please find and draw the minimum-cost spanning tree for this graph.

#### 科目:離散數學【電機系碩士班丙組選考】

共 2 頁 第 / 頁

(每題均需將推導過程敘述清楚)

1. (15%) Given an axiom that

$$p \rightarrow q \equiv \sim p \vee q$$

prove the following expression is true by enumerating and examining all possible cases in the truth table.

$$(a \ v \ b) \ ^ (\sim \ b) \rightarrow a$$

(We restrict you to prove in above specified way (via truth table). Prove in other way will at most get half score of this question.)

- 2. (10%) Given a bag containing three balls (one red ball, one blue ball, and one white ball), the probabilities to draw the red ball, the blue ball, and the white ball out from the bag are 0.5, 03, and 0.2, respectively. We draw a ball from the bag and put it back to the bag repeatedly. Compute the probability of drawing a ball for ten times and having the red ball, the blue ball, and the white ball being drawn exactly for twice, three times, and five times, respectively.
- 3. (10%) Given a rule

$$like(A,B)$$
 ^  $like(B,C)$   $\rightarrow$   $like(A,C)$ 

it says that if A likes B and B likes C, then we can deduce that A likes C.

Given some facts as follows:

like(John, Mary), like(Peter, Jenny), like(Tom, Ivy), like (Ivy, Mary),

like(Mary, Jenny), like(Peter, John), like(Mary, Peter)

derive all possible like relationship between these 6 persons with transitive closure of relation like.

4. (15%) Given a connected graph G(V,E) where each vertex v is labeled with a weight  $w_i$ , write a recursive algorithm based on depth-first search that traverses all vertices, add up their vertex weights, and finally returns the total weight. We select one vertex s in V as the source vertex. The recursive function is in the form of

int calculate weight(vertex IN\_VERTEX);

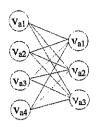
(Hint: You can utilize a Boolean visit\_flag (initially false) on each vertex to decide whether it has been traversed. You can also utilize the function

vertex get\_next\_adjacent\_vertex(vertex CURRENT\_VERTEX);

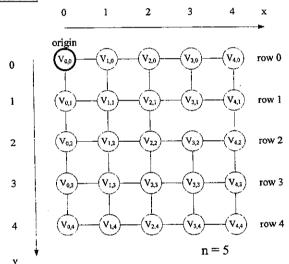
to get next adjacent vertex from the adjacency list of CURRENT\_VERTEX in constant time. Note that this function may return a vertex that you just traversed.)

- 5. (15%) A complete bipartite graph  $K_{m,n}$  has a vertex set divided into two disjoint subset  $V_1$  and  $V_2$  that contain m and n distinct vertices, respectively. In either  $V_1$  or  $V_2$ , there are no edges connecting vertices inside. However, between any vertices  $v_a$  in  $V_1$  and  $v_b$  in  $V_2$ , there is an edge connecting between them.
  - (a) (5%) Calculate is its maximum cut set size.
  - (b) (10%) Given that  $m \ge 3$ ,  $n \ge 3$ , m is even, and n is odd, determine whether there is an <u>Eulerian path</u> in the graph. <u>Prove it</u>.

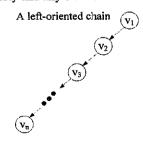
(An example of  $K_{4,3}$  is shown in the following.)



- 6. (20%) A 2D  $n \times n$  mesh is a graph as shown in the following figure (e.g. n = 5). It is composed of  $n^2$  vertices organized in a 2-dimensional  $n \times n$  array. Each vertex has edges connecting with adjacent vertices horizontally and vertically.
  - (a) (5%) Derive the average path length between the origin vertex  $v_{0,0}$  and all vertices (including itself) in row 0 (i.e. y = 0) as a function of n.
  - (b) (5%) Derive the average path length between the origin vertex  $v_{0.0}$  and all vertices in row i (i.e. y = i) as a function of n and i.
  - (c) (10%) Derive the average path length between the origin vertex  $v_{0,0}$  and all vertices (including itself) in the mesh as a function of n.



- 7. (15%) Given a binary rooted tree with n nodes such that its postorder traversal sequence is the same as its inorder traversal sequence, prove that the tree satisfy above property must be a left-oriented chain consisting of only left child nodes.
  - (i.e. Such chain satisfies above property and any other rooted binary tree violates above property.)



## 科目:電力系統【電機系碩士班丁組】

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- Figure 1 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 2. The voltage at bus 1 is V = 1.0∠0° per unit. Voltage magnitude at bus 2 is fixed at 1.05 pu with a real power generation of 400 MW. A load consisting of 500 MW and 400 Mvar is taken from bus 3. Line admittances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected. (20%)
  - (a) Show that the expression for the real power at bus 2 and real and reactive power at bus 3 are

$$\begin{split} P_2 &= 40 |V_2| |V_1| \cos(90^\circ - \delta_2 + \delta_1) + 20 |V_2| |V_3| \cos(90^\circ - \delta_2 + \delta_3) \\ P_3 &= 20 |V_3| |V_1| \cos(90^\circ - \delta_3 + \delta_1) + 20 |V_3| |V_2| \cos(90^\circ - \delta_3 + \delta_2) \\ Q_3 &= -20 |V_3| |V_1| \sin(90^\circ - \delta_3 + \delta_1) - 20 |V_3| |V_2| \sin(90^\circ - \delta_3 + \delta_2) + 40 |V_3|^2 \end{split}$$

(b) Using Newton-Raphson method, start with the initial estimates of  $V_2^{(0)} = 1.0 + j0$  and  $V_3^{(0)} = 1.0 + j0$ , and keeping  $|V_2| = 1.05$  pu, determine the phasor values of  $V_2$  and  $V_3$ . Perform two iterations.

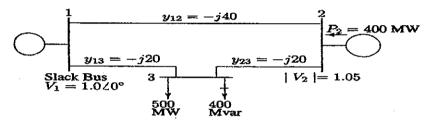


Figure 1

2. For a power system with positive and negative sequence network impedance matrices as follows

$$Z_{bus}^2 = Z_{bus}^1 \begin{bmatrix} j0.145 & j0.105 & j0.130 \\ j0.105 & j0.145 & j0.120 \\ j0.130 & j0.120 & j0.220 \end{bmatrix}.$$

Find the phase fault currents for a double line-to-ground fault at bus 3 through a fault impedance  $Z_f = j \ 0.1$  p.u. (20%)

- 3. For the circuit shown in Figure 2, the generator is delivering real power  $P_e = 0.8$  p.u. and Q = 0.074 p.u. to the infinite bus at a voltage V = 1.0 p.u. Find the power angle equations ( $P_e = P_{max} \sin \delta$ , note that  $E' = |E| \angle \delta$ ) (20%)
  - (a) During is a fault at the middle of one of the lines.
  - (b) After the fault is cleared.
  - (c) Before the fault occurs (normal condition).

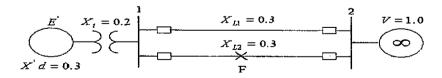
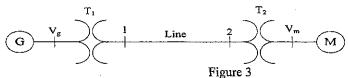


Figure 2

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4 The three-phase power and line-line ratings of the electric power system shown in Figure 3 are given below. (20%)



G1: 60MVA 20kV X=9% T1: 50MVA 20/200kV X=10% T2: 50MVA 200/20kV X=10% M: 43.2MVA 18kV X=8% Line: 200kV  $Z=120+j200\Omega$ 

- (a) Draw an impedance diagram showing all impedances in per-unit on a 100-MVA base. Choose 20kV as the voltage base for generator. (15%)
- (b) The motor is drawing 45MVA, 0.80 power factor lagging at a line-to-line terminal voltage of 18kV. Determine the terminal voltage and the internal emf of the generator in per-unit and in kV.
- 5 Three loads are connected in parallel across a 12.47kV three-phase supply. Find the total complex power, power factor, and the supply current. Also find the capacitance per phase in μF to improve the overall power factor to 0.8 lagging. (20%)

Load 1: Inductive load, P=60 kW and Q=660 kVAR

Load 2: Capacitive load, 240 kW at 0.8 power factor

Load 3: Resistive load of 60 kW

- 1. A parallel RLC load is connected to a sinusoidal voltage source which has the output resistance of 10 ohm, as shown in Fig. 1.
  - (a) (10%) Choose the capacitance value C so that the load has unity power factor.
  - (b) (10%) With the value of C chosen in problem (a), find the value of R such that the average power dissipation on resistor R is maximized. Also, what is the maximum average power delivered to the resistor R?

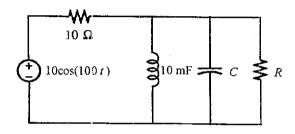


Fig. 1

- 2. For the network in Fig. 2,
  - (a) (10%) find the input irapedance Z(s) (the transfer function from i to v) and show its pole-zero locations.
  - (b)(10%) Connect a sinusoidal voltage source to Z(s), what frequency  $\omega$  of sinusoidal input v will result in zero current i in steady state.

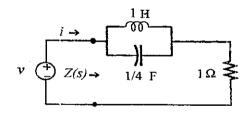


Fig. 2

- 3. In Fig. 3, the switch has been at the position A for a long time and is moved to position B at *t*=0.
  - (a) (10%) Find i(t) for t>0.
  - (b) (10%) Find the maximum instantaneous energy stored in the capacitor for t>0.

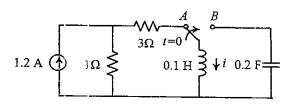


Fig. 3

4. (15%) Suppose the v-i characteristic of the nonlinear resistor satisfies the relationship  $v=2i+i^2$ , find its total harmonic distortion from the input  $i=\cos(\omega t)$  to the output  $v_o$ .

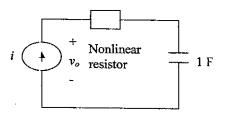


Fig. 4

5. (15%) For the circuit in Fig. 5 find the value of C that cases oscillation at  $\omega = 2$  rad/sec for some initially stored energy.

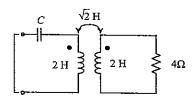


Fig. 5

5. (10%) Design an inverting amplifier which has the gain of -10 and an integrator which has the unity gain at  $\omega = 1000$  rad/s, using ideal OP-Amps.

1. A conducting cone is separated from a ground plane by an infinitesimal insulating gap, as shown in Fig. P1. The axis of the cone is perpendicular to the conducting ground plane. The potential of the cone is maintained at  $V_0$  while the potential of the ground is 0. Solve Laplace equation in the spherical coordinates for the potential distribution  $\Phi$  in the region  $\theta_1 < \theta < 90^\circ$  and the surface charge density on the cone. Hint: You may need the integral formula  $\int (1/\sin\theta) d\theta = \ln(\tan\theta/2)$ .

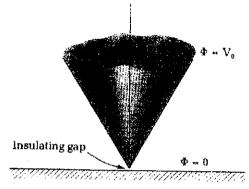


Fig. P1.

2. A constant current source of 10 A along the z-direction is placed at the corner of two perfectly conducting plates, as shown in Fig. P2. The material filling the region between the two plates has a conductivity of  $\sigma = 0.01$  mho/m. Find the potential at point B shown in the figure. (20%)

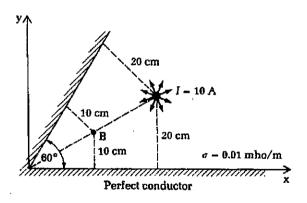


Fig. P2.

3. A transmission line with characteristic impedance  $Z_0$  is terminated with an unknown load impedance  $Z_L$ . Through measurements it is known that the standing wave ratio S and the first voltage minimum nearest to the load is located at  $z_m/\lambda$ . Express  $Z_L$  in terms of S and  $z_m/\lambda$ .

(20%)

### 科目:電磁學【電機系碩士班戊組】

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4. Find the polarization (linear, circular, or elliptical and left-handed or right-handed) of the following fields:

(4% each)

(a) 
$$\mathbf{E} = (j\hat{\mathbf{a}}_x + \hat{\mathbf{a}}_y)e^{-jkz}$$

(b) 
$$\mathbf{E} = (\hat{\mathbf{a}}_{x} - j\hat{\mathbf{a}}_{y})e^{+jkz}$$

(c) 
$$\mathbf{E} = [(1+j)\hat{\mathbf{a}}_y + (1-j)\hat{\mathbf{a}}_z]e^{-jkx}$$
,

(d) 
$$\mathbf{E} = [(2+j)\hat{\mathbf{a}}_x + (3-j)\hat{\mathbf{a}}_z]e^{-jky}$$
,

(e) 
$$\mathbf{E} = (j\hat{\mathbf{a}}_x + j2\hat{\mathbf{a}}_y)e^{+jkz}$$
.

5. Consider a rectangular waveguide with cross section  $a \times b$ , shown in Fig. P5. The region z < 0 is air and the region z > 0 is filled with lossless material characterized by  $\varepsilon_2$  and  $\mu_2$ . A TE<sub>10</sub> mode with amplitude  $E_0$  is incident from z < 0 on the boundary at z = 0. Assume that the reflected wave is TE<sub>10</sub> with amplitude E<sub>1</sub> and the transmitted wave is also TE<sub>10</sub> with amplitude  $E_2$ . Find the ratio  $E_1 / E_0$  in terms of a,  $\varepsilon_0$  and  $\mu_0$ ,  $\varepsilon_2$  and  $\mu_2$ .

(20%)

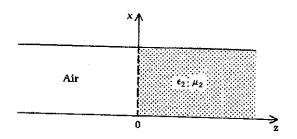


Fig. P5.

### 科目:通訊理論【電機系碩士班己組】

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- 1. Explain the basic principle of Differential Pulse Code Modulation (DPCM)? What is the statistical condition under which the quantization error performance of this method is better than that of PCM? (10 points)
- 2. What is noise figure? How can we compute the overall noise figure of a 3-stage system? Derive your answer in detail. (10 points)
- 3. Define clearly "Additive White Gaussian Noise" (AWGN). (10 points)
- 4. Derive the optimal detection receiver for binary signals at base band by the use of cost functions, prior probabilities and class conditional probabilities. (10 points)
- 5. What kinds of modulations are used for the audio signal and video signal reception for the commercial TV? Explain their principles in detail, state the reasons why different modulations are used? (10 points)
- Define the energy utilization efficiency and the bandwidth utilization efficiency of a digital communication system. Derive the relation for these two measures for the ideal channel and explain clearly your results. (10 points)
- 7. Explain the operation principles of the Wiener filter. State the reasons why we use it in the communication system. (10 points)
- 8. Explain the operation principles of the matched filter. State the reasons why we use it in the communication system. (10 points)
- A recorded conversation is to be transmitted by a pseudo-noise spread spectrum system. Assuming the spectrum of the speech waveform is band-limited to 3 KHz, and using 128 quantization levels. Find the chip rate required to obtain a processing gain of 20 db. (10 points)
- 10. For the above PN Spread Spectrum System, given that the sequence length is to be greater than 5 hours, find the number of shift register stages required.(10 points)

#### 科目:機率【電機系碩士班已組】

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1. Let the random variables X and Y be independent and Gaussian, and let each have a mean of zero and a variance of  $\sigma^2$ .

$$f_X(x) = f_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{x^2}{2\sigma^2}\right]$$

If a new random variable Z is defined by

$$Z = \frac{X}{Y}$$

- (a) Find the conditional probability density function of Z given Y. (15%) (b) What is the probability density function of Z (15%)?
- 2. Let X and Y be the jointly Gaussian random variables with joint density function defined as

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left\{\frac{-1}{2(1-\rho^2)} \left[\frac{(x-\overline{X})^2}{\sigma_X^2} - \frac{2\rho(x-\overline{X})(y-\overline{Y})}{\sigma_X\sigma_Y} + \frac{(y-\overline{Y})^2}{\sigma_Y^2}\right]\right\}$$

- (a) Find the conditional density functions  $f_X(x|Y=y)$  and  $f_Y(y|X=x)$ . (15%)
- (b) Show that X and Y are both Gaussian random variables, where  $\overline{X} = E[X]$  and  $\overline{Y} = E[Y]$ ) (8%) (Note that  $\rho$  is the correlation coefficient of X and Y)
- (c) Assume that  $\sigma_X = \sigma_Y = \sigma$ , show that the locus of the maximum of the joint density is a line passing through the point  $(\overline{X}, \overline{Y})$  with slope  $\frac{\pi}{4}$  (or  $-\frac{\pi}{4}$ ) when  $\rho = 1$  (or  $\rho = -1$ ) (7%)
- 3. Consider the probability density function of the random variable X to be defined by

$$f_x(x) = \begin{cases} \frac{1}{\sqrt{2\pi}} & x^{-1/2}e^{-x/2}, & \text{if } x > 0 \end{cases}$$
 (The p.d.f. of Chi-square with n=1)

- (a) Please evaluate E[X],  $E[(X-E[X])^2]$ . (10%)
- (b) Find the moment generating function,  $M_X(t) = E[e^{tX}]$ . (10%) (Note:  $\Gamma(n) = (n-1) \Gamma(n-1)$ ,  $\Gamma(n+1) = n!$ ,  $\Gamma(1/2) = (\pi)^{1/2}$  and  $\Gamma(1) = 1$ )
- 4. Let  $X_1$  and  $X_2$  be two independent *Poisson* random variables with identical distribution. (a) Find  $P[X_1 = x_1 \mid X_1 + X_2 = y]$  (10%) and (b)  $E[X_1 \mid X_1 + X_2 = y]$  (10%). Note that:  $P(X_1 = x_1) = \frac{e^{-\theta_1}\theta_1}{x_1!}$

#### Linear Algebra

#### 1. (15%)

- (a) (3%) A is a 4 × 4 matrix and has eigenvalues -4, -2, 2, and 4. Is A orthogonally diagonalizable? Answer YES, NO, or NOT ENOUGH DATA. (You do NOT have to give explanations.)
- (b) (6%) Answer TRUE, FALSE or NOT ENOUGH DATA. (You do NOT have to give explanations.)
  - i.  $\lambda = 0$  is never an eigenvalue.

ii. 
$$\mathbf{A} = \begin{bmatrix} 4 & 5 \\ -5 & 4 \end{bmatrix}$$
 is an orthogonal matrix.

(c) (6%) Find 
$$c_1 = \begin{vmatrix} 2 & 1 & \spadesuit & 0 \\ 0 & 0 & 8 & 0 \\ 1 & 3 & \clubsuit & 2 \\ 0 & 2 & \heartsuit & 2 \end{vmatrix}$$
, and  $c_2 = \begin{vmatrix} 7 & 3 & 8 & 9 & 2 \\ 1 & 2 & 3 & 4 & 2 \\ 6 & 1 & 3 & 9 & 7 \\ 4 & 1 & 7 & 3 & 1 \\ 2 & 4 & 6 & 8 & 4 \end{vmatrix}$ .

- 2. (15%) Suppose u, v, and w are nonzero orthogonal vectors.
  - (a) (5%) Show that  $\|\mathbf{u} + \mathbf{v} + \mathbf{w}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2$ .
  - (b) (5%) Show that u, v, and w are linearly independent.
  - (c) (5%) Suppose  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are linearly independent vectors. Show that  $\mathbf{u} + \mathbf{v} + \mathbf{w}$ ,  $\mathbf{v} + \mathbf{w}$ ,  $\mathbf{v} \mathbf{w}$  are linearly independent.
- 3. (20%) Assume that

$$\mathbf{A} = \left[ \begin{array}{ccc} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{array} \right].$$

- (a) (5%) Find the eigenvalues and all the real eigenvectors of  $\bf A$ . It is a symmetric Markov matrix with a repeated eigenvalue.
- (b) (5%) Find the limit of  $A^k$  as  $k \to \infty$ . (You may work with  $A = S\Lambda S^{-1}$  without computing every entry.)
- (c) (6%) Choose any positive number r, s, t so that  $\mathbf{A} r\mathbf{I}$  is positive definite,  $\mathbf{A} s\mathbf{I}$  is indefinite, and  $\mathbf{A} t\mathbf{I}$  is negative definite.
- (d) (4%) Suppose this **A** equals  $\mathbf{B}^T\mathbf{B}$ . What are the singular values of  $\mathbf{B}$ ?
- 4. (15%) Assume that  $y_1(0) = y_2(0) = y_1'(0) = 4$ ,  $y_2'(0) = -4$ . Solve the initial value problem.

$$y_1'' = 2y_1 + y_2 + y_1' + y_2' y_2'' = -5y_1 + 2y_2 + 5y_1' - y_2'$$

科目:線性代數【電機系碩士班已組】

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5. (15%)

(a) (8%) If  $a \neq c$ , find the eigenvalue matrix  $\Lambda$  and eigenvector matrix  ${\bf S}$  in

$$\mathbf{A} = \left[ \begin{array}{cc} a & b \\ 0 & c \end{array} \right] = \mathbf{S} \Lambda \mathbf{S}^{-1}$$

(b) (7%) Find the four entries in the matrix A<sup>100</sup>.

6. (20%) Consider the matrix 
$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$
.

- (a) (6%) Give a factorization  $\mathbf{A} = \mathbf{Q}\mathbf{R}$ , where  $\mathbf{R}$  is an upper-triangular matrix and  $\mathbf{Q}$  is a matrix with orthonormal columns.
- (b) (7%) Find the least square solution to the system

$$\mathbf{A}\underline{x} = \underline{b}, \quad \text{for } \underline{b} = \begin{bmatrix} 4 \\ 8 \\ 6 \end{bmatrix}.$$

(c) (7%) The projection matrix  $\mathbf{P} = \mathbf{A}(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$  projects all vectors onto the column space of  $\mathbf{A}$ . Find a vector  $\underline{q}$ , not in the column space of  $\mathbf{A}$  such that

$$\mathbf{P}\underline{q} = \left[ \begin{array}{c} 1 \\ 4 \\ 4 \end{array} \right].$$

 $\sim$ End $\sim$ 

#### 科目:數位電路【電機系碩士班庚組(含丙組選考)】

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[Problem 1] By Boolean algebra to determine and prove whether or not the following expressions are valid, i.e. whether the left- and right-hand sides represent same function. (10%)

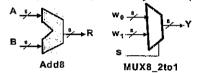
a. 
$$x_2x_3x_4 + x_1x_2x_4 + x_1x_2x_3 + x_1x_2x_3 = x_2x_4 + x_1x_2 + x_2x_3$$
  
b.  $(x_1 + x_3)(x_1 + x_2 + x_4)(x_2 + x_3 + x_4) = x_1x_3 + x_2x_3 + x_3x_4 + x_1x_2 + x_1x_4$ 

[Problem 2] Construct a NAND-gate circuit to implements a negative-edge-trigged D flip-flop. Let the basic gates take the same delay time as 1d. Determine the delay time of the D flip-flop critical path. (15%)

[Problem 3] Given the 4-bits fast adder (named Add4), the 2-to-1 1-bit multiplexers (named MUX\_2to1), the 2-to-1 4-bit multiplexers (named MUX4\_2to1) and the basic gates such as NOT, AND, OR, NAND, and NOR, construct a 2-digit BCD adder. (10%)

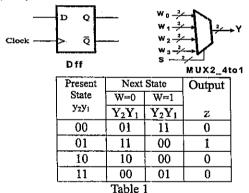
[Problem 4] Given the 8-bits fast adder (named Add8), the 2-to-1 8-bits multiplexers (named MUX8\_2to1) and the basic gates such as NOT, AND, OR, NAND, and NOR, you are asked to design an ALU in function block diagrams, which must match the following requirements:

- (1) Support add, sub, and slt (set on less than) functions. Their operation selection bits (op\_sel) are as follows: add(00), sub(01), slt(11),
- (2) Report the result status in sign, zero, overflow, and carry bits. (20%)



[Problem 5] In VHDL or Verilog HDL, write an 8-bit up/down counter with synchronous clear. (10%)

[Problem 6] Given the positive-edge-triggered D flip-flop (named Dff), the 4-to-1 2-bits multiplexers (named MUX2\_4to1) and the basic gates such as NOT, AND, OR, NAND, and NOR, derive a circuit by the state-assigned table as table 1. (20%)



[Problem 7] In VHDL or Verilog HDL, write an 8-bit multiplier with the constant 7, i.e. function for X\*7, but can not use the multiply operator "\*". (15%)