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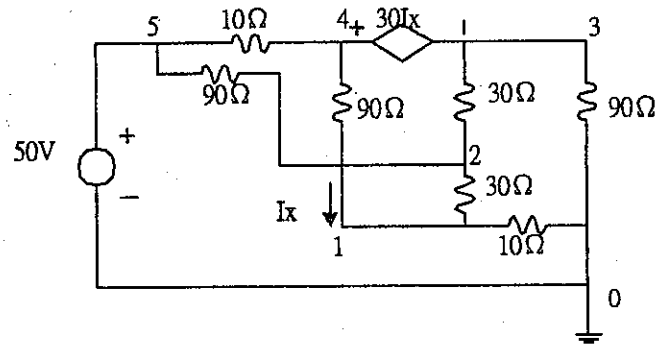
Basic Circuit Analysis (20% each)

【Problem 1】 For the following circuit, a current-control voltage source is attached between node 3 and 4. Try to find all the node voltages



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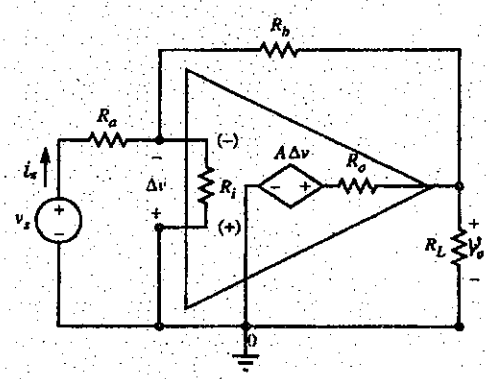
【Problem 2】
For the following OPAMP, Ra=1kΩ, Rb=10kΩ, Ri=10kΩ, Ro=100Ω
With A=100, Find
(a) G=Vo/Vs, Ri=Vs/is
(a) The close-loop output resistance Ro



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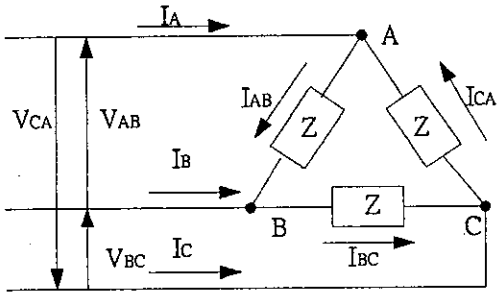
【Problem 3】 A 3-phase 3-wire 240V network ACB is connected to a △ load as shown in the figure below. Try to find (1) the phase current (2) the line current, and (3) the plot in polar axis.



$Z_{AB}=10\angle 30°\ \Omega$

$Z_{BC}=20\angle 90°\ \Omega$

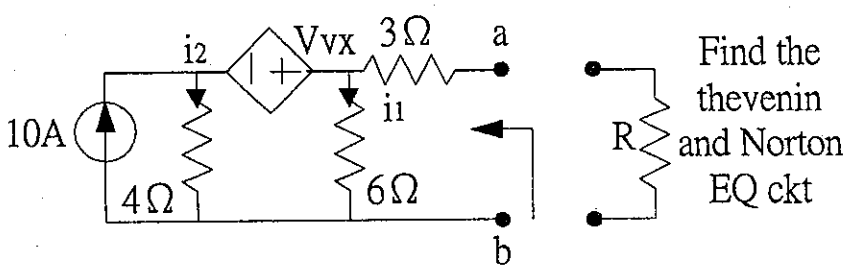
$Z_{AB}=15\angle -45°\ \Omega$

【Problem 4】 With a parallel RLC circuit, Try to find

(1) The unity-power-factor frequency。

(2) the voltages on the L and C elements

【Problem 5】



國立中山大學九十四學年度碩士班招生考試試題

科目：電力系統【電機系碩士班丁組】

1. For the power system in Fig. 1, (all units are in p.u with 100 MVA base). Bus 1 (swing bus) and Bus 2 are connected by a transformer with impedance of j 0.25 p.u and tap ratio 1:1.05. (25%)
   1) Find the equivalent circuit of the regulating transformer.
   2) Solve the voltage at Bus 2 by Newton Raphson method for one iteration. The initial voltage of Bus 2 is $1.0\angle 0^\circ$, The voltage level of swing bus $|V_1| = 1.0$)

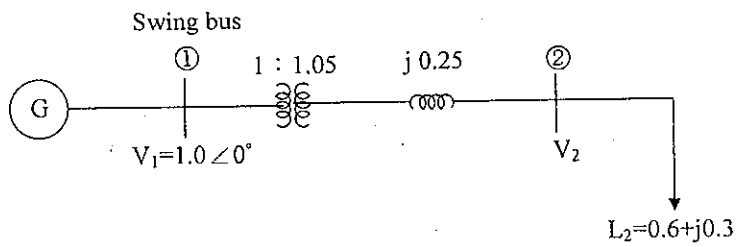


Fig 1

2. With load flow analysis for the system in Fig. 2 (all units in p.u), the bus voltage has been solved as $V_2=0.98183\angle -3.5035^\circ$ and $V_3=1.00125\angle -2.8624^\circ$. Solve the power delivered by swing bus and total system power loss. (15%)

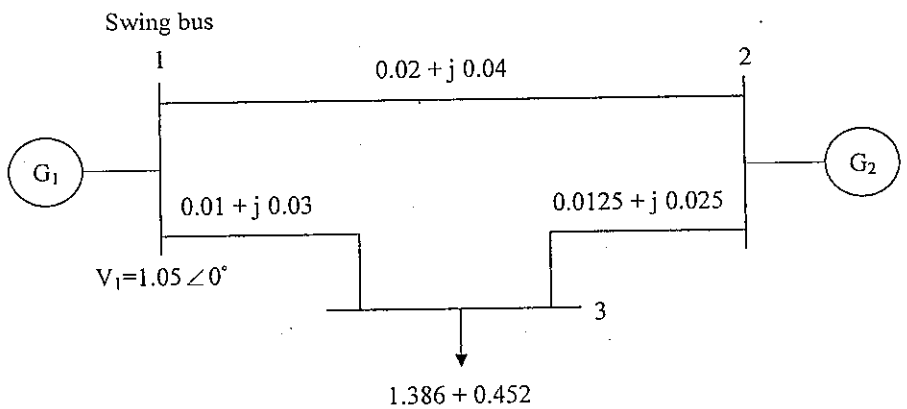


Fig 2

3. A 69KV, three phase transmission line is 40 km long with per phase series impedance of (0.125 + j 0.50) Ω per km. Determine the voltage regulation and the transmission efficiency when the line delivers 100 MVA with power factor 0.8 lagging at 66KV. (20%)

4. (a) Describe the critical clearing time and how the equal area criterion is applied for power system stability analysis. (10%)
   (b) Describe the unit commitment, economic dispatch control for power system operation. (5%)

5. For a power system in Fig. 3, the neutral of each generator is grounded with a reactor of 0.1 p.u on a 100 MVA base. The system data are shown as the tables below. What's the fault current for a double line to ground fault at Bus 2 through a fault impedance Z_f=48.4Ω . (25%)

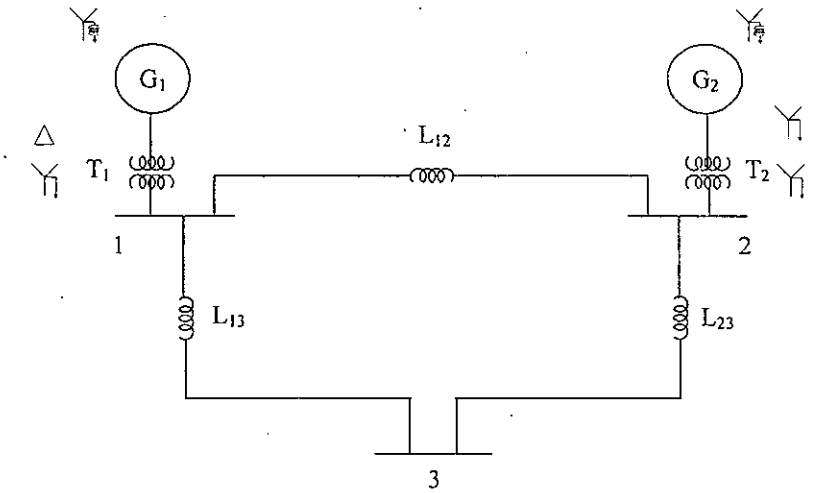


Fig 3

Item	Capacity Rating (MVA)	Voltage Rating (KV)	X^1	X^2	X^0
G_1	100	20	0.15	0.15	0.05
G_2	150	20	0.15	0.15	0.05
T_1	100	20/220	0.10	0.10	0.10
T_2	200	20/220	0.10	0.10	0.10

Line	X^1 (Ω)	X^2 (Ω)	X^0 (Ω)
L_{12}	60	60	150
L_{13}	60	60	150
L_{23}	60	60	150

1. A spherical capacitor is filled with a dielectric material of ε_1 in half of the space and with another material ε_2 in the remaining space, as shown in Fig.P1.
   (10%) (a) Find the potential distribution in the region $a < r < b$ assuming the potential at $r = a$ is V_0 and at $r = b$ is 0.
   (5 %) (b) Find the electric field in the region $a < r < b$.
   (5 %) (c) Find the electric flux density in the region $a < r < b$.

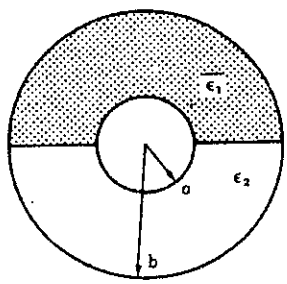


Fig. P1.

2. (20%) A rectangular loop is located in the vicinity of an infinitely long wire carrying a direct current I as shown in Fig. P2. Center of the rectangular loop is located at $x = d + a/2$. Find the voltage induced in the loop if it is rotating about the axis parallel to the z-axis with an angular frequency ω.

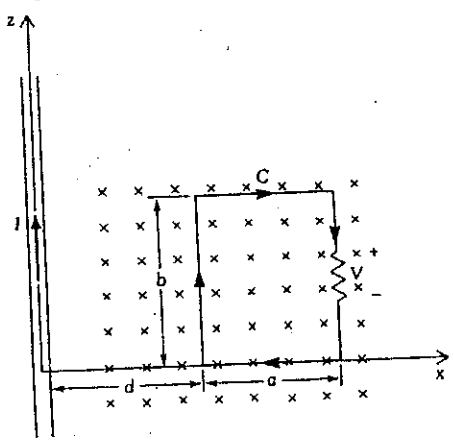


Fig. P2.

3. (20%) A light ray is obliquely incident from one side of a prism at an angle θ_i as shown in Fig P3. The angle θ_i is adjusted to a specific angle ψ_c such that the incident angle on the other side of the prism is equal to the critical angle of incidence θ_{ic}. Derive the expression for refractive index of the prism in terms of the angles A and ψ_c.

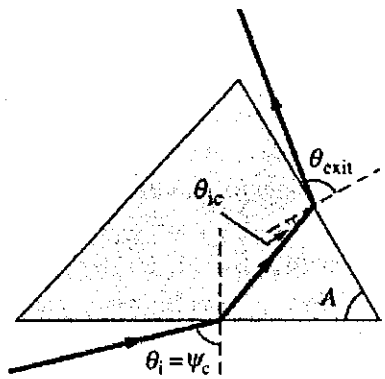


Fig. P3.

4. (10%)(a) What is a microstrip line? How to control the characteristic impedance of a microstrip line?

   (10%)(b) A 100 Ω microstrip line has an effective dielectric constant of 1.65. Find the shortest open-circuited length of this line that appears at its input as a capacitor of 5 pF at 2.5 GHz. Also find the shortest short-circuited length for an inductor of 5 nH at 2.5 GHz.

5. A standard air-filled *K*-band rectangular waveguide has dimensions a = 1.07 cm and b = 0.43 cm.

   (10%)(a) Determine the first two propagating modes and their cutoff frequencies.

   (10%)(b) If this rectangular waveguide is made from perfect conductor and filled with a dielectric material having $\varepsilon_r = 2.2$ and $\tan\delta = 0.002$, compute the TE_{10} mode attenuation in dB/m at 20 GHz.

計分說明：

作答 請依順序，一頁一題

本題計分將採用一種新創的"市場需求調整計分法"，即計分高低(價值)與獲解率(市場需求率之倒數，愈易獲解，市場上需求愈低)成反比。所以考生除了要把握多數人會的題目外，還要著重於自己獨特的能力，即別人不容易會的問題。在入學徵才錄取率不高的情況下，希望考生能發揮出自己卓越的特點。

你的得分($S_1', S_2', S_3', \ldots$)公式計算如下：令$R_1, R_2, R_3, \ldots$爲小題各自原始分數給分範圍，$R_1', R_2', R_3', \ldots$爲依市場需求調整後的分數給分範圍，$M_1, M_2, M_3, \ldots$爲小題各自的原始平均得分，$S_1, S_2, S_3, \ldots$爲小題各自原始得分：

則$S_1' = S_1 \times \frac{R_1'}{R_1}$

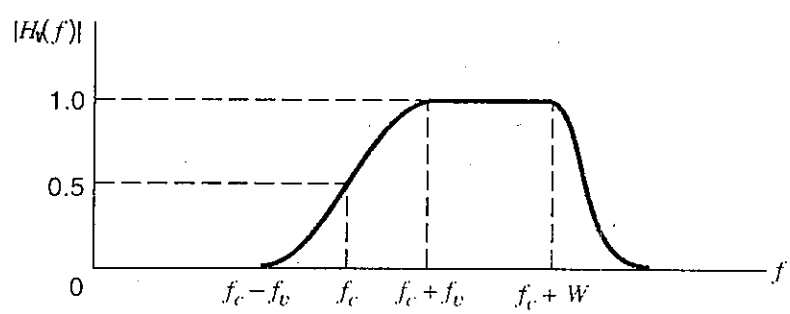
而$R_1', R_2', R_3', \ldots$的計算，在$M_1, M_2, M_3, \ldots$的各小題平均分數算後，

依據$R_1' : R_2' : R_3', \ldots = \frac{R_1}{M_1} : \frac{R_2}{M_2} : \frac{R_3}{M_3}, \ldots$得到，且$R_1' + R_2' + R_3', \ldots = 100$

注意$R_1, R_2, R_3, \ldots$的原始範圍可以任意設定，爲著讓考生易所依循，我仍可以做一次不必要的設定：$R_1 = R_2 = R_3, \ldots = \frac{100}{5}$

(考卷附圖，皆爲示意，不必拘泥)

1. The following plot demonstrates the magnitude response of VSB filter.



(a) Why the VSB (Vestigial Sideband Modulation) are demanded in communications such as TV applications?

(b) Draw the plot of $Hv(f - f_c)$.(only $Hv(f - f_c)$)

(c) Derive the requirement for the response of VSB that

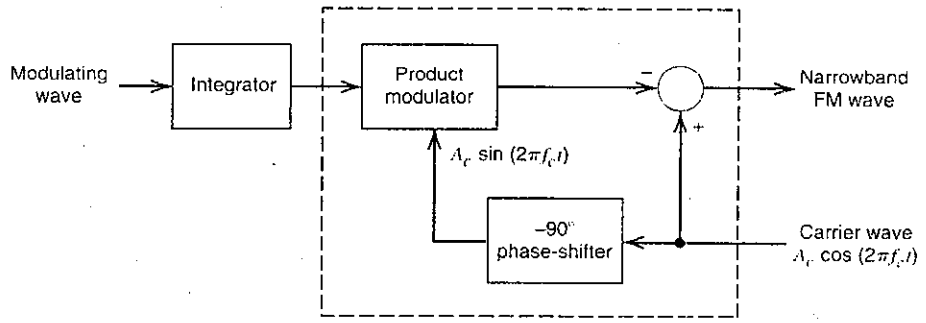
$Hv(f - f_c) + Hv(f + f_c) = 1$.

You must start by the modulated signal denoted as the following

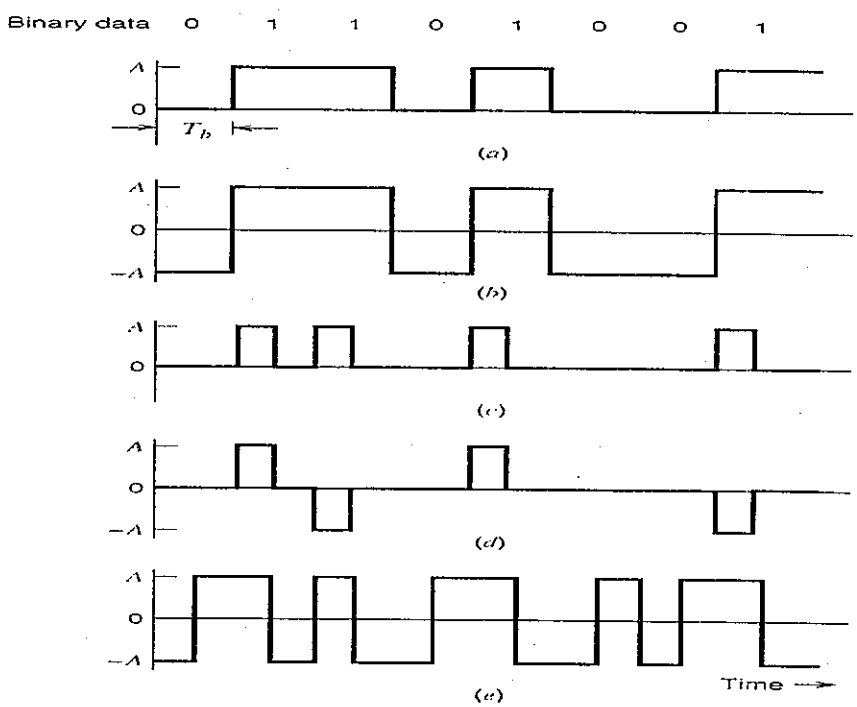
$$\Phi_{VSB}(\omega) = \left[\frac{1}{2}F(\omega - \omega_c) + \frac{1}{2}F(\omega + \omega_c)\right]H_v(\omega)$$

Where $\omega = 2\pi f$.

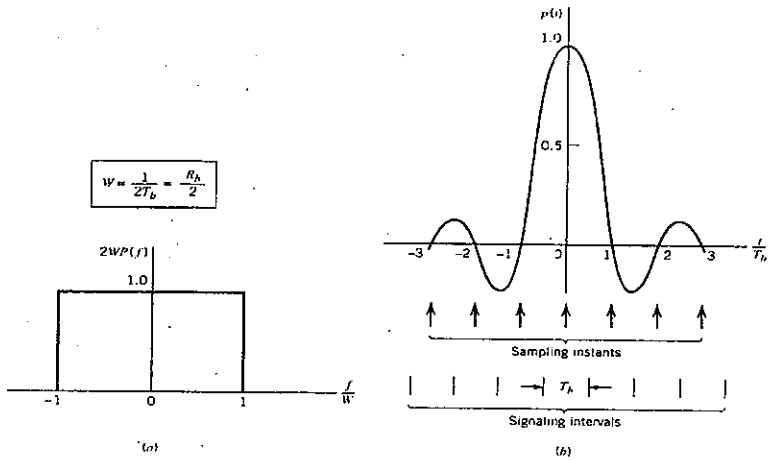
2. (a) What is the function of the following block diagram.
(b) Prove your answer by appropriate mathematics.



3. The following diagram demonstrates some famous line codes.
(a) Which one has the best bandwidth performance? Please explain.
(b) Which one has the best error probability performance? Please explain.
You don't need to give the names of codes. Concentrate on explaination.



4. The following diagram demonstrates the ideal basic pulse shape of an ideal Nquist channel.

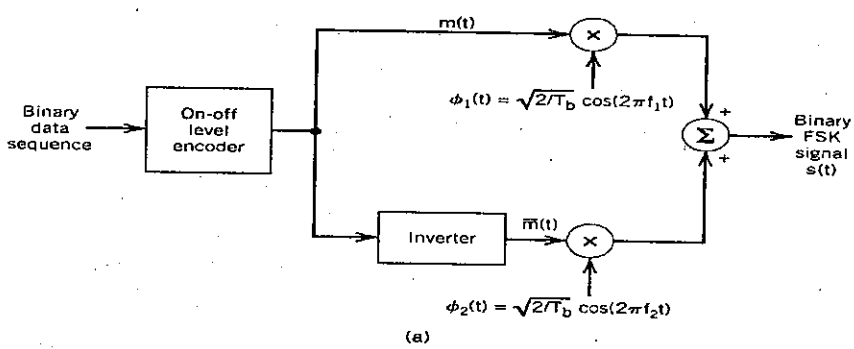


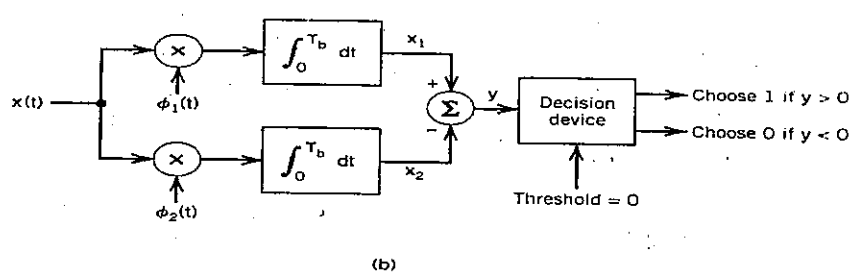
(a) What is the idea of basic pulse shape?

(b) What should be the basic pulse shape if the channel response (transfer function) is triangle instead of rectangle? Please explain.

5. (a) What are the functions of the following block diagrams?
What is the function of the block of on-off level encoder?

(b) Give the outlines for computing the bit error rate in terms of the E_b and N_o as commonly defined for signal and noise.





Linear_Algebra_2005 Each Problem counts for 20 points Page 1/1

1. What is singular value decomposition (SVD)? Why do we need it? Explain your concepts in detail and then compute the SVD of the following matrix:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

2. What are the eigenvalues, eigenvectors and the eigenspaces of a matrix? Why do we study them? Explain your concepts in detail and then compute the eigenvalues, eigenvectors and the eigenspaces of the following matrix:

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{bmatrix}$$

3. Find the $N(A)$, $R(A^T)$, $N(A^T)$, and $R(A)$ of the following matrix:
(where N, R and T denote null space, range space and transpose of a matrix.)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 4 \end{bmatrix}$$

4. Compute the Gram-Schmidt QR factorization of the matrix:

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 0 & 1 \\ 2 & -4 & 2 \\ 4 & 0 & 0 \end{bmatrix}$$

5. Let A be a symmetric tridiagonal matrix (i.e., A is symmetric and $a_{ij} = 0$ whenever $|i - j| > 1$). Let B be the matrix formed from A by deleting the first two rows and columns. Show that (where det and M denote determinant and sub-matrix.)

$$\det(A) = a_{11} \det(M_{11}) - (a_{12})^2 \det(B)$$

1. (15%) Binary Search
   (a) (10%) Given the following list, show the steps of locating the target '357' by binary search:

   10 25 78 97 105 131 189 207 210 241 323 357 366 465 501 567 622 793

   In particular, show at least the 'top' and 'bottom' indices in each step.

   (b) (5%) For this particular example, how much faster (in terms of percentage in reduction of the number of comparisons made) does binary search perform compared to sequential search?

2. (10%) Sorting
   You are told that a list of 10,000 words is already in order but you wish to check it to make sure and sort any words found out of order. Which of the following sorting algorithms would you choose: insertion sort, selection sort, mergesort, quicksort or bubblesort? Explain your answer clearly.

3. (25%) Tree / Binary (Search) Tree
   (a) (5%) How many edges are there in a tree if the tree has n vertices? Explain your answer clearly.
   (b) (5%) 2-tree (or extended binary tree) is useful in analyzing the efficiency of a searching algorithm. If the height of a 2-tree is 3, what are the largest and the smallest numbers of vertices that can be in a tree? Use one example each to demonstrate your answers.
   (c) (10%) An *AVL tree* is a binary search tree in which the heights of the left and right subtrees of the root differ by at most 1 and in which the left and right subtrees are again AVL trees. As in Figure 1, after deleting node p, rearrange the nodes such that the resulting tree is still an AVL tree. Show the steps of such a rearrangement. Try to make as few moves as possible.
   (d) (5%) Construct a binary tree such that its preorder sequence is: ABCDEFGHI and its inorder sequence is: BCAEDGHFI.

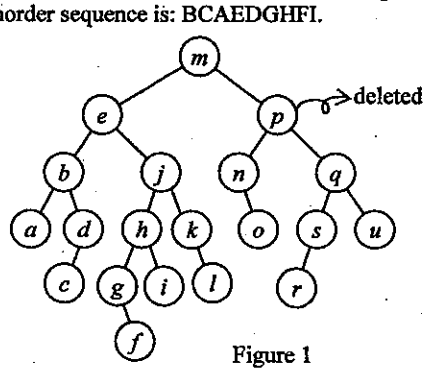


Figure 1

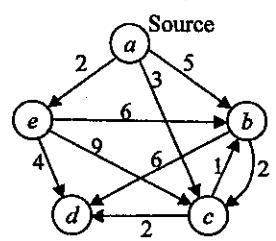


Figure 2

4. (10%) Graph
   Given a directed graph (as in Figure 2), find its shortest paths from vertex a to every other vertex in the graph. Show the running steps clearly.

5. (20%) List
   (a) (5%) Given a linked list (as in Figure 3), write pseudocode for inserting a node (pointed by pointer *q*) after the node pointed by pointer *p*. Use **null** to represent the value of a pointer that points to nothing.
   (b) (5%) Continued from (a), write pseudocode for deleting the node after the node pointed by pointer *p*.
   (c) (10%) Consider using circular array to implement a queue. The '**front**' and '**rear**' indices are used to indicate the beginning and the end of the queue. Describe the problem of distinguishing the condition of an empty queue from that of a full queue. Present one approach that solves the problem.

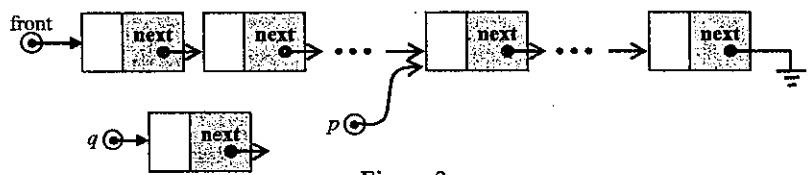


Figure 3

6. (20%) Hash Table
   (a) (5%) Given the following pairs of elements and associated key values:

   ('Jim', 234) ('Tom', 159) ('Amy', 933) ('Ron', 659) ('Ann', 054) ('Kim' 131)

   If we put them into a hash table one by one using the hash function:

   H = (sum of all the digits in the key) (mod 11)

   What does the resulting hash table look like? Suppose the size of the hash table is 11 and collision is resolved by chaining.

   (b) (10%) Design a new hash function such that the elements are more evenly distributed than in (a). Show the resulting table.
   (c) (5%) There are two lists. One list contains 10,000 elements and the other contains 25 elements. By sequential search, how much longer does it take, in average (in terms of number of comparisons), to retrieve an element from the former list than from the latter. If we put the elements of the former list into Hash Table 1 of size 40,000, and the elements of the latter into Hash Table 2 of size 100. Suppose the elements are evenly distributed in both hash tables. How much longer does it take, in average, to retrieve an element from Hash Table 1 than from Hash Table 2?

[Problem 1] Given the basic gates such as NOT, AND, OR, NAND, NOR and XOR, you are asked to answer following questions.

(a) Implements a 4-bit carry-lookahead adder, named CLA4. (10%)

(b) Uses the CLA4, designed in the problem 1.a, to implement a tow-digit BCD adder. (15%)

[Problem 2] Prove the validity of the expression Overflow = C_n XOR C_{n-1} for addition of n-bit signed numbers (Hint: The C_{n-1} is the carry-out from the MSB position and the C_n is the carry-out from the sign-bit position) (15%)

[Problem 3]

(a) Show a circuit that implements the gated SR latch using NAND gates only. (5%)

(b) Describe the truth table of the gated SR latch, designed at 3.a, and express what conditions can be an unpredictable behavior. (10%)

[Problem 4] Figure 1 shows a state table for a particular FSM. Minimize the number of states. (15%)

Present state	Next State		Output
	w = 0	w =1	z
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0

Figure 1

[Problem 5]

(a) Write the code for a negative-edge-triggered D flip-flop with asynchronous reset in VHDL or Verilog HDL (10%)

(b) In VHDL or Verilog HDL, write the code for the traffic light controller, whose lighting sequences are as Figure 2. After the red lamp lights 2 cycles, the green lamp lights 3 cycles. After the green lamp lights, the yellow lamp lights 1 cycles. (20%)

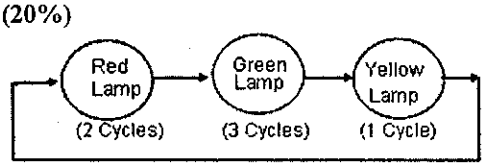
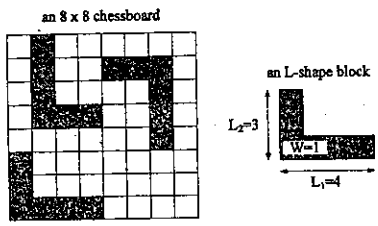


Figure 2

（答題時，每個推導步驟均需寫出明確的原由，不然不予計分）

1. (15%) In a company of 90 employees, there are 15 managers, 44 persons in engineering division, 35 female employees. 7 managers are in engineering division. 14 female employees are in engineering division. 20 employees are male, not managers, and not in engineering division. Find the number of managers who are male and not in engineering division.

2. (15%) In an 8 x 8 chessboard, compute the number of all possible L-shape blocks with width W=1, side length L_1=4, and side length L_2=3 in any multiple of 90° angles at any positions within the chessboard.



3. (15%) Given an unfair 6-face die, on each toss of the die, the probabilities of getting face values 1, 2, 3, 4, 5, and 6 are 0.2, 0.2, 0.2, 0.2, 0.1, and 0.1, respectively. When we toss the die n times, calculate the probability of getting a total face value = $6n - 4$. (You need only write the formula and explain the cases. You need not perform the calculation of summing up the values.)

4. (15%) Prove that the transitive closure of a symmetric relation is a symmetric relation.

5. In a complete graph of n vertices,
   (a) (10%) Calculate the number of Hamiltonian circuits exist in the graph.
   (b) (10%) Calculate the number of all possible spanning trees in the graph.
   (c) (10%) If n is an even number, in order to divide the graph into two equal size subgraphs, what is the size of such a cut set in the graph?
   (d) (10%) Calculate the total number of all possible cut sets for equal-size bi-partition of the graph as in subprobelm (c) given that n is even.

1. Let X be a random variable, nonnegative or not. **(a)** For any real $t > 0$, *show* the *Chernoff inequality*: (10%)

$$P\{X \geq a\} \leq e^{-ta} M_X(t)$$

where $M_X(t)$ is defined in (P-1). Next, if $X_1, X_2, \ldots, X_N$ are considered to be independent random variables with *moment generating functions*, $M_{X_1}(t), M_{X_2}(t), \ldots, M_{X_N}(t)$, respectively, where $M_{X_i}(t)$, is defined as

$$M_{X_i}(t) = E[e^{tX_i}] = \int_{-\infty}^{\infty} e^{tx_i} f_{X_i}(x_i) dx_i, \qquad \text{(P-1)}$$

(b) If we let the random variable $Y = X_1 + X_2 + \ldots + X_N$, please find $M_Y(t)$, in terms of $M_{X_i}(t)$, i =1,..., N. (10%)

(c) Now, assume that X_i is with Gaussian distribution (or *Normal variable*), and the probability density function (*p.d.f.*) is given by

$$f_{X_i}(x_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{(x_i-\mu_i)^2}{2\sigma_i^2}},$$

where the *mean* and *variance* are defined as μ_{X_i} and σ_{X_i}, respectively. For N=2, please compute $M_Y(t)$? (10%).

2. (*Sum of Poisson random variables*) Let X and Y are random variables with *Poisson* PMF as follows:

$$P_X(k) = \frac{1}{k!} e^{-\lambda_1} \lambda_1^{\,k}$$

and

$$P_Y(j) = \frac{1}{j!} e^{-\lambda_2} \lambda_2^{\,j}$$

(a) Find P[Z=m] where Z=X+Y (10%) (b) Compute P[Z≦5] for λ_1=2 and λ_2=3. (10%)

3. Three uncorrelated random variables X_1, X_2, and X_3, have *means* μ_1 =1, μ_2 = -3, and μ_3 =1.5 and *second order moments* $E[X_1^2] = 2.5$, $E[X_2^2] = 11$, and $E[X_3^2] = 3.5$. Let $Y = X_1 - 2X_2 + 3X_3$ be a new random variable and find: **(a)** the *mean* value (10%), and **(b)** the *variance* of Y. (10%)

4. Let the random variables X and Y be independent and *Gaussian*, and let each have a mean of zero and a variance of σ^2.

$$f_X(x) = f_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-\frac{x^2}{2\sigma^2}]$$

If a new random variable Z is defined by

$$Z = XY$$

(a) Find the *conditional probability density function* of Z given Y. (15%) (b) What is the *probability density function* of Z (15%)?

國立中山大學九十四學年度碩士班招生考試試題

科目：工程數學乙【電機系碩士班乙組】

1. (25%)本題之計分僅以最後答案為準，不考慮計算過程。答案請寫在計算題部份，註明小題號，由(1)、(2)、…、依序列出。

Part I (15pts). A periodic signal $x(t)$ has a period $T=2$ and

$x(t)=\begin{cases}1, & |t|<\frac{1}{2}\\ 0, & \frac{1}{2}\le|t|<1\end{cases}$. Represent $x(t)$ in complex Fourier series $x(t)=\sum_{k=-\infty}^{\infty}X[k]e^{+jk\omega_0 t}$,

where ω_0 is the fundamental angular frequency. Find

(a) $X[0]=$ ___(1)___. (b) $X[0]-X[1]+X[2]-\cdots=$ ___(2)___. (c) $\sum_{k=-\infty}^{\infty}X^2[k]=$ ___(3)___.

Part II (10pts). The Fourier transform of $x(t)$ is defined as

$X(j\omega)\triangleq\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$. Suppose $x(t)=\begin{cases}1, & |t|<\frac{1}{2}\\ 0, & \text{otherwise}\end{cases}$.

(a) Find $X(j\pi)=$ ___(4)___. (b) Calculate the following convolution integral

$\int_{-\infty}^{\infty}\frac{\sin(\pi\tau)\sin[2\pi(t-\tau)]}{\pi^2\tau(t-\tau)}d\tau=$ ___(5)___.

2. (40%) This problem contains two parts with 20 points in each part. ONLY ANSWERS WITHOUT PROOF are required in both parts.

Part I 是選擇並改正題。此部份中含有八個敘述，其中至少有四個敘述是錯誤的。請任選四個**錯誤的**敘述、並很精確地指出錯誤處、以及其正確的樣式。舉例如下：在下面三個敘述中

(S1) If $a>b$ and $b>c$, then $a\ge c$.

(S2) Let $A\in\mathbb{R}^{m\times n}$. Then matrix A is singular if and only if $\det(A)=0$.

(S3) Let $A\in\mathbb{R}^{m\times n}$. Then all columns of matrix A are linearly dependent if and only if null space $N(A)=\{\mathbf{0}\}$.

根據對數學和線性代數的了解，你知道(S1)是對的敘述、而(S2)和(S3)是錯誤的敘述。因此你寫的答案如下：

=======

answer for Part I:

wrong statement	correction (may be written in Chinese)
(S2)	The statement is true only when A is a square matrix, i.e. when $m=n$.
(S3)	$N(A)=\{\mathbf{0}\}$ should be corrected as $N(A)\neq\{\mathbf{0}\}$.

=======

※※※注意：

1. 更正的敘述必須簡潔、精確；因為每一個敘述中錯誤處的更正都僅需書寫一行即可完成。
2. 當然同學注意到(S3)也可以更正成：將"only if part"敘述中的 linearly dependent 改成 linearly independent。因此為便於閱卷起見，統一規定：**當你認為某一個若且惟若的敘述是錯誤的，而且更改"only if part"或者"if part"中的敘述後即變為正確時，請更正"if part"中的敘述（意即：假設"only if part"中的敘述永遠是對的）。**不遵守此一規定的答案一律視為錯誤的答案。

此部份的分數計算方式為：

- 每一個正確的選擇為 2 分，如果你所提供的更正也正確、則可再得 3 分（因此，每一個正確的選擇及正確的更正共可得到 5 分，而如此答對四個正確的選擇及更正、便可得到 20 分）。如果選擇正確，但是所提供的更正不正確，則會因為錯誤的更正被扣 1 分，而僅得到 1 分。如果只寫了正確的選擇、但沒有提供任何更正、則得分數為 2。
- **每一個錯誤的選擇**(無論有否提供更正)**均會被扣 4 分**。然而，扣分的計算僅限於此部份的 20 分內，不會影響到 Part II 的得分數。

因此，如果你的答案如上所述，則可得到 10 分。但是，如果你的答案如下

=======

answer for Part I:

wrong statement	correction (may be written in Chinese)
(S1)	$a\ge c$ should be corrected as $a>c$.
(S3)	$N(A)=\{\mathbf{0}\}$ should be corrected as $N(A)\neq\{\mathbf{0}\}$.

則你僅會得到 1 分。其餘情況可自行類推。

Part I (20%): Let m and n be any two positive integers and let $A\in\mathbb{R}^{m\times n}$ and $\mathbf{b}\in\mathbb{R}^m$ be arbitrary. We use $R(A)$ to denote the range or column space of A and $N(A)$ to denote its null space. In the following (S1) to (S8) eight statements, at least four of them are wrong. Please choose arbitrary four statements that you think are WRONG and write the correction as simple and precise as possible in your answer.

(S1) Either exist $\mathbf{x}\in\mathbb{R}^n$ to satisfy the linear system $A\mathbf{x}=\mathbf{b}$ or exist $\mathbf{y}\in\mathbb{R}^m\cap N(A^T)$ such that $\langle\mathbf{y},\mathbf{b}\rangle\neq 0$.

(S2) Either $\langle\mathbf{b},A\mathbf{x}\rangle=0$ for $\forall\mathbf{x}\in\mathbb{R}^n$ or $\langle\mathbf{b},A\mathbf{x}\rangle>0$ for every nonzero $\mathbf{x}\in\mathbb{R}^n$.

(S3) Matrix A has a right inverse, i.e. $\exists B\in\mathbb{R}^{n\times m}$ such that $AB=I_m$, if and only if A is full row rank, i.e. all rows of A are linearly independent. In that case, the linear system $A\mathbf{x}=\mathbf{b}$ is always consistent, i.e. it is always solvable.

(S4) The linear system $A^TA\mathbf{x}=A^T\mathbf{b}$ is always consistent if and only if matrix A is full row rank.

(S5) Matrix A^TA is nonsingular if and only if matrix A is full row rank. In that case, the linear system $A\mathbf{x}=\mathbf{b}$ has at least one solution whenever it is consistent. When the linear system is inconsistent, however, it has a least squares solution described by the vector $\hat{\mathbf{x}}:=(A^TA)^{-1}A^T\mathbf{b}$, i.e. $\|A\hat{\mathbf{x}}-\mathbf{b}\|_2\le\|A\mathbf{x}-\mathbf{b}\|_2$ for $\forall\mathbf{x}\in\mathbb{R}^n$.

(S6) Suppose A is not a full rank matrix, i.e. assume $rank(A)=k<\min(m,n)$. Then $\exists Q\in\mathbb{R}^{m\times k}$ and $R\in\mathbb{R}^{k\times n}$ such that $A=QR$, where all columns of Q are orthonormal and R is a *non-square* upper triangular matrix.

(S7) From the QR factorization of a not full rank matrix A, i.e. $A=QR$ as mentioned in (S6), obviously we know $R(A)\subset R(Q)$. But the reverse inclusion $R(Q)\subset R(A)$ does not hold because of the existence of matrix R.

(S8) Let L be the linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ defined by A, i.e. $L(\mathbf{x}):=A\mathbf{x}$ for any $\mathbf{x}\in\mathbb{R}^n$. Let L^* be the linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$ defined by A^T, i.e. $L^*(\mathbf{y}):=A^T\mathbf{y}$ for any $\mathbf{y}\in\mathbb{R}^m$. Then $\exists\mathbf{v}\in\mathbb{R}^n$ such that $L(\mathbf{v})=\mathbf{b}$ if and only if $\mathbf{b}\in Ker(L^*)$.

Part II 含有四個子題，前三個子題是填充題，而第四個子題是選擇題。選擇題若有多於一個選項的答案，則全選對才給分。此部份答錯不倒扣。

Part II (20%): Let A be an $n\times n$ real matrix. Denote S and T as the symmetric and skew-symmetric parts of A, and let the eigenvalues of AA^T, S, and T be α_i, β_i, and γ_i for $i=1,\cdots,n$, respectively. Please answer following questions without giving any proof.

(a) (6%) What are S and T respectively?

(b) (2%) What is the value of $trace(ST-TS)$?

(c) (6%) What is the relationship between $\sum_{i=1}^{n}\alpha_i$, $\sum_{i=1}^{n}\beta_i^2$, and $\sum_{i=1}^{n}\gamma_i^2$?

(d) (6%) Without loss of generality, suppose $n=3$ and let $\beta_1\neq\beta_2$ and $\gamma_1\neq\gamma_2$ be two pairs of distinct eigenvalues of S and T, respectively. Let $\mathbf{u}_i$ be a nonzero vector in the null space $N(S-\beta_iI)$ and, similarly, let $\mathbf{v}_i$ be a nonzero vector in the null space $N(T-\gamma_iI)$ for $i=1,2$, respectively. Then what are possible pairs of $\mathbf{u}_i$'s and $\mathbf{v}_i$'s?

(A) $\mathbf{u}_1=[1\ \ 0\ \ 0]^T$, $\mathbf{u}_2=[0\ \ 1\ \ -1]^T$ and $\mathbf{v}_1=[1\ \ i\ \ -1]^T$, $\mathbf{v}_2=[0\ \ 1\ \ -i]^T$

(B) $\mathbf{u}_1=[1\ \ 0\ \ 0]^T$, $\mathbf{u}_2=[0\ \ i\ \ 1]^T$ and $\mathbf{v}_1=[i\ \ 1\ \ -1]^T$, $\mathbf{v}_2=[0\ \ 1\ \ 1]^T$

(C) $\mathbf{u}_1=[1\ \ 0\ \ -1]^T$, $\mathbf{u}_2=[\sqrt{2}\ \ 1\ \ \sqrt{2}]^T$ and $\mathbf{v}_1=[i\ \ 1\ \ 0]^T$, $\mathbf{v}_2=[0\ \ 0\ \ 1]^T$

(D) $\mathbf{u}_1=[1\ \ 0\ \ 1]^T$, $\mathbf{u}_2=[-\frac{1}{2}\ \ 1\ \ \frac{1}{2}]^T$ and $\mathbf{v}_1=[1\ \ 1\ \ 1]^T$, $\mathbf{v}_2=[0\ \ -1\ \ 1]^T$

(E) All of above statements are correct.

(F) None of above statements is correct.

3. (15%) Find the general solution of $(x^2+xy+y^2)\,dx-x^2\,dy=0$.

4. (20%) Find the Laplace transform of the following function: $t\,e^{-t}\sinh 2t$

國立中山大學九十四學年度碩士班招生考試試題

科目：計算機結構【電機系碩士班丙組】

I. (20%) Identify two differences between the following terminology pairs.

(1) (5%) Distributed-Memory multicomputer vs Cluster Computer

(2) (5%) PowerPC architectures vs Itanium architectures

(3) (5%) Write-Through Cache vs Copy-Back Cache

(4) (5%) RAID-1 vs RAID-2

II. (20%) Instruction Set Architectures

(1) (10%) Assume an instruction set that uses a fixed 16-bit instruction length to provide three types of instructions: zero-operand, one-operand, and two-operand. Operand specifiers are 6 bits in length. There are **K** two-operand instructions and **L** zero-operand instructions. What is the maximum number of one-operand instructions that can be supported.

(2) (10%) Design a variable-length opcode to allow all of the following to be encoded in a 36-bit instruction.

7 instructions with two **15**-bit addresses and one **3**-bit register number.

500 instructions with one **15**-bit address and one **3**-bit register number.

50 instructions with no addresses or registers.

III. (20%) Assume the exponent **e** is constrained to lie in the range **0 <= e <= X**, with a bias of **q**, that the base is **b**, and that the significand is **p** digits in length.

(1) (10%) What are the largest and the smallest positive values that can be written?

(2) (10%) What are the largest and the smallest positive values that can be written as normalized floating-point numbers?

IV. (20%) Cache Memory Design

Consider a 32-bit microprocessor that has an on-chip 16 Kbytes four-way set associative cache. Assume that the cache has a line (or block) size of four 32-bit words.

(1) (5%) Show the 32-bit physical address (Show how many Tag bits, Set bits, and Offset bits).

(2) (5%) Where in the cache (by indicating the set number) is the double word from memory location ABCDE8F8 mapped?

(3) (10%) Draw a block diagram of this cache showing its organization and how the different address fields are used to determine a cache hit or miss.

V. (20%) A two-level memory system has eight (page 0 to page 7) virtual pages on a disk to be mapped into four page frames (PFs) in the main memory. Assume each page has 4 words.
The i-th page in disk consists of word address from $4i$ to $4i+3$, where $i = 0$ to 7.
A certain program trace generated the following word address.
0, 1, 2, 5, 6, 7, 8, 9, 3, 4, 9, 10, 4, 5, 10, 11, 6, 7, 13, 12, 16, 17, 14.

(1) (10%) Show the successive virtual pages residing in the four page frames using FIFO (First-In-First-Out). Assuming that the main memory PFs are empty initially. Compute the Hit Ratio.

(2) (10%) Repeat the above using LRU (Least Recently Used). Compute the Hit Ratio.

國立中山大學九十四學年度碩士班招生考試試題

科目：工程數學甲【電機系碩士班甲、丁、戊組(含丙組選考)】

1. (35%)本題之計分僅以最後答案為準，不考慮計算過程。答案請寫在計算題部份，註明小題號，由(1)、(2)、...、依序列出。

Part I (15pts). A periodic signal $x(t)$ has a period $T=2$ and

$x(t)=\begin{cases}1, & |t|<\frac{1}{2}\\ 0, & \frac{1}{2}\le|t|<1\end{cases}$. Represent $x(t)$ in complex Fourier series $x(t)=\sum_{k=-\infty}^{\infty}X[k]e^{+jk\omega_0 t}$,

where ω_0 is the fundamental angular frequency. Find

(a) $X[0]=$ ___(1)___. (b) $X[0]-X[1]+X[2]-\cdots=$ ___(2)___. (c) $\sum_{k=-\infty}^{\infty}X^2[k]=$ ___(3)___.

Part II (10pts). The Fourier transform of $x(t)$ is defined as

$X(j\omega)\triangleq\int_{-\infty}^{\infty}x(t)e^{-j\omega t}\,dt$. Suppose $x(t)=\begin{cases}1, & |t|<\frac{1}{2}\\ 0, & \text{otherwise}\end{cases}$.

(a) Find $X(j\pi)=$ ___(4)___. (b) Calculate the following convolution integral

$\int_{-\infty}^{\infty}\frac{\sin(\pi\tau)\sin[2\pi(t-\tau)]}{\pi^2\tau(t-\tau)}d\tau=$ ___(5)___.

Part III (10pts). Let field $\vec{G}(x,y,z)=\vec{a}_x(x-yz)+\vec{a}_y(y^2-zx)+\vec{a}_z(z^2-xy)$.

(a) Find the line integral $\int_{C_1}\vec{G}(x,y,z)\cdot d\vec{l}=$ ___(6)___, where C_1 is a segment of the curve $y=x^2$, $z=x$ from (0,0,0) to (1,1,1).

(b) Compute the sum of line integrals $\int_{C_2}\vec{G}(x,y,z)\cdot d\vec{l}+\int_{C_3}\vec{G}(x,y,z)\cdot d\vec{l}$ where C_2 is the straight line from (1,1,1) to (0,0,1) and C_3 is along the z-axis from (0,0,1) to (0,0,0).

Answer: ___(7)___.

2. (30%) This problem contains two parts with 15 points in each part. ONLY ANSWERS WITHOUT PROOF are required in both parts.

Part I 是選擇並改正題。此部份中含有六個敘述，其中至少有三個敘述是錯誤的。請任選三個**錯誤的**敘述、並很精確地指出錯誤處、以及其正確的樣式。舉例如下：在下面三個敘述中

(S1) If $a>b$ and $b>c$, then $a\ge c$.

(S2) Let $A\in\mathbb{R}^{m\times n}$. Then matrix A is singular if and only if $\det(A)=0$.

(S3) Let $A\in\mathbb{R}^{m\times n}$. Then all columns of matrix A are linearly dependent if and only if null space $N(A)=\{\mathbf{0}\}$.

根據對數學和線性代數的了解，你知道(S1)是對的敘述、而(S2)和(S3)是錯誤的敘述。因此你寫的答案如下：

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answer for Part I:

wrong statement	correction (may be written in Chinese)
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(S3)	$N(A)=\{\mathbf{0}\}$ should be corrected as $N(A)\neq\{\mathbf{0}\}$.

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※※※注意：

1. 更正的敘述必須簡潔、精確；因為每一個敘述中錯誤處的更正都僅需書寫一行即可完成。
2. 當然同學注意到(S3)也可以更正成：將"only if part"敘述中的 linearly dependent 改成 linearly independent。因此為便於閱卷起見，統一規定：**當你認為某一個若且惟若的敘述是錯誤的，而且更改"only if part"或者"if part"中的敘述後即變為正確時，請更正"if part"中的敘述。**不遵守此一規定的答案一律視為錯誤的答案。

此部份的分數計算方式為：

- 每一個正確的選擇為 2 分，如果你所提供的更正也正確、則可再得 3 分 (因此，每一個正確的選擇及正確的更正共可得到 5 分，而如此答對四個正確的選擇及更正、便可得到 20 分)。如果選擇正確，但是所提供

的更正不正確，則會因為錯誤的更正被扣 1 分，而僅得到 1 分。如果只寫了正確的選擇、但沒有提供任何更正、則得分數為 2。

- 每一個錯誤的選擇(無論有否提供更正)均會被扣 4 分。然而，扣分的計算僅限於此部份的 20 分內，不會影響到 Part II 的得分數。

因此，如果你的答案如上所述，則可得到 10 分。但是，如果你的答案如下

=======

answer for Part I:

wrong statement	correction (may be written in Chinese)
(S1)	$a \geq c$ should be corrected as $a > c$.
(S3)	$N(A) = \{0\}$ should be corrected as $N(A) \neq \{0\}$.

=======

則你僅會得到 1 分。其餘情況可自行類推。

Part I (15%): Let m and n be any two positive integers and let $A \in \mathbb{R}^{m \times n}$ and $\mathbf{b} \in \mathbb{R}^m$ be arbitrary. We use $R(A)$ to denote the range or column space of A and $N(A)$ to denote its null space. In the following (S1) to (S6) six statements, at least three of them are wrong. Please choose arbitrary three statements that you think are WRONG and write the correction as simple and precise as possible in your answer.

(S1) Either exist $\mathbf{x} \in \mathbb{R}^n$ to satisfy the linear system $A\mathbf{x} = \mathbf{b}$ or exist $\mathbf{y} \in \mathbb{R}^m \cap N(A^T)$ such that $\langle \mathbf{y}, \mathbf{b} \rangle \neq 0$.

(S2) Either $\langle \mathbf{b}, A\mathbf{x} \rangle = 0$ for $\forall \mathbf{x} \in \mathbb{R}^n$ or $\langle \mathbf{b}, A\mathbf{x} \rangle > 0$ for each nonzero $\mathbf{x} \in \mathbb{R}^n$.

(S3) Matrix A has a right inverse, i.e. $\exists B \in \mathbb{R}^{n \times m}$ such that $AB = I_m$, if and only if A is full row rank, i.e. all rows of A are linearly independent. In that case, the linear system $A\mathbf{x} = \mathbf{b}$ is always consistent, i.e. it is always solvable.

(S4) The linear system $A^T A\mathbf{x} = A^T\mathbf{b}$ is always consistent if and only if matrix A is full row rank.

(S5) Matrix $A^T A$ is nonsingular if and only if matrix A is full row rank. In that case, the linear system $A\mathbf{x} = \mathbf{b}$ has at least one solution whenever it is consistent. When the linear system is inconsistent, however, it has a least squares solution described by the vector $\hat{\mathbf{x}} := (A^T A)^{-1} A^T \mathbf{b}$, i.e. $\| A\hat{\mathbf{x}} - \mathbf{b} \|_2 \leq \| A\mathbf{x} - \mathbf{b} \|_2$ for $\forall \mathbf{x} \in \mathbb{R}^n$.

(S6) Let L be the linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ defined by A, i.e. $L(\mathbf{x}) := A\mathbf{x}$ for any $\mathbf{x} \in \mathbb{R}^n$. Let L^* be the linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$ defined by A^T, i.e. $L^*(\mathbf{y}) := A^T\mathbf{y}$ for any $\mathbf{y} \in \mathbb{R}^m$. Then $\exists \mathbf{v} \in \mathbb{R}^n$ such that $L(\mathbf{v}) = \mathbf{b}$ if and only if $\mathbf{b} \in Ker(L^*)$.

Part II 含有三個子題，它們都是填充題。此部份答錯不倒扣。

Part II (15%): Let A be an $n \times n$ real matrix. Denote S and T as the symmetric and skew-symmetric parts of A, and let the eigenvalues of AA^T, S, and T be α_i, β_i, and γ_i for $i = 1, \cdots, n$, respectively. Please answer following questions without giving any proof.

(a) (6%) What are S and T respectively?

(b) (2%) What is the value of $trace(ST - TS)$?

(c) (7%) What is the relationship between $\sum_{i=1}^{n} \alpha_i$, $\sum_{i=1}^{n} \beta_i^2$, and $\sum_{i=1}^{n} \gamma_i^2$?

3. (15%) Evaluate $\oint_C \frac{z+8}{z^2+z-2} dz$ by using the residue theorem,

where C : $|z| = 3$ and $z = x + iy$

4.(20%) Find the general solution of $(x^2 + xy + y^2)\,dx - x^2\,dy = 0$.

1. For the circuit shown in Fig 1, (a) break the feedback loop at X and find the loop gain βA; then (b) find the frequency of oscillation f_0 and (c) the minimum required value of R_f for oscillations to start in the circuit. 20%

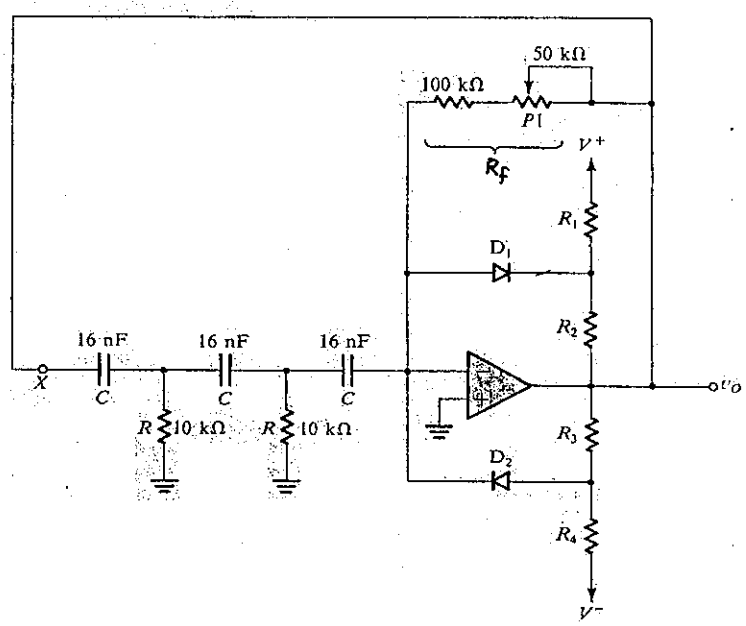


Figure 1.

2. In Figure 2 an inverter fabricated in a 0.12μm CMOS technology uses the minimum possible channel lengths (i.e. $L_n = L_p = 120$nm). (a) If $W_n = 180$nm, find the value of W_p that would result in Q_N and Q_P being matched. (b) For this technology, $k'_n = 160\mu A/V^2$ and $k'_p = 54\mu A/V^2$ with the supply voltage $V_{DD} = 1.5$ V and the threshold voltage $V_{tn} = 0.5$V. Calculate the value of the output resistance of the inverter when $v_O = V_{OL}$ ("Low" level at the output). 20%

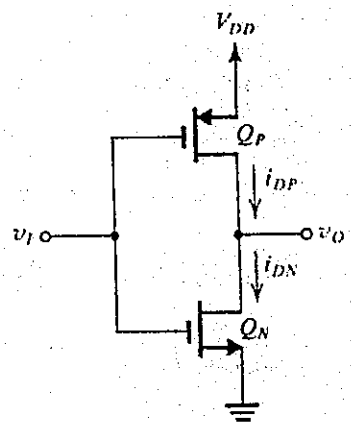


Figure 2.

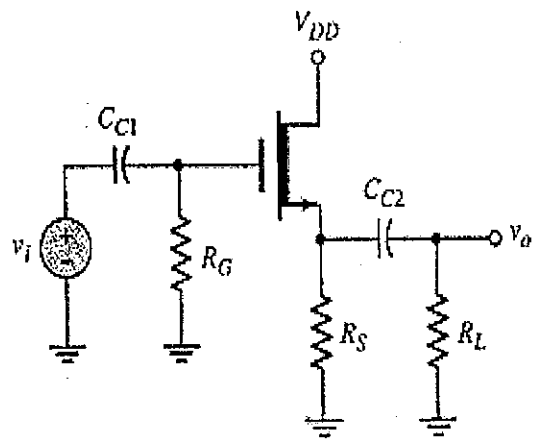


Figure 3.

3. Consider the source-follower circuit shown in Figure 3. The most negative output signal voltage occurs when the transistor just cuts off. Show that (a) this output voltage $v_o(min)$ and (b) the corresponding input voltage $v_i(min)$ with respect to I_{DQ}, g_m, R_S and R_L. 20%

4. Figure 4 shows an op amp connected in the inverting configuration. The op amp has an open-loop gain $\mu = 10^4$, a differential input resistance $R_{id} = 100\text{k}\Omega$, and an output resistance $r_o = 1\text{k}\Omega$. Use the feedback method to find (a) the voltage gain V_o/V_s, (b) the input resistance R_{in}, and (c) the output resistance R_{out}. 20%

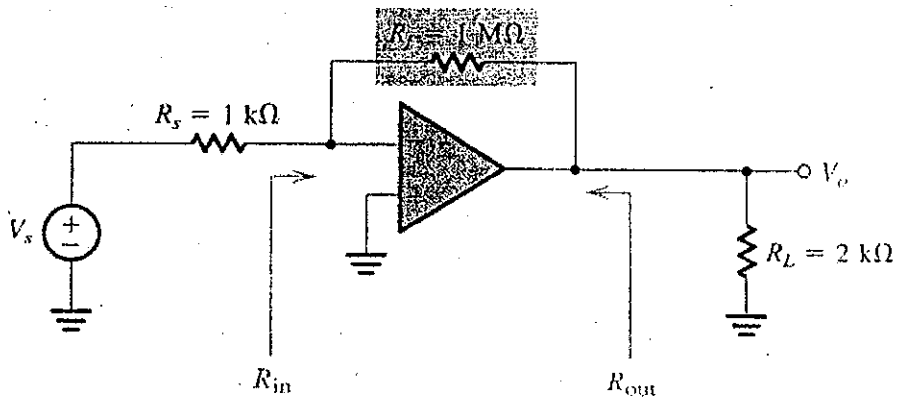


Figure 4.

5. (a) Using a simple (r_π and g_m) model for each of the two transistors Q_{18} and Q_{19} in Figure 5.a, find the small-signal resistance between A and A'. Where $I_{C18} = 165\ \mu A$ and $I_{C19} = 16\ \mu A$. 10%

(b) Figure 5.b shows the circuit for determine the 741 op-amp output resistance when v_O is positive and Q_{14} is conducting most of the current. Using the resistance of the Q_{18} and Q_{19} network calculated in 5.(a) and neglecting the large output resistance of Q_{13A}, find R_o when Q_{14} is sourcing an output current of 5mA. 10%

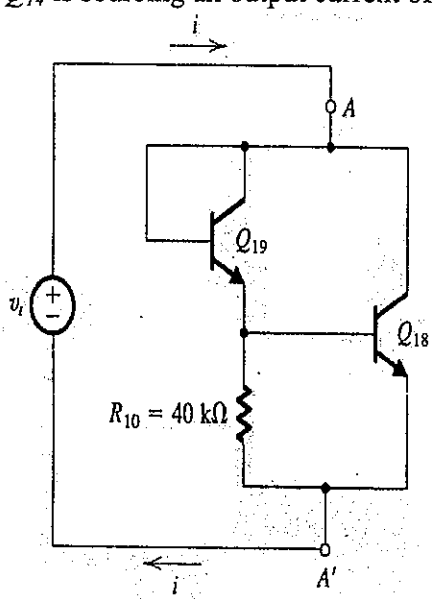


Figure 5.a

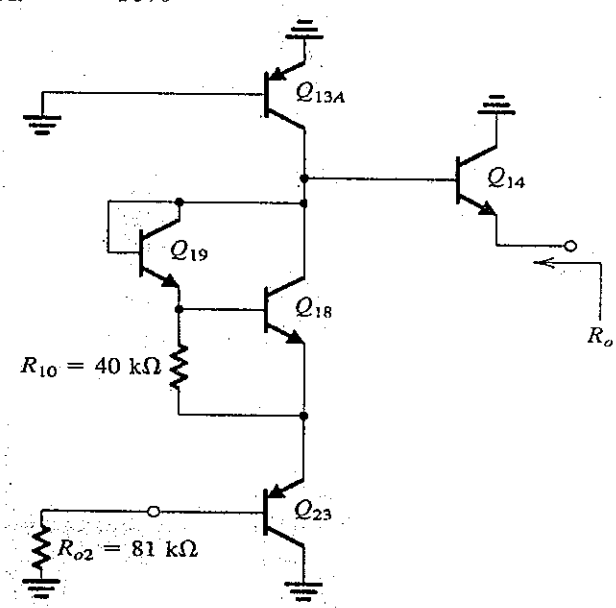


Figure 5.b

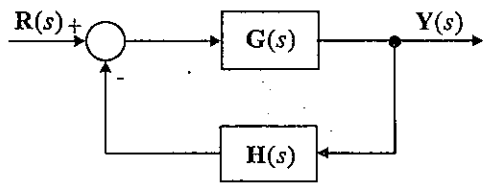
1. Answer the following true-and-false problems. Correct each statement if your answer to that statement is "false". No credit will be given if you reply to the question only with the answer "false", or your corrected answer is not correct.

(a)(4%) The initial-value theorem is applicable to linear time-varying systems.

(b)(4%) The final-value theorem is applicable to any linear time-invariant systems.

(c)(4%) Consider the following MIMO feedback control system, where $\mathbf{R}(s) \in \mathcal{R}^{p\times 1}$, $\mathbf{Y}(s) \in \mathcal{R}^{q\times 1}$. We can obtain the closed-loop transfer function matrix as

$$\mathbf{M}(s) = [\mathbf{I} + \mathbf{H}(s)\mathbf{G}(s)]^{-1}\mathbf{G}(s).$$



(d)(4%) If the transfer function of a control system has pole-zero cancelation, then this system is not controllable, but is still observable.

(e)(4%) When we apply parallel decomposition technique to a transfer function (no pole-zero cancelation) of a control system, we can always obtain a set of state equations with diagonal canonical form.

(f)(4%) Adding a zero to a second-order prototype closed-loop transfer function will generally decreases both the rise time and the maximum overshoot of the step response.

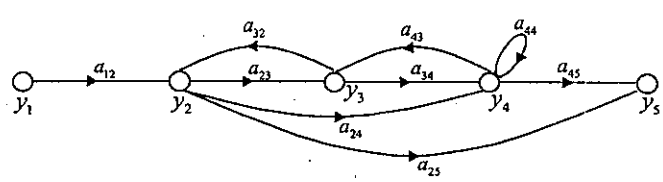
(g)(4%) The asymptotes of the root loci $1 + kG(s)H(s)$ refer to the angles of the root loci when $k = \pm\infty$.

(h)(4%) Bode plot can be used for stability analysis both for minimum and nonminimum-phase transfer function.

(i)(4%) Assigning all the eigenvalues to the left-half of s-plane still can not guarantee the stability of a linear time-varying system.

(j)(4%) Nichols chart can be used to find bandwidth and the resonant peak value of an open-loop system.

2.(10%) Suppose that a designer wants to analyze a control system whose signal-flow graph(SFG) is show below.



Now he wants to use the Mason rule $M = y_{out}/y_{in} = \sum_{k=1}^{N} M_k\Delta_k/\Delta$ to compute the transfer function y_4/y_2, where N is the total number of forward paths between y_{in} and y_{out}, M_k is the gain of the kth forward paths between y_{in} and y_{out}, and

Δ = 1 − (sum of the gains of all individual loops)
+ (sum of products of gains of all possible combinations of two nontoching loops) − ⋯

Δ_k = the Δ for that part of the SFG that is nontoching with the kth forward path

The computation results of this designer are shown as follows:

$$
\begin{aligned}
N &= 2, \\
M_1 &= a_{23}a_{34}, \quad M_2 = a_{24} \\
\Delta &= 1-(a_{23}a_{32}+a_{34}a_{43}+a_{24}a_{43}a_{32}+a_{44})+a_{23}a_{32}a_{44} \\
\Delta_1 &= 1, \quad \Delta_2 = 1
\end{aligned}
$$

Are the computations correct or not? Point out which step you think that is not correct, explain your reason, and re-compute the transfer function y_4/y_2.

3. (10%) For the feedback system of Fig. 1, please design a second-order $G(s)$ such that the overall feedback system is stable and the steady-state tracking error e is zero when applied with $r(t)=\sin(2t)$.

4. Consider the feedback system of Fig. 1 with the Nyquist diagram of $G(j\omega)$ sketched in Fig. 2, where point A is -0.2+j0 ($\omega=800$ rad/sec), point B is $-(\sqrt{3}+j)/4$ ($\omega=600$ rad/sec), point C is $-(1+j)/\sqrt{2}$ ($\omega=200$ rad/sec). Assume $G(s)$ has no poles in the right half s-plane.

(1) (5%) Please determine the stability of the feedback system from its Nyquist diagram.

(2) (10%) Find its gain margin and phase margin.

(3) (5%) Suppose input r is a unit step function, please find its steady-state regulation error $e(t)$.

(4) (5%) Suppose $r(t)=\cos(600t)$, please find its steady-state output response $y(t)$.

5. Consider the system of Fig. 1, where G is described by

$$\dot{x}=\begin{bmatrix}-5 & -6\\ 1 & 0\end{bmatrix}x+\begin{bmatrix}1\\ 0\end{bmatrix}e,\quad y=\begin{bmatrix}k & 0\end{bmatrix}$$

(1) (5%) Find the transfer function $G(s)$.

(2) (5%) Determine the range of k for a stable closed-loop system.

(3) (5%) Determine the range of k so that no overshoot in output $y(t)$ is found with a step input r.

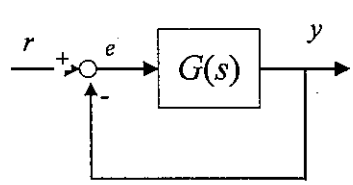


Fig. 1

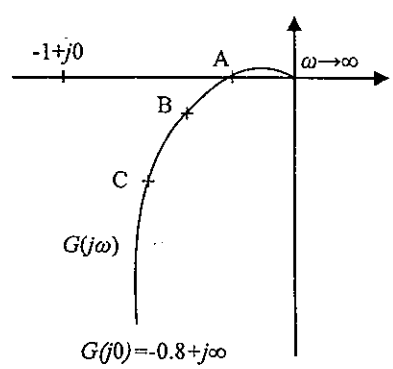


Fig. 2

Physical constants:

K= 1.38 x 10^{-23} J/K e = 1.60 x 10^{-19} C ε_o = 8.85 x 10^{-12} F/m

kT (300 K) = 0.0259 V h = 6.625 x 10^{-34} J-s Eg; Si = 1.12 eV (300 K)

$\varepsilon_{Si} = 11.7\ \varepsilon_o$ (Si; $m_n^* = 1.08\ m_o$; $m_p^* = 0.56\ m_o$)

GaAs; $N_C = 4.45 \times 10^{17}\ cm^{-3}$; $N_V = 7.72 \times 10^{18}\ cm^{-3}$

1). Consider a GaAs MESFET with a gold Schottky barrier with a barrier height of 0.8 V. The n-channel doping is $10^{17}\ cm^{-3}$ and the channel thickness is 0.25 μm. Calculate the 300 K threshold voltage for the MESFET.(15%)

2). To calculate the built-in potential barrier, the space charge width and electric field in a p-n junction. Consider a silicon p-n junction at T=300 K with doping densities; Na=1x10^{16} cm^{-3} and Nd = 1x10^{15} cm^{-3}. (20 %)

3). Consider a GaAs p-n diode with the following parameters at 300 K:

Electron diffusion coefficient	30 cm^2/V-s
Hole diffusion coefficient	15 cm^2/V-s
p-side doping	5 x 10^{16} cm^{-3}
n-side doping	5 x 10^{17} cm^{-3}
Electron mobility carrier lifetime	τ_n = 10^{-8} s
Hole mobility carrier lifetime	τ_p = 10^{-7} s

Calculate the injection efficiency of the LED assuming no recombination due to traps. (15%)

4). Calculate the probability that a state in the valence band is occupied by a hole; assume the Fermi energy is 0.25 eV below the conduction band. and calculate the thermal equilibrium hole concentration in Silicon at 300 K. .(20%)

5). Derive the Einstein relation. (10%)

6). To determine the diffusion coefficient given the carrier mobility. Assume that the mobility of a particular carrier is 1000 cm^2 /V-s at T = 300 K. (10%)

7). Compare the properties of the p-n diode and the Schottky diode. (10%)